Interferometers, Information, & Synthesis — Tony Hewish.

Topic e.g. Necessary sampling rate on an $f(t)$? Economy vs. efficiency.

## Fourier Ideas.

"Computer" form $\quad f(x) = \sum a_n \cos 2\pi n x + b_n \sin 2\pi n x \quad$ Periodic

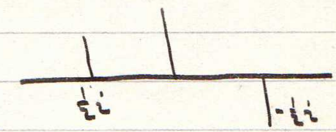
"Thinking" form $\quad = \int_{-\infty}^{+\infty} G(n) e^{2\pi i n x} \, dx \quad$ General

Spectrum. Generally cplx even if $f(x)$ real

-ve frequencies : +ly & -ly rotating vectors → simple cosine.

cos


sin

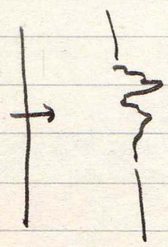



Only pictorial version of ordinary exp$^l$ rep$^n$

$$G(n) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi i n x} \, dx$$

Notation $\quad f(x) \leftrightarrow G(n)$

Fraunhofer Diff$^n$:



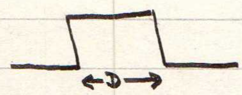
Aperture → $f(x)$

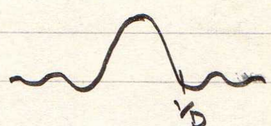
Angular Spectrum $\quad G(\theta) = \int f(x) e^{-2\pi i X S} dX$

$G(S)$ $\qquad S = \sin\theta$ $\qquad X = x/\lambda$

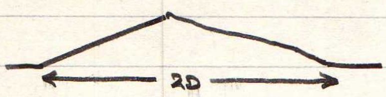
## Compendium of Fourier Transforms.

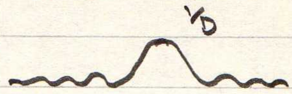
$f(x)$ ⟶ $G(S)$



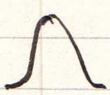

$\dfrac{\sin \pi D S}{\pi D S}$



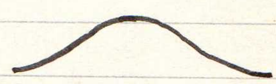



$\left[\dfrac{\sin \pi D S}{\pi D S}\right]^2$

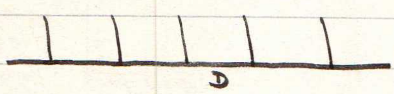
Gaussian



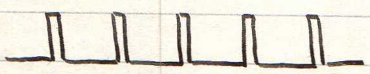
$e^{-x^2/D^2}$



$e^{-\pi^2 D^2 S^2}$







$\left[\dfrac{\sin \pi D S}{\pi D S}\right]$

## Convolution

$f(x)$ Running average $f'(x)$

Define Averaging fn $A(X)$ → method of weighting $f(x)$ to → w. mean.

Not$^n$ $f'(x) = f(x)\overrightarrow{A(X)}$

Eg. Sky $f(x)$ Aerial beam $A(X)$ Aerial o/p $f'(x)$

Aerial o/p $\left\{\begin{array}{l}\text{Time-const.}\\ \text{Filter}\end{array}\right\}$ Record.

Math$^l$ rel$^n$ $$f'(x) = \int A(X) f(X-x)\,dX$$

Conv. Thm.
$$f(x) \leftrightarrow G(n)$$
$$A(X) \leftrightarrow H(n)$$
$$f'(x) = f(x)\overrightarrow{A(X)} \leftrightarrow G(n)H(n).$$

e.g.

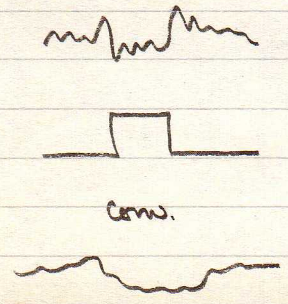
$$\left[\int_{-\infty}^{+\infty} f_n \cos 2\pi n X \, A_n \cos 2\pi n (X-x)\,dx = \frac{f_n A_n}{2}\cos 2\pi n x \quad \text{as sin terms} \to 0 \text{ in } \int^n\right]$$

$f(x)$

$A(X)$

Conv.

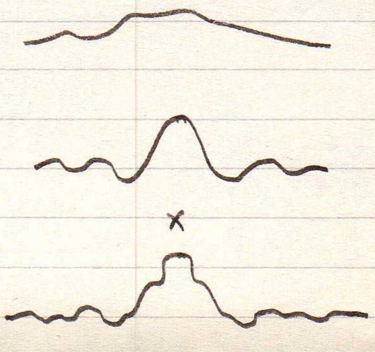
$f'(x)$



$G(n)$

$H(n)$

×

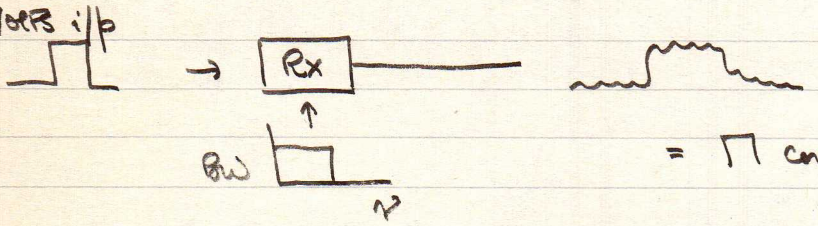


As $A(X)$ broadens, $G(n)$ becomes eliminated by narrowing $H(n)$ → — ↔ |

Convolving loses HF cpts unless the convolving fn. is v. narrow.

changes these actually ler than.

Volts i/p

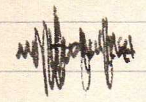
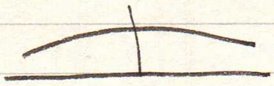


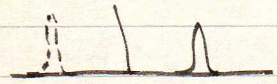
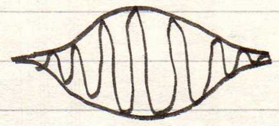

= ⊓ conv. with

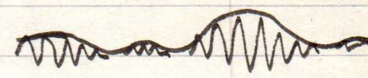
Spectrum of i/p o/p

Radar. Need BW $4 \times 1/T_{pulse}$ @ least to preserve pulse shape.

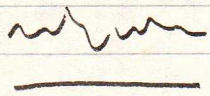
Random signal (noise) through filter.

. Cannot define ord? spectrum (only power spectrum), but think of it as having a spectrum 

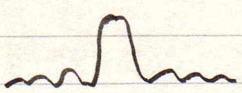
Filter  → f(t) 

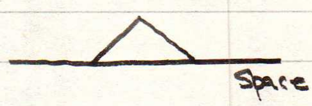
∴ o/p of filter = noise conv. with f(t) →  Quasi-periodic filter → q-periodic o/p from random i/p.

Aerial Smoothing. Bracewell & Roberts Aust J. Phys. 1954 7, 615

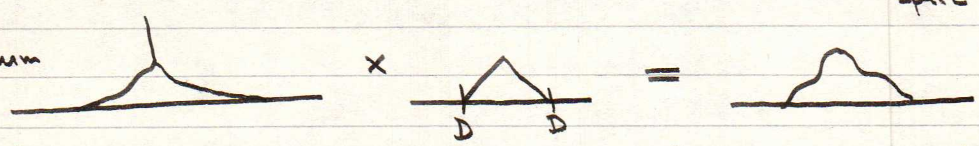
Sky  f(x)

Fourier: Large ocls near 0 ∵ of large isotropic regions
Broad near 0 ∵ of Galaxy
Some high freqs. ∵ of sources.

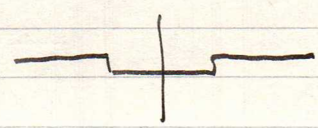
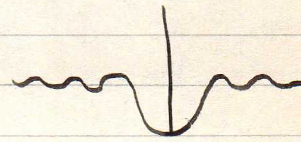
Scan Polar Diag  Angle



F.T. of aerial pattern. (Spectral sensitivity fn.)

