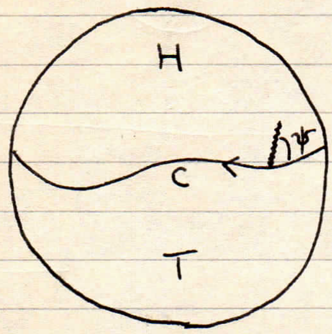
Observational Cosmology Jerer

Theorem. No spherical cat is everywhere strokable.

Must be some sort of singularity somewhere, bald spot, parting, etc!

Consider closed curve C drawn on cat → divides into 2 regions H, & T
arbitrarily small louse L circumnavigates C + observes ∡ ψ made by fur with curve. In one c/nav., $\sum \Delta\psi$ must $= 2\pi N$, supposing fur to be cont's.



Cont'sly deform curve C
Provided it crosses no singularities, $\sum \Delta\psi$ must remain constant, still a whole no. of turns.

Run C small enough → arbitrarily. In H say
Fur must be at ~~[illegible]~~ const. dir^n if it is an arbitrarily small C_h

$\Delta\psi = \underline{\underline{+1}}$ turn

Do it on the T side → C_t, on which $\Delta\psi = \underline{\underline{-1}}$

∴ Assumption must be false

World restrictions.

i) Isotropy & homogeneity. → At given time looks same to all obs.
↓
Same in all dir^{ns} to given obs.
↓
"Trouble". S.Rel. as soon as we decide on "expansion".

Isotropy difficult ∵ it's a f. of v_{obs} ←

Isotropy. At any pt ∃ an observer velocity s.t. world is isotropic to such an obs^r.
Homogeneity. World history same to all observers defined by isotropy cond^n.
Call these locally stationary?
Assume ordinary physics locally valid everywhere. S.Rel.

Isotropic Universe cannot rotate.
Plot galaxies on celestial sphere + observe them for a long while. Each gal's motion can be plotted on the sphere. This diagram looks like fur of spherical cat.
Must be distinguishable region, by theorem, defeats isotropy.
Apart from random motions, then, galaxies remain at rest rel to "Foucault pendulum frame".

Also show light-rays reversible.

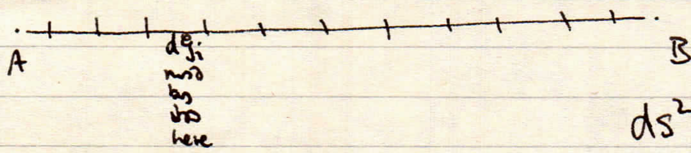
Has to decide if 2 gals are at same distance?
Can order them on a light-path. by i) red shift.
ii) apparent mag. of Cepheids of given period

not necessarily "distance", ∵ this period as seen differs from local period. → a "distance indicator" too.

iii) radar time — local clock travel time unambiguous.

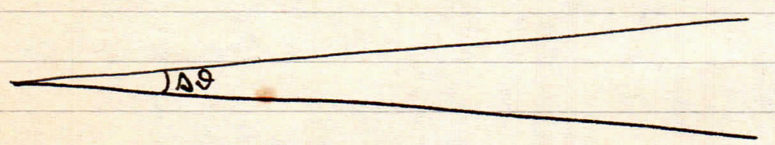
All these indicators must be consistent with one another, by isotropy.
If they weren't, could distinguish bits of world by rel^{ns} betw diff^t indicators.
Can synchronise clox @ same d.i.'s.
This is consistent ∵ clox will keep time thereafter by isotropy.
Ring ABC will also keep synchronised.

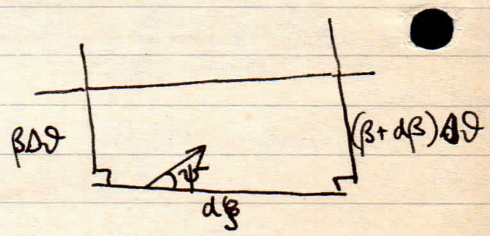
Distance $\xi = \sum d\xi_i$ over all segments of light-path
Each $d\xi_i$ msd. by local stat^y observer at given univ. time. Proper length.



$$ds^2 = dt^2 - \frac{1}{c^2} d\xi^2$$

$\xi \to$ proper length in limit of $\xi \to$ small.