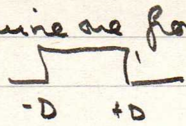
Sky Spectrum 


Spatial freqs above some D are simply not available, & freqs. near D are distorted.
"Invisible distrib." is one with no spectral cpts in ±D. e.g. sinusoidal oscill. of $T_b$ in sky of period < 1/D in sky.

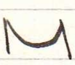
Suppose ∃ spectrum like this  Corresponds to 

Source in an absorption trough might → this distrib. Point is that one would not assume such a distrib. in analysing records.

Reasonability of a record?  Record  Spectrum

If spectrum is genuine one from the aerial, it contains no freqs > ±D.
∴ Convol. with  should not modify spectrum.


∴ Scanning record with a  fn. = voltage polar diagram of aerial width 2D would show up features produced other than by aerial.

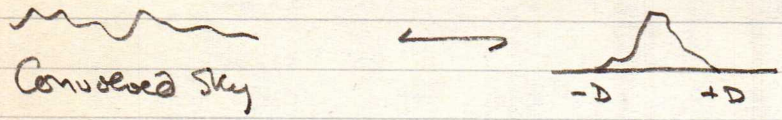
To correct △ sensitivity, × by  to give = wt. to the spatial frequencies within the capability of the aerial aperture. Can ∴ produce a modified map ("Principle Sol.") that might be regarded as the best map from an aerial → narrowest poss. beam. Degrades S:N possibly tho! ∵ noise amp by the correcting spectral fn.

⊓ has ± sidelobes tho, ∴ get better resoln. but @ expense of sidelobes being increased. Good for doubles, e.g. but not in surveys where dim sources near bright ones interesting.

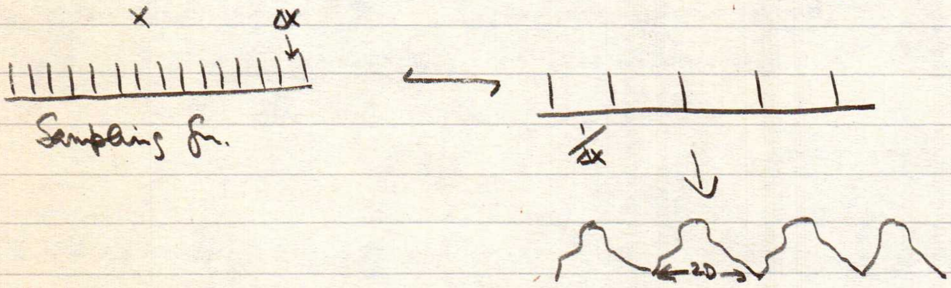
Gaussian grading lowers sidelobe level also → bumps on records are there, not lobes.
Refce: Jacquinot, Progress in Optics 1964 3, 31 (optical analogues.)

Data Processing - Sampling Theorem.

Shannon Proc. I.R.E. (1949) 37, 10.
If $\exists f(x) \longleftrightarrow G(n)$. $G(n) = 0 \quad n > n_0$
$f(x)$ properly defined if sampled @ $\Delta x \leq 1/2n_0$.



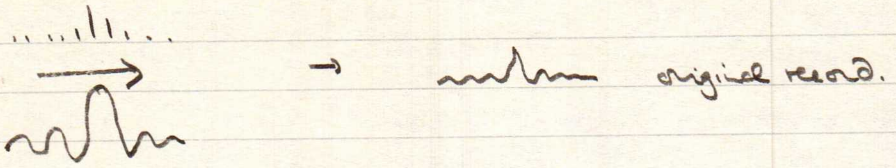

Sample at some arbitrary rate ≡t. ($\times^n$) by set of unit impulses.



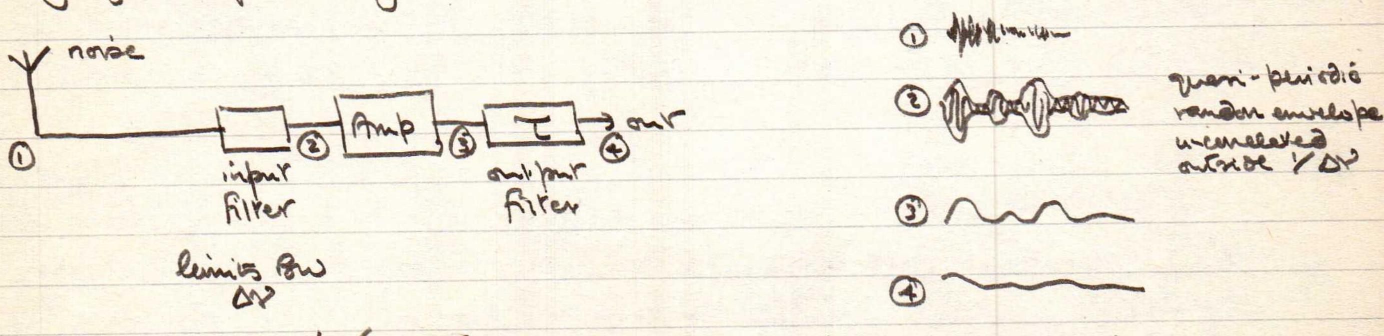

Provided the spectra in the analysis of the sampled sky record are far enough apart, no info. is lost. Spectrum of whole sky being folded over on itself in an unravellable manner w.d. lose info.
∴ $\Delta x$ must be $\leq 1/2D$. Min. no. of pts if $\Delta x = 1/2D$.
≡t. to 1/4 of beamwidth of a polar diagram.

Source appears as ......||...... Ampde & posn? Not obvious, unless the sample rate is over-redundant.

Remove unwanted hf spectral cpts by freqy. filter
≡ convolve with "voltage" polar diag. of aerial width 2D on record.




Detection of Signal in presence of Noise.



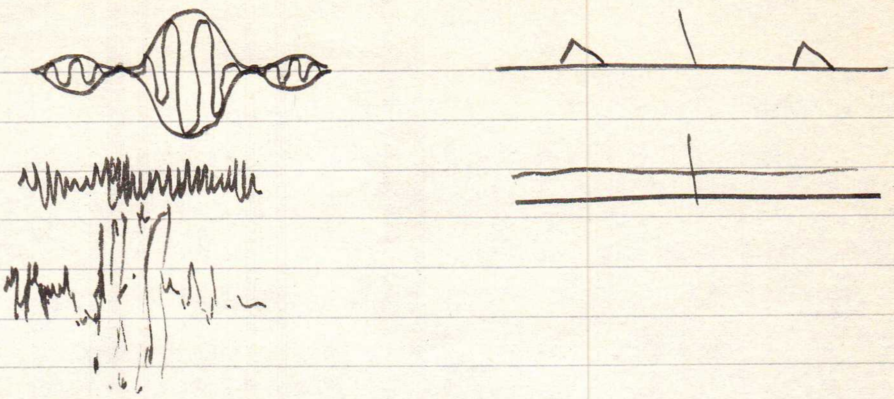

Jitter? on record $\propto 1/\sqrt{\tau\Delta\nu}$ (Info. every $1/\tau\Delta\nu$ → $\tau\Delta\nu$ nos.)
Don't want to force the signal out of existence tho!

Signal (eg. interferometer)

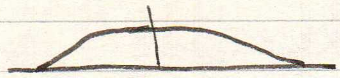
+ Noise

Record

RC filter



$\frac{1}{1+2\pi i n\tau}$ ←



make this → 0 just where signal finishes in spectrum. Discriminates against noise.

Make $\tau = 1/2\pi n$ to do this

If signal has period T, $\tau = T/2\pi$

The integn is ∴ to convolving record with expl  $e^{-t/CR}$

This lets the … noise thru too.

Better to make filter with characteristic

where signal is

Obviously.

Max. pos S:N should have identical characteristics to ~~the~~ signal itself.

(Principles of Coding, Filtering, & Info. Th? Schwarz 1963)

More properly, want conjugate cplex. filter. (Matched filter)

Mustn't muck up phase of signal tho. Filter operating in real time must have a phase characteristic.

Bode's Thm.

Sampling + punching however can get over this. Sample & convolve. The convolving fn. ohs. is the ideal signal" itself. Computer can reproduce the ideal filter, without loss of "Bode's Theorem phase". Out of real time.

Having gone thru optimum filter, noise has same characteristics as signal & cannot be recognised as such. Large noise spike now looks like a source.

Final spectrum of signal is the input spectrum$^2$ →

∴ loss of resoln. Now broader than it was before.

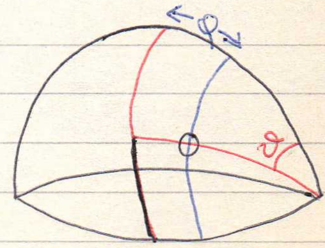
In practice compromise bet. opt. filtering & loss of resoln.

## <u>The Phased Interferometer Machine</u>


λ (only)

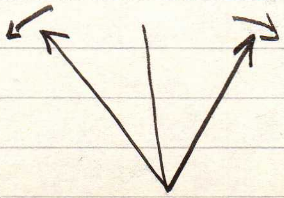
Isotropic aerial patterns.

Stationary.




P.D. for signal @ angle to cel centre $= \pm \frac{1}{2} D \sin\varphi$

Ph.D. " " " $= \pm \pi D \sin\varphi$ (D in λ units)

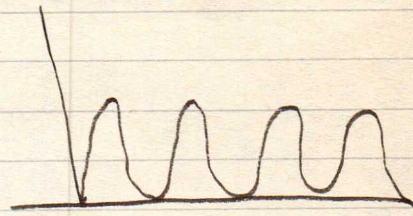


Voltage vector is sum of rotating vectors.

$\cos(\pi D \sin\varphi)$

Power is $K.\cos^2(\pi D \sin\phi)$   Power polar diagram.

$= K.\,[1 + \cos 2\pi D \sin\phi]$



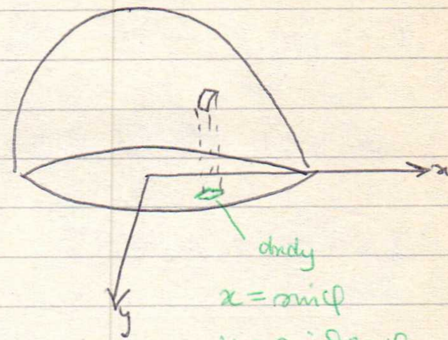
Let Sky Brightness be $I(\theta,\phi)$

Power o/p from Rx $= \iint_{\text{all sky}} I(\theta,\phi)\,[1 + \cos 2\pi D \sin\phi]\, d\theta d\phi \,.\, \cos\phi$

Project into new coordinate system on ground plane.

$dx = \cos\phi\, d\phi$

$dy = \cos\theta \cos\phi\, d\theta$




Area ratio $= \dfrac{dxdy}{\cos\phi\, d\theta\, d\phi} = \sqrt{1 - x^2 - y^2}$

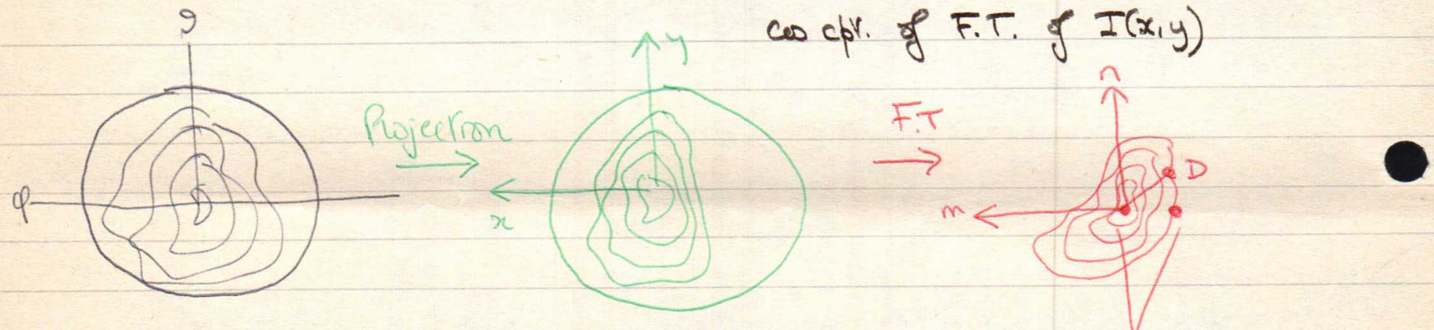
$I(x,y) \equiv I(\theta,\phi) \Big/ \sqrt{1 - x^2 - y^2}$

Power o/p $= \int_{-1}^{+1}\int_{-1}^{+1} I(x,y)\,[1 + \cos 2\pi D x]\, dx dy$

Assume that sky brightness $\to 0$ at horizon. Then $\iint \to -\infty \to +\infty$.
Doesn't worry in practice ∵ the aerials are directive & $\sim \to 0$ on horizon if it conducts !!!!!!

Power $= \int_{-\infty}^{+\infty}\int I(x,y)\, dx dy + \int_{-\infty}^{+\infty}\int I(x,y) \cos 2\pi D x\, dx dy$

cos cpt. of F.T. of $I(x,y)$




Interferometer samples FT @ 2 places
zero order + that det. by spacing.

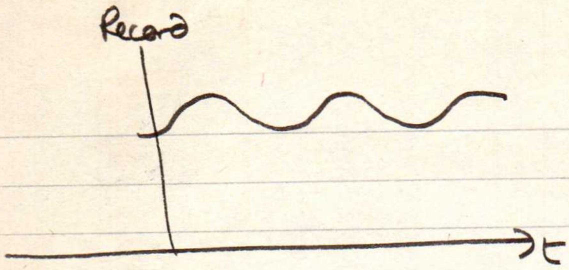
Got cos cpt by taking middle as phase refce.
If put $\lambda/4$ in one side $\to$ sin cpt.
If used 1 aerial only $\to$ 0 cpt ∴ between 2 obs can get 0 & cos
$\lambda/4$ extra as well   0 & sin
Rotate them 90° on gnd $\to$ y part. weighting fn. is 0ths symmetrical.

This is the basis of all systems. 3 measurements reqd. in principle.

Rotⁿ of Earth   $x' = x - \alpha t$

$$o/p = \iint I(x,y)\, dx dy + \iint I(x,y) \cos 2\pi D(x - \alpha t)\, dx dy$$

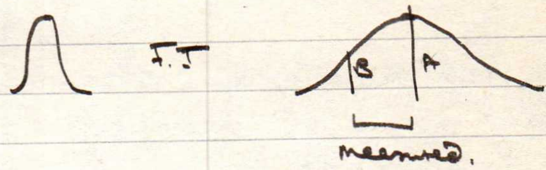
$$= \;\; .. \;\; + \cos 2\pi D\alpha t \iint I(x,y) \cos 2\pi D x\, dx dy$$
$$+ \sin 2\pi D\alpha t \iint I(x,y) \sin 2\pi D x\, dx dy$$




Amp. & phase of record → sin & cos cpts.
Time var^n → sin & cos cpts.

This cannot be done in 2 dimensions @ once. Can only scan in x using rot^n of sky!
Use of simple arrangement → sin & cos one way only?
$\star^r$ diameter. Assume ⊙ sym^m of source.

$$I(x,y) = e^{-\left(\frac{x^2+y^2}{a^2}\right)}$$




Ratio $\gamma = B/A \to a$ ∵ $B/A = e^{-\pi^2 D^2 a^2}$
"First shot" method of getting $\star^r$ diams.