$\beta\Delta\theta = r\,d\theta$ in ord physics

$\oint d\psi$ around loop? $\Delta\psi = 0$ along light-path, first, ~~[illegible]~~.

$$\oint d\psi = 0 + \left(\frac{d\beta}{d\xi}\right)_{\xi + d\xi} \Delta\theta - 0 - \left(\frac{d\beta}{d\xi}\right)_{\xi} d\theta$$

$$= \frac{1}{\beta} \frac{d^2\beta}{d\xi^2} \beta\, d\xi\, \Delta\theta = \frac{1}{\beta} \frac{d^2\beta}{d\xi^2} \cdot [\text{area of loop}]$$

measurable by local observer.

Must be same everywhere by homogeneity. $\Delta\psi$ / unit area.

$$\frac{1}{\beta}\frac{d^2\beta}{\partial \xi^2} = -a^2 \text{, const., say} \quad (1.1)$$

$$\left(\frac{d\beta}{d\xi}\right)^2 = \text{const} - a^2\beta^2$$

Must → local physics. $\beta \sim \xi$ for ξ small. $C = 1$ ∴

Now, $\sin^{-1}(a\beta) = \text{const} + a\xi$

Again $\beta \sim \xi$ for small → $\beta = \dfrac{\sin a\xi}{a}$

Note: a is a(t). Calcⁿ done for particular instant of U.T.

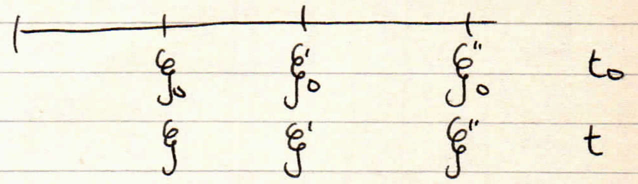
Hence $$ds^2 = dt^2 - \frac{1}{c^2}\left\{d\xi^2 + \beta^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right\}$$

$$= dt^2 - \frac{1}{c^2}\left\{d\xi^2 + \frac{\sin^2 a\xi}{a^2}(d\theta^2 + \sin^2\theta\, d\phi^2)\right\} \quad (1.2)$$

If we had taken const → $+a^2$, get $\sinh^2 a\xi$.
$a = 0$ → Euclidean space, proper distance reduces to sph. polar case.

ξ is $\xi(r,t)$ r is a label – any r(t) which monotonically < will describe expanding universe.
All distances ξ_0 or multiples of any one ξ_0 must change by same factor in same Δt, or else universe not homogeneous + isotropic.

$$\frac{\xi}{\xi_0} = \frac{\xi'}{\xi'_0} = \frac{\xi''}{\xi''_0} = \frac{\xi''-\xi'}{\xi''_0-\xi'_0} = \frac{R(t)}{R(t_0)}$$



Define $r = \xi(t_0)$ $\quad \xi = \dfrac{R(t)}{R(t_0)}r$

Proper distance bet 2 galaxies @ t_0 is
whose $\not\Delta^r$ sepⁿ in $d\theta$

$$\frac{\dfrac{\sin[A(t_0)\xi(t_0)]}{A(t_0)}d\theta}{\dfrac{\sin[A(t)\xi(t)]}{A(t)}d\theta} = \frac{R(t_0)}{R(t)}$$

@ t

$$\therefore \sin\left[\frac{A(t)R(t)r}{R(t_0)}\right] . A(t_0) = \frac{\sin A(t_0) r}{R(t_0)} A(t)R(t)$$

Must have $\dfrac{A(t)R(t)}{R(t_0)} = A(t_0)$ → $A(t) = A(t_0)\dfrac{R(t_0)}{R(t)}$ (1.3)

Subst into ds^2, $$ds^2 = dt^2 - \frac{1}{c^2}\left\{\left(\frac{R(t)}{R(t_0)}dr\right)^2 + \frac{\sin^2 A(t_0)\frac{R(t_0)}{R(t)}\cdot\frac{R(t)}{R(t_0)}r}{\left[A(t_0)\,R(t_0)/R(t)\right]^2}\,d\Omega^2\right\}$$

$$ds^2 = dt^2 - \frac{1}{c^2}\frac{R^2(t)}{R^2(t_0)}\left\{dr^2 + \frac{\sin^2 A(t_0)r}{A^2(t_0)}\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right\} \qquad (1.4)$$

Take t_0 = now
$A(t_0) = A$
Define $R(t_0) = 1$. ∵ ratio $R(t)/R(t_0)$ is all that matters.