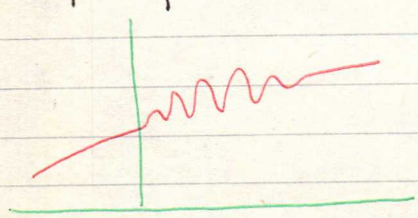
ΦS Interf.
Periodic $\lambda/2$ chgs bet. one of the aerials & centre. Set ms. power diffce.

$P_1$ in ph. $P_2$ a/ph. $P_{o/p} = P_1 - P_2$

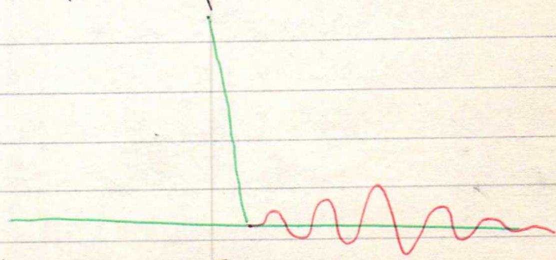
$$P_1 = \iint I(x,y)\left[1+\cos 2\pi Dx\right]dx\,dy$$
$$P_2 = \iint \cdots \left[1-\cos 2\pi Dx\right]dx\,dy$$

$$P_1 - P_2 = \iint I(x,y)\cos 2\pi Dx\,dx\,dy.$$
→ cpt. without sampling @ origin as well.

Simple Interferometer



ΦS Interferometer.

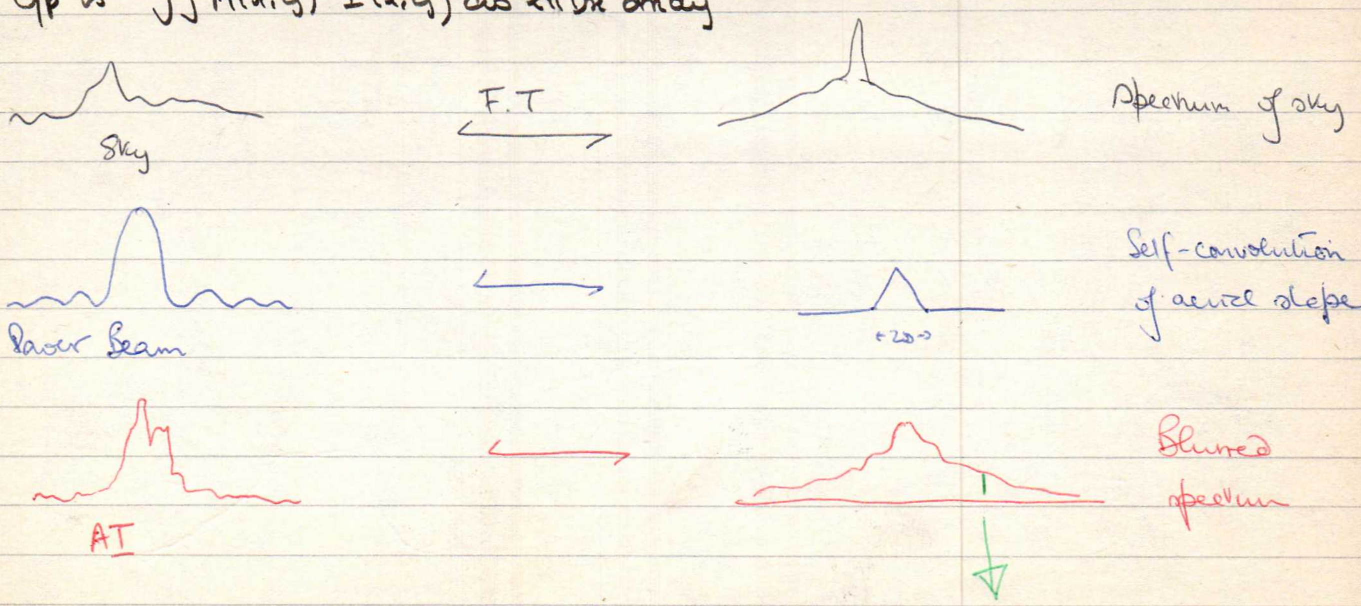


Can use huge sensitivities without having to back off the Galaxy.

Aerials of finite size.

Let power polar diagram be $A(\theta,\varphi) \longrightarrow A(x,y)$

O/p is $\iint A(x,y)\, I(x,y)\cos 2\pi Dx\,dx\,dy$

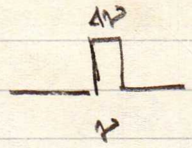


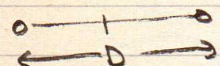

ΦS Interferometer samples CONVOLVED spectrum!

Finite BWidth ?

855b.

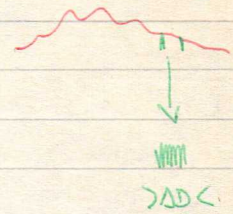
Suppose p/b of Rx is



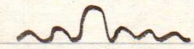




Simple Ifferometer

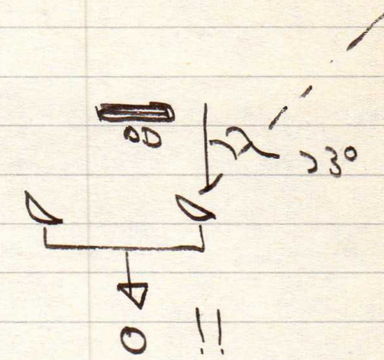
Now have D "spread" or diff $\nu$'s in p/b.
Sample over a range of D  $\Delta D$
O/p is sum of all Fourier cpts contained in $\Delta D$




Transform back the range of sample →

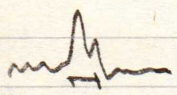


Exactly same as directional props. i.e.
→ "Bandwidth polar diagram"
For 4 Mc/s @ 408 Mc/s → 3° beam.
∴ Anything more than 3° off meridian invisible



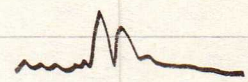

Think of it as correlation.

$V_1(t)$



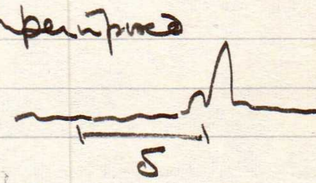
Correl$^n$ $\Delta t \sim 1/\Delta\nu$

$V_2(t)$ no delay

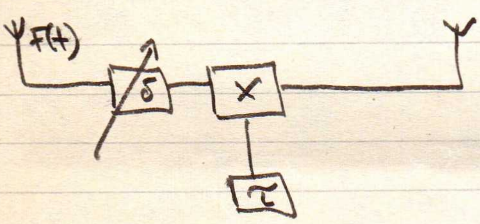


Superimposed

Delayed

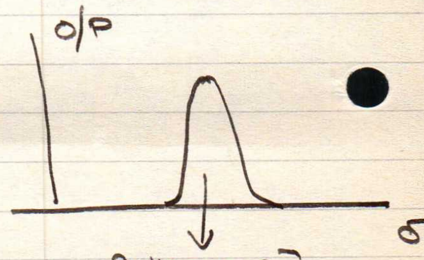



If $\delta > \Delta t \sim 1/\Delta\nu$ → no correl$^n$ bet. signals $V_1(t)$ + $V_2^{\delta}(t)$ & no o/p.
Put in compensating cables to eliminate $\delta$. Ev. to "steering" BW polar diagram.
Could happen that BW polar diag < "actual" polar diagram.
Might be able to do "broad BW" "high resol$^n$" surveys. Exact position finding.




o/p $\overline{F(t)\,F(t+\delta)}$

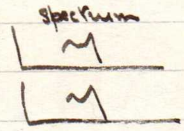
If $\delta\nearrow$, o/p is fn. of $\delta$



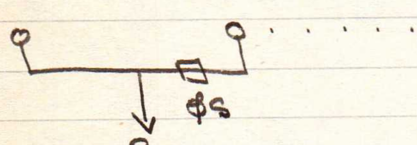

Autocorrel$^n$

Spectrum is Energy spectrum

$F(t)$
$F(t+\delta)$




<u>Principle of Synthesis Procedure</u>   Variable spacing Ifferometer.



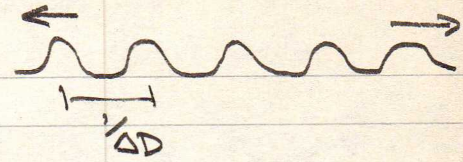

Isotropic aerials.

Is there preferred position separation for moving aerial?
Let source be 1-d $I(x)$



If aerial moved $\Delta D$ each time, sample $G(u)$ @ pts $\Delta D$ apart.

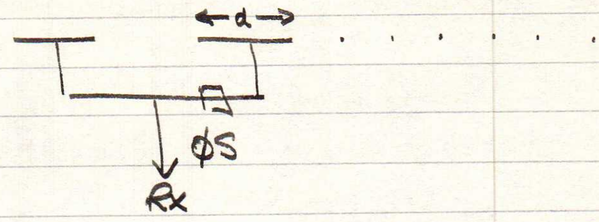
F.T. back sampled F.T. → repetitive form of $I(x)$
$1/\Delta D$ apart.



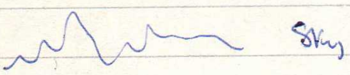

∴ $1/\Delta D$ must be > width of object of study.
Then no pt. in putting aerials' posns closer together than '1/x' size.

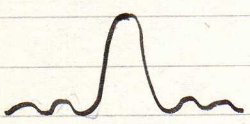
Sun 1/57 radian → 57 $\lambda$ synthesis. !! Not realised at first.
Whole sky? Object is so wide → −1 to +1 in S plane.
∴ $1/\Delta D \geq 2 \underline{\underline{n}}$ → $\Delta D = \lambda/2$

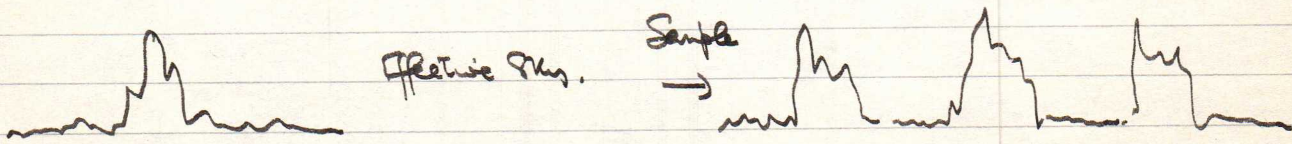
Antennae of finite size.




Any spacing → F. cpt averaged over given range. Sample spectrum convolved with polar diagram. ∵ Convolution blurs distribn anyway no pt. sampling so often. ∴ less spacings.
Sky effectively "localised", alternatively, ∵ of directional primary pattern.

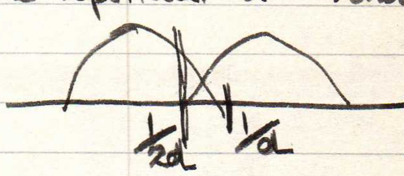


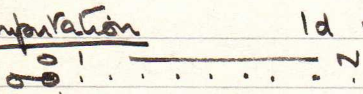




Always some reception, but consider it → 0 @ < 1% level. Practical!
Now sample spectrum s.t. repetition separates beamwidths of antenna.
Interval must correspond to 2 × polar diagram width to zero No. 1.
∴ $\frac{1}{\Delta D} \geq \frac{2}{d}$   $\Delta D \leq d/2$

Positions must physically overlap, in principle.
In practice, picture not v. much use to edges of polar diagram ∵ we have to get back to "sky" by getting polar pest convolved out. Towards edges of beam, sky weighted out by polar response falling off. With noise + glitches the re-weighting to get sky back is dodgy.
∴ Only work out to ½-power pts. Weighting not so important in → answers back for real sky. ½-power ~ $1/2d$
Move to ~~[illegible]~~ $2d/3$




Computation   1d still.




Sin Sn + Cos Cn from each posn (No form of Σ)
2N nos.

$$I(x) = \int_{-\infty}^{+\infty} G(n) e^{2\pi i n x} \, dn$$

$\equiv \sum G(n) e^{2\pi i n x}$

$I(m\Delta x) = \sum G(n) e^{2\pi i n \Delta D m \Delta x}$

$m \rightarrow$ sample rate on sky (final)

$\Delta D \rightarrow$ spectrum sample.

$G(n) = C_n + iS_n$

Sum goes from $n = -N$ to $+N$

↓

Change of sign for $-n$ only $\rightarrow$ $G(-n) = C_n - iS_n$

Real ex. to utility of sky gradⁿ $\rightarrow$ $\Delta x \le 1/2N\Delta D$ from Shannon S.Thᵐ.

$= \sum G(n) e^{2\pi i n m / 2N}$

Repeats every $m = 0, 2N, 4N$ —— . etc.
No pt making $m > 2N$ $\rightarrow$ $2N$ useful samples of sky. Reasonable enough.
Having applied S.Thᵐ $\rightarrow$ same no. of data out as in.
This is indepᵗ. of aerial length.

<u>Phased Array for Comparison</u>

etc.

$-\phi, -2\phi, -3\phi$ ...

<u>$\phi, 2\phi, 3\phi$ in turn.</u>

Volts out $= \sum V_n e^{in\phi}$

$= \sum V_n e^{2\pi i n \Delta D \sin\theta}$

$\phi = 2\pi \Delta D \sin\theta$

Put $\sin\theta = m\Delta x$ $\rightarrow$ same sum.
Computⁿ in Fourier synthesis $\equiv$ real phased array.

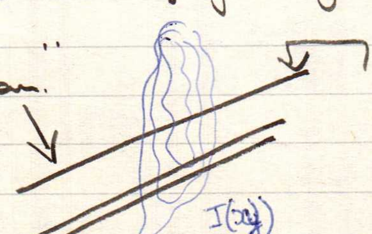
Regarding array as diff. grating slit width = slit sep:
$\rightarrow$ single beam in forward dirⁿ ∵ primary pattern zeros $\rightarrow$ array maxᵃ.

But phasing displaces the array pattern (not the primary pattern)
Now $\rightarrow$ sidelobes ∵ array maxᵃ do not get chopped by primary pattern.
"Grating sidelobes" of an array.

Analogy betw grating sidelobes & repetitive pattern in synthesis.
Spacing $\lambda/2$, or $2\lambda/3$ $\rightarrow$ elim. of g. sidelobes.

∴ Synthesis better than phased array from grating sidelobe standpoint.

1-d synthesis. "Line Scan" Ideal pencil beam




Strip scan (Abel transform) Ideal fan beam.

Info. in line scan of spectrum in system.

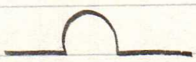
$\delta_{\parallel}$ = 2-d $\delta$-fn. $I(x,y) \times \delta_{\parallel}$ = line scan

F.T. of $\delta_{\parallel}$ = another $\delta_{\parallel}$ at 90°. $\delta_{=}$.

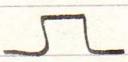
$I(x,y) \times \delta_{\parallel} = G(n,m) \cap \delta_{=}$

∴ $\mathcal{F}$(line scan) → strip scan.

$\mathcal{F}$ line of $\frac{G}{I}$ → strip of $\frac{I}{G}$

 Abel trans → 

 .. .. 

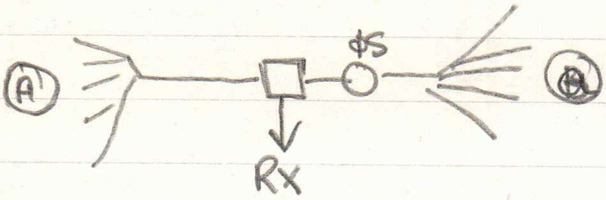
∴ Strip scan  → shell source ? 