$$ds^2 = dt^2 - \frac{R^2(t)}{c^2}\left\{dr^2 + \frac{\sin^2 Ar}{A^2}\,d\Omega^2\right\} \qquad (1.5)$$

Caveat. This looks different under different authorships. Different distance labels
Exx.

$$r^* = Ar \qquad R^*(t) = \frac{R(t)}{Ac}$$

$$\rightarrow ds^2 = dt^2 - R^{*2}(t)\left(dr^{*2} + \sin^2 r^*\, d\Omega^2\right)$$

Or, $u = \tan\frac{1}{2}r^* = \tan\frac{1}{2}Ar$.

$$ds^2 = dt^2 - \frac{4R^{*2}(t)}{(1+u^2)^2}\left\{du^2 + u^2 d\Omega^2\right\}$$

Or $u = \tanh\frac{1}{2}r^*$ in the $+a^2$ constr. case. $\rightarrow$ $(1 \pm u^2)^2$

Usually written $(1+Ku^2)^2$ $K = 1, 0, -1$
May seem strange that only 3 types of universe, but we have absorbed scale of Universe curvature through absorbing A altogether. Only $\pm$ or $0 \times a^2$ really.
No black magic, just conjuring.

H.P. Robertson Ap. J. 82, 284 (1935)
83, 187, 257 (1936)
A.G. Walker Proc. Lond. Math. Soc (2) 42, 90 (1936)

2.
Relation to observables? Remember that r, u, etc. are merely parameters, labels.

Consider galaxy on $\theta = 0$. $d\Omega^2 = d\theta^2$.

Line of sight phenomena. $d\theta = 0$. $$ds^2 = dt^2 - \frac{R^2(t)dr^2}{c^2} \qquad (2.1)$$

t_0 $\longleftarrow$ t_1, $t_1 + \Delta t$ $\qquad ds^2 = \Delta t^2$
$t_0 + \Delta t'$

No. of waves emitted $= \nu\Delta t$. ν meas'd by emitter.

$ds = 0$ on a light-ray. $\therefore\ dt = \frac{R(t)}{c}\,dr$

$dr = \frac{c\,dt}{R(t)}$, $\therefore$ $\int dr = \int \frac{c\,dt}{R(t)}$ $\qquad$ $\int dr$ is $\int_0^r$ if r is label of emitter

$$\therefore \quad r = \int_{t_1}^{t_0} \frac{c\,dt}{R(t)} \qquad (2.2)$$

Must also $= \int_{t_1+\Delta t}^{t_0+\Delta t'} \frac{c\,dt}{R(t)}$ $\quad\therefore$ all sources keep their distance labels for ever.

r = same.

Spec[?] $\int c$, keep $\Delta t <<$ travel time, $\rightarrow$ $\frac{\Delta t'}{\Delta t} = \frac{R(t_0)}{R(t_1)} = \frac{1}{R(t_1)}$ by $R(t_0)=1$

$$\Delta t' = \Delta t / R(t) \qquad (2.3)$$

$$\nu' = \frac{\nu \Delta t}{\Delta t'} = \nu R(t) \qquad (2.4)$$

$$\frac{\nu}{\nu'} = 1 + z = \frac{\lambda'}{\lambda} = \frac{\text{received}}{\text{emitted}} \qquad z = \Delta\lambda / \lambda_{\text{emitted}} \qquad (2.5)$$

Angular Diameter.

Take θ axis along line of sight to simplify line element.

$$ds^2 = dt^2 - \frac{R^2(t)}{c^2}\left(dr^2 + \left(\frac{\sin Ar}{A}\right)^2 d\theta^2\right)$$

Want interval between 2 events @ F & G, $\Delta t = 0$, $\Delta r = 0$.

$$ds^2 = -\frac{R^2(t)}{c^2} \cdot \frac{\sin^2 Ar}{A^2} \Delta\theta^2 \qquad t = t_{\text{emission}}$$

$$ds = i \frac{R(t)}{c} \frac{\sin Ar}{A} \Delta\theta = \frac{i}{c}\left\{\frac{R(t) \sin Ar}{A} \Delta\theta\right\}$$