If can assume $I(r)$ only → Fourier-Bessel Trans $\int_0^\infty G(n)\, J_0(2\pi r n)\, n\, dn$.

Tony.

<u>Aperture Synthesis.</u>

Compound Ifferometer




Let voltage @ one element be $\underset{\sim}{A}e^{i\Phi}$

Total voltage @ rcr $=$ (in $\phi$) $\sum A_m e^{i\Phi_m} + \sum a_n e^{i\varphi_n}$

Power coming out $\propto V^*V$

$$= \sum A_m^2 + \sum a_n^2 + \sum A_m A_{m'} e^{i(\Phi_m - \Phi_{m'})} + \sum a_n a_{n'} \cdots$$

$$+ \sum_{m,n} A_m a_n e^{i(\Phi_m - \varphi_n)}$$

$$= \sum A_m^2 + \sum a_n^2 + 2\sum A_m A_{m'} \cos(\Phi_m - \Phi_{m'})$$

$$+ 2\sum a_n a_{n'} \cos(\varphi_n - \varphi_{n'})$$

$$+ 2\sum A_m a_n \cos(\Phi_m - \varphi_n)$$

Antiphase power $a \to -a$

$$= \text{------} - 2\sum A_m a_n \cos(\Phi_m - \varphi_n)$$

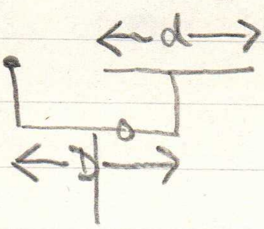
$$\therefore \text{o/p} \propto 4\sum A_m a_n \cos(\Phi_m - \varphi_n)$$

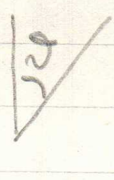
Product of voltage polar diagrams

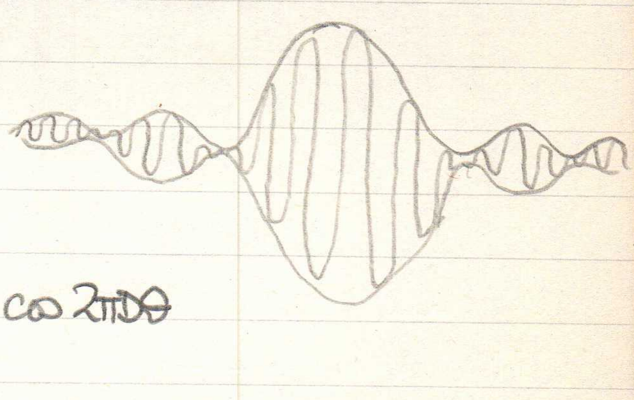
Ifference pattern of 2 points

$\therefore$ o/p of an arbitrary phase-switched array $= \sum$ all possible $\phi S$ interferometer combinations.

i)

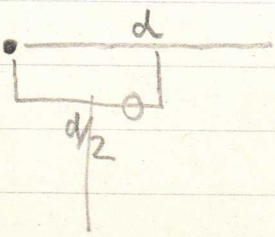


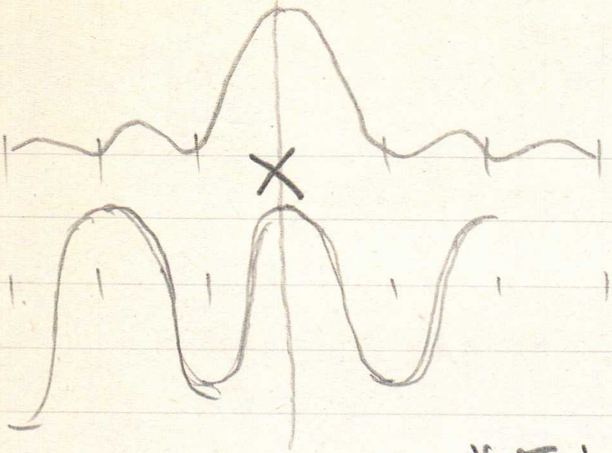



Reception pattern $= 1 \times \dfrac{\sin \pi d\vartheta}{\pi d\vartheta} \times \cos 2\pi D\vartheta$

ii)



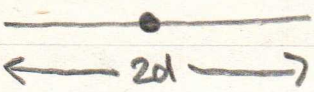

$R = 1 \times \dfrac{\sin \pi d\vartheta}{\pi d\vartheta} \times \cos \pi d\vartheta$



$$= \qquad$$

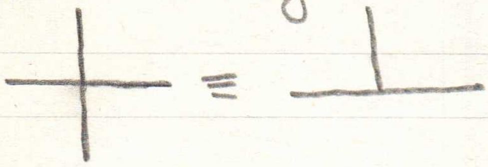
$$= \alpha \quad \frac{\sin 2\pi d\theta}{2\pi d\theta}$$

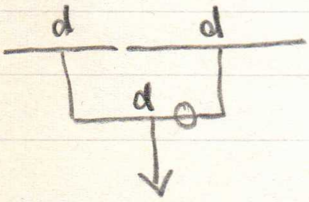
Yields polar diag = that of a 2d aperture !

= that of 

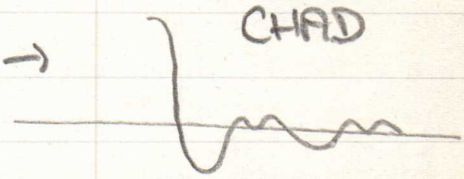
$= 1 \times \frac{\sin 2\pi d\theta}{2\pi d\theta} \times 1$

Only gain in extra length is sensitivity – no more Fourier cpts.

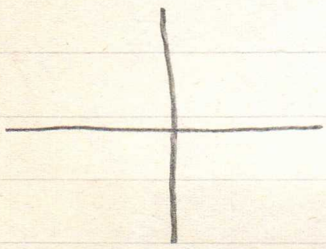





$$R = \frac{\sin^2 \pi d\theta}{(\pi d\theta)^2} \times \cos 2\pi d\theta \rightarrow$$

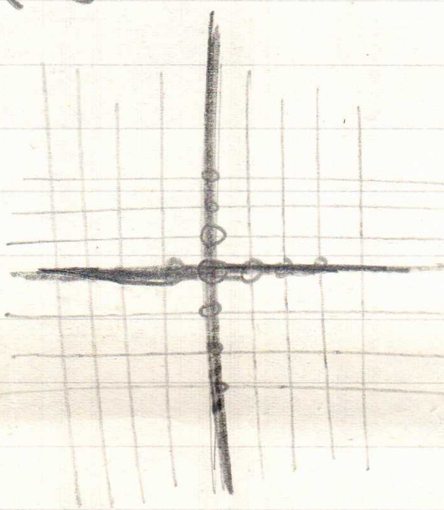



Mills Cross



Antenna

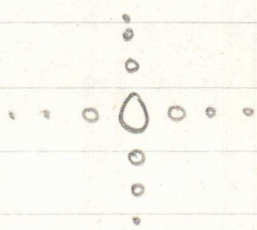
→



Polar diag

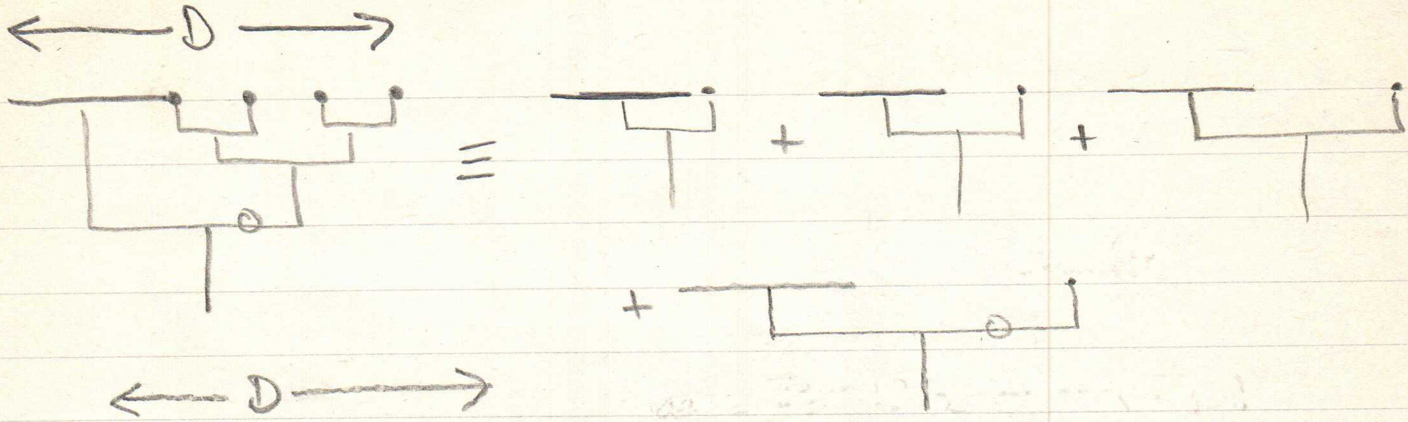
Sidelobes die out gradually. Then one ×s principal response → sidelobe

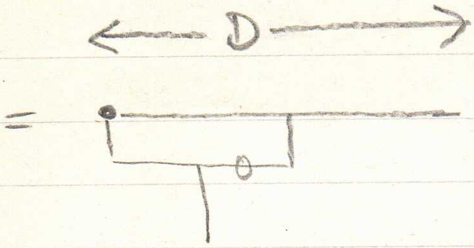
→



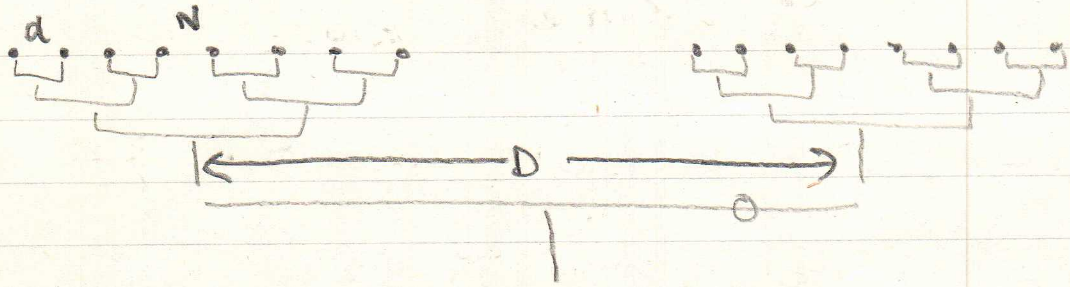
Reduce as far as pos. by giving primary pattern as low a sidelobe level as you can. Grading – apodisation.

Gaussian distribⁿ of excitⁿ → no sidelobes.
Wasteful in power + truncate Gaussian @ 15% level. Reintroduces the lobes, but makes it v. much better.

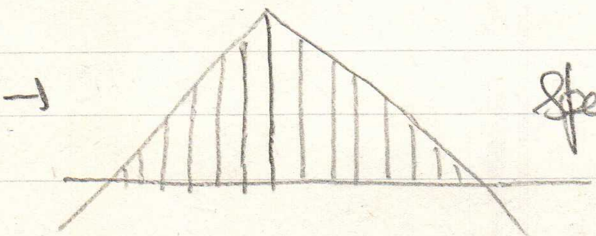



Saving in hardware!




$\equiv N$ pairs @ $D$ $+ (N-1)$ pairs @ $(D \pm d)$ $+ (N-2)$ @ $(D \pm 2d)$
$+ \; - - - \; +$ 1 pair @ $(D \pm (N-1)d)$

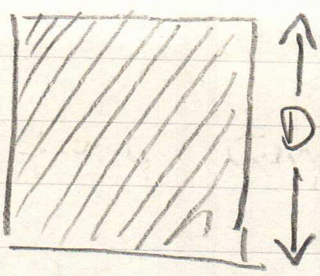


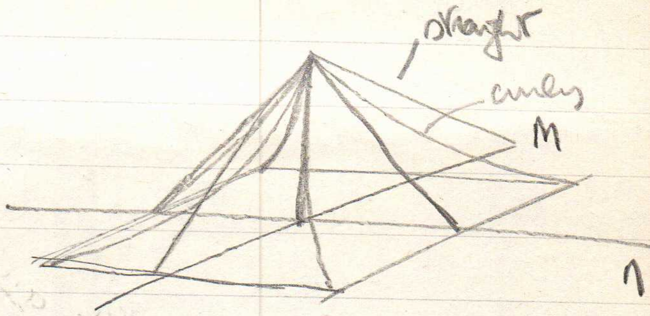
Spectral sensitivity.

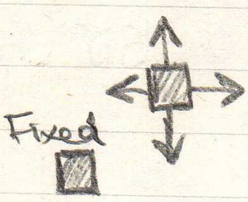
## APERTURE SYNTHESIS

How to make

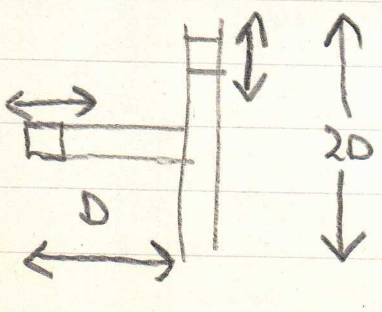
?

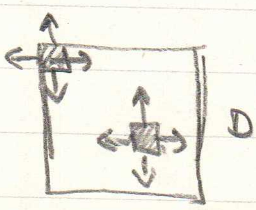



Need only move over $D \times 2D$. Still cover $2\times$ area of orig'd

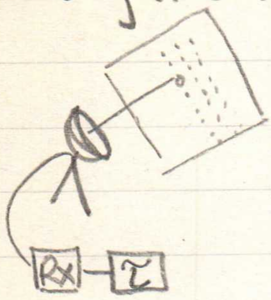


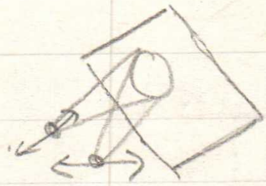

Move other

Move 1 in each dim. Move both One there T Synth

Observing Time & Signal:Noise in Synthesis?




Parabola must scan.



← Same Rx →

← $\tau$ →

Area of sky surveyed $A > a$ (beam area of small aerials)

2/3 overlap of posns. for moving aerials.

Optimum sampling.

Conventional aerial □ D $\quad A' = 1/D^2$

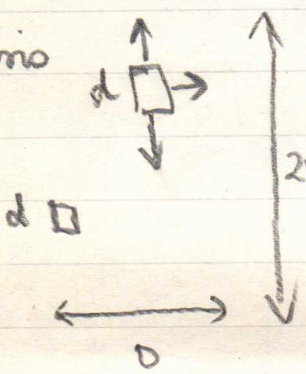
Sample intervals → grid of pts $1/2D$ apart

No of obs $\quad A \cdot 4D^2 \rightarrow$ Obs. Time $A4D^2\tau = T$

Signal strength $\propto$ gain $\propto D^2$

RMS noise $\Sigma$. $\quad S:N = D^2/\Sigma$

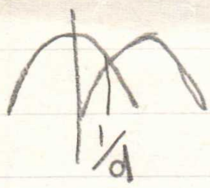
Synthesis




No of aerial posns (1 pointing) $= \dfrac{2D^2 \cdot 9}{4d^2}$

Total $= \dfrac{2D^2 \cdot 9}{4d^2}\tau$

Primary beam




Cannot analyse too far out.
Must sample @ ~ 1/d

$\therefore$ No of settings $= Ad^2$

Total Time $= \dfrac{2D^2 9\tau}{4d^2} \cdot Ad^2 = 4.5D^2\tau A$

~ 10% longer.

A.H.