Proper diameter FG $= \frac{R(t) \sin Ar\ \Delta\theta}{A}$ $\qquad$ $\Delta\theta_{\text{obs}} = \frac{\Delta\theta_{\text{proper}}}{\left[\frac{\sin Ar}{A}\right]} \cdot (1+z)$ $\qquad (2.7)$

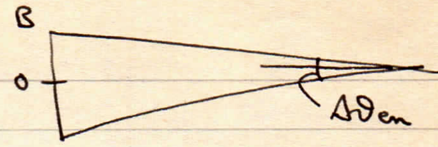
In freq. band $(\nu, \nu+d\nu)$, n photons emitted in $(t_1, t_1+\Delta t)$ / ster
$(\nu', \nu'+d\nu')$, ... arrive in $(t_0, t_0+\Delta t)$

o $\qquad\qquad$ · G
t_0 $\qquad\qquad$ t_1
$t_0 + \Delta t'$ $\qquad\qquad$ $t_1+\Delta t$

Energy emitted $= nh\nu$ $\qquad$ Power $= nh\nu / \Delta t$
rec^d. $= nh\nu' / \Delta t'$

$$L(\nu_{\text{emitted}}) = \frac{4\pi n h\nu}{\Delta t\, \Delta\nu} \qquad (2.8)$$

Energy radiated over star appears to be a cone of semi-∠ angle $\Delta\theta$. Solid ∠ $\pi\Delta\theta^2$



No. of photons same for receiver & source.

$$\frac{\nu'}{\nu} = (1+z)^{-1} = R(t), \quad \frac{\Delta t'}{\Delta t} = 1+z, \quad \Delta\nu' = (\Delta\nu' + \nu') - \nu' = \frac{\Delta\nu}{1+z}$$

Power received $\dfrac{nh\nu}{\Delta t(1+z)^2}\pi\Delta\theta^2 = \dfrac{L(\nu)\Delta\nu}{4\pi}\dfrac{\pi\Delta\theta^2}{(1+z)^2}$

Over what area does this arrive? Almost ∠ᵛ same sum, but now the rays diverge from past, not converge on us.

OB is proper length bet. $\Delta t = 0$, $R(t) = R(t_0) = 1$, $\Delta r = 0$

$$ds = -\frac{i}{c}\left[\frac{\sin Ar}{A}\Delta\theta\right] \qquad \pi OB^2 = \frac{\pi\sin^2 Ar\,\Delta\theta^2}{A^2}$$

∴ Power/unit area $= \dfrac{L(\nu)\Delta\nu}{4\pi(1+z)^2\left(\frac{\sin Ar}{A}\right)^2}$ (2.13)

Note role of $\left(\dfrac{\sin Ar}{A}\right)$ instead of r of flat space throughout. Also limit $A \to 0$.

Total power/area $= \displaystyle\int L(\nu)d\nu \cdot \frac{1}{4\pi(1+z)^2}\left[\frac{A^2}{\sin^2 Ar}\right]$. → Bolometric magnitude.

Not practical. Need u/v, i/r, inaccessible freq. bands.

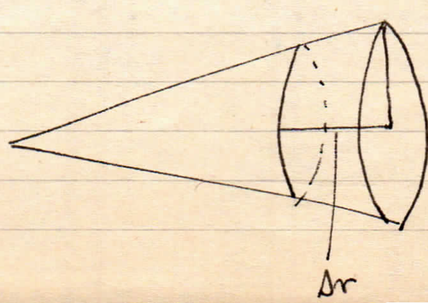
Power/m²/sec/Hz $S = \dfrac{L(\nu)\Delta\nu . A}{4\pi\sin^2 Ar(1+z)^2} \cdot \dfrac{1}{\Delta\nu'} = \dfrac{L(\overset{\text{emitted}}{\nu})A}{4\pi\sin^2 Ar\,(1+z)}$ (2.14)

Power law spectrum. $\nu^{-\alpha}$

$$\to S(\nu_{obs}) = \frac{L(\nu_0)A}{4\pi\sin^2 Ar\,(1+z)^{1+\alpha}} \quad (2.14a)$$

How many galaxies with $r(<r_0)$?