Conventional system $S/N = D^2/\varepsilon$

Synth:

Defl.ⁿ on single record $\sim 2d^2$   Noise is again $\varepsilon$ if same Rx, $\tau$.

Total signal $= 2d^2 \times$ no. of diff. posns.

$= 2d^2 \times \frac{4.5D^2}{d^2} \times F$ ⟵ numerical factor due to weighting factor → tapered spectral sensitivity fn.

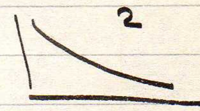
For 2d triangle taper $F = 1/4$.

1d $F = 1/2$

$\therefore$ Total signal $= \frac{9D^2}{4}$.

Total noise = appropriately weighted random noise.

Not weighted, noise $= \varepsilon\sqrt{\frac{4.5D^2}{d^2}}$   if weighted, $F' = 1/3$.



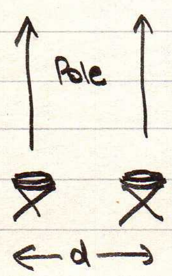

Final S/N out $= \frac{9D^2}{4} \times 3 \Big/ \varepsilon\sqrt{4.5}\,\frac{D}{d} = \frac{D^2}{\varepsilon} \cdot \frac{27}{4} \cdot \frac{1}{\sqrt{\text{No. of posns}}}$

Compare these: synthesis is worse by (depending on $\sqrt{\text{No.}}$)

Equivalent collecting area of synthesis system $= \frac{\text{area synthesized}}{\sqrt{\text{No. of posns}}} \times \sim 6$

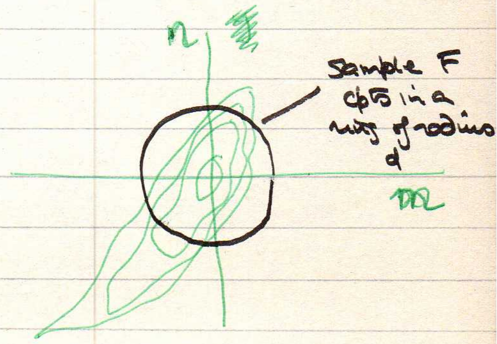
But factor isn't factor of real areas present, much better than that.

Thupersynthethith.




Relative phase diffce matters.

By symmetry, only need cpts over ½ plane. 12 hrs only needed for record.




Vary d according to overlaps, etc., → fill in the map in Fourier space. Complete 2d transform sampled.

Synthesise circular ap. ∥ equatorial plane of the Earth. Diam 2d max.

At lower declinations, effective aperture reduced → elliptical × section.

Phase from one pt. changes contly around sky if the pt is not on pole.

Path compensator added to overcome this. Means that signal need not be recorded so often. To follow fringes must sample @ T/4 @ least. This is more rapid than the rate needed for optimum filter. ∴ Path compensⁿ slows fringes ⇒ also aids sampling.

On a N-S baseline ⟶ synthesises surface of a cone. This has some advantages.

Towards pole, eff $0^\circ$. Same as E-W synthesis.

From side, $\delta = 0$, there is a real aperture. E-W one is simply a strip, of width = width of the simple aerial.

Effects of errors in recording.

Need to keep things very constant in spec. Approx random phase errors are most likely. (Ionosphere, preamps, cables, etc.) Suppose $\Delta\phi_{rms} = \Delta\phi \ll 1$ radian.

Samples $G(n) \equiv C(n) + iS(n)$

$\equiv G(n)e^{i\Delta\phi_n}$ when phase in error.

$= G(n)(1 + i\Delta\phi_n)$

adds a cpt. into F.T. $iG\Delta\phi$.

$$\text{Map} = \sum_{\substack{-N \\ n,m}}^{+N} G(n)\, e^{2\pi i nm/2N}$$

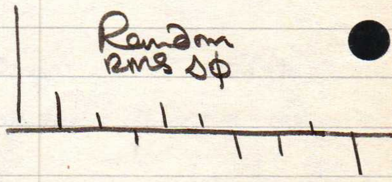
$$\text{now} = \sum \text{-------} + \sum G(n).\, i\Delta\phi_n\, e^{2\pi i nm/2N}$$

"True map" + "Error map"

Same as convolving map with [F.T. of $i\Delta\phi_n$]. — phase error fn.

?

If $\Delta\phi$ is fn of $n$



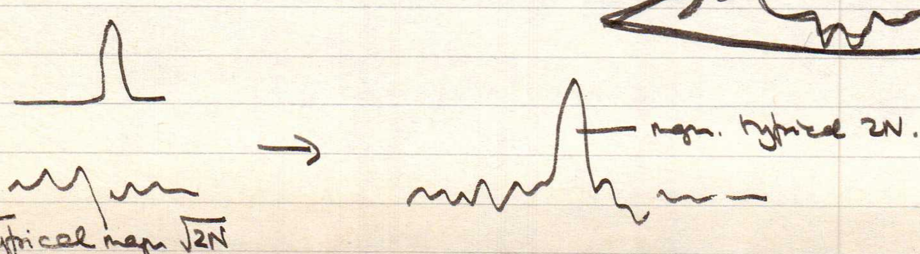

What is F.T. of such a random fn.

= diffn. pattern of regular array of random emitters.

F.T. has av. magnitude $\sqrt{2N}\,\Delta\phi_{rms}$. limited overall by width of el. used in synth.


∴ Pt. source

conv

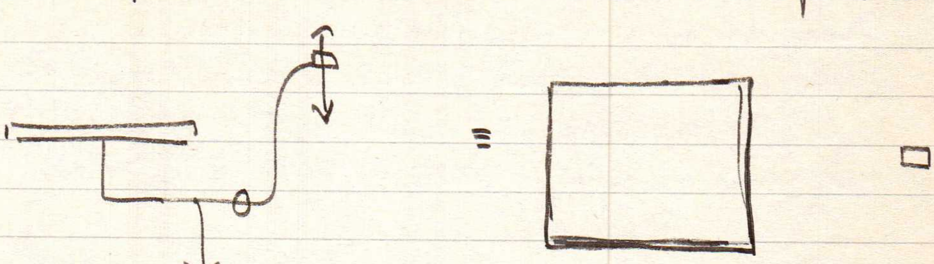



% sidelobe level is $\sim \dfrac{\sqrt{2N}\,\Delta\phi_{rms}}{2N}$

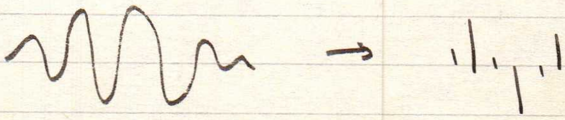
If $\Delta\phi \sim 10^\circ \sim .2$ radns. $2N = 100$, sidelobe level 2%.

Systematic phase error, moves each pm. = rel. to. where it should be → tips beam.

The Glorious 178.



Sample read @ rate T/4 on clock




→ 24 data tapes. 1 punch every ~ 3 or 4 secs

Edit. Right no. of holes, no worst bits of junk. Checks phase by standard sources.

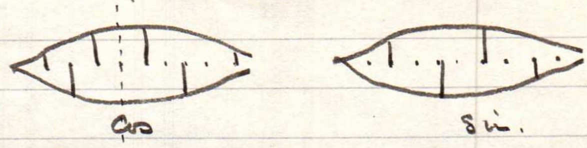
Sampled much more often than optimum.

Output filter is done by convolution to get opt. filter & best S/N.

No need for sin Rx & cos Rx ∵ sin is only cos a bit later for an E–W system.

Convolve with fn. like aerial beam sampled @ the rate on data tape, alt $\pm$

Cos & sin take same envelope




Conv. takes out the cos → Cn & Sn nos.

F.T. a matrix 24 × ~3000 nos → $24^h \times 4^\circ$ matrix 24×3000 nos.

$I(m,n)$

$$I(m) = \sum_{-N}^{+N} G(n)\, e^{2\pi i nm/2N}$$

(−12 to +12.)

1d synth, other is scanned in sky