Proper volume of spherical shell.
Radius of el^t at emission: $R(t)\dfrac{\sin Ar}{A}\Delta\theta$ $\qquad d\Omega = \pi d\theta^2$



$\delta V = \left[\dfrac{R^2(t)\sin^2 Ar\,\Delta\theta^2}{A^2}\right]\pi\,\Delta r . R(t)$

Elementary vol $= R^3(t)\,\dfrac{\sin^2 Ar}{A^2}\,\Delta\Omega_1\,\Delta r$

Proper vol. of sph. shell $= \dfrac{4\pi}{(1+z)^3}\left(\dfrac{\sin Ar}{A}\right)^2 \Delta r$ (2.15)

Must now need to take in cosmology philosophical-type. steady state vs not.

i) Relativistic cosmologies. Each $dr\,d\Omega_1$ carries its galaxies with it for all time. But these labels will 'expand' & ∴ observed ρ of galaxies will vary.
No. density / unit proper volume must be everywhere same by H.

$$\rho_n(t_0) = \rho_n. \quad = \text{proper density of galaxies in neighbhd of us.}$$

No of gals in r to $(r+\Delta r)$ always = no. of gals in that shell now.

$$= 4\pi\left(\frac{\sin Ar}{A}\right)^2 R^3(t_0)\,\Delta r\,\rho_n$$

$$= 4\pi\left(\frac{\sin Ar}{A}\right)^2 \Delta r\,\rho_n \quad \text{———} \quad (2.16)$$

Steady state ρ_n = constant → $4\pi\left(\dfrac{\sin Ar}{A}\right)^2 \dfrac{\Delta r}{(1+z)^3}\,\rho_n$ — [Proper density always constant. (2.17)

All these formulae contain the label r. Can relate r to observables thru $r = \displaystyle\int_{t_1}^{t_0} \frac{c\,dt}{R(t)}$

Hubble's Constant.

Determine $\dfrac{c\,\Delta\lambda}{\lambda}$ = vely of recession. $= \dfrac{c\,[R(t) - R(t-dt)]}{R(t-dt)}$

$$= c\,\frac{\dot R(t)\,\Delta t}{R(t)}$$

Distance $\xi \sim r R(t)$ (small ξ)

∴ In limit $H_0 = \dfrac{\dot R(t)}{R(t)} = H(t)$ (2.19)

∴ $\xi(t) = \xi(t_0) + r H(t_0)\, t$ in absence of i/galactic forces.

Gravity might well have been important in the past to modify this cosmological expansion, even if it isn't now.

Most of features of these relativistic cosmologies don't use anything specially relativistic — can get these formulae from Newtonian mechanics.

Difficulty. Creation of it all at H^{-1} ago, if not shorter ∵ of gravitation.
1930. $H \sim 500$ km/sec/Mpc → 2.10^9 years, not more, prob. less.
Radioactive minerals → ~ same, laid down in present form.
Stellar evolution → longer times.

Dodge 1. Cosmological term $\lambda \to$ repulsion. Expansion slower long ago.
Also static state when $\frac{GM}{r^2} \sim$ repulsion, $\to$ "only long state" before expansion started.

Respectable at time, dodgy now? Einstein wanted it for Mach's Princ., put it in before cosmology anyway.

Einstein $\rho \neq 0$, static. Metric only "exists" if $\rho \neq 0$. Cannot apply bdy cond^s (sph. space) if $\rho \neq 0$.

But de Sitter showed that if $\rho \neq 0$ can get expanding sol^ns, non-static. based on $\rho = 0$. Friedmann $\to$ expanding models with matter. $\lambda = 0$ now possible still preserving Mach. ∴ It's only put in for cosmology, not from "1st principles".

Baade $\to$ 100 Km/sec/Mpc $\to$ 10^{10} years.

Dodge 2 B., G., & H.
Homogeneity & isotropy + stationarity in time.
Must expand to hold down Olbers-wise, + redshift expt.
H. introduced term into Einstein's law, ad hoc $\to$ dynamics, and density of universe related to H & G. B. & G. doesn't do this & is easier to handle.

Bondi-Gold Steady State Theory.

$A(t)$ in principle constant ∵ it is no. / unit area — measurable.

(1.3) $\to$ $R(t)\,A(t) =$ constant.

But $A(t) =$ const $= A$. $R(t)$ must vary to keep Olbers' Paradox quiet.
∴ $R(t)$ varies.
Only way out is $R(t)A(t) = 0 \longrightarrow A(t) = 0$, flat space.

$$\frac{\dot R(t)}{R(t)} = H, \text{ constant.} \qquad (3.1)$$

$$\to R(t) = \text{const.}\, e^{Ht}$$

$$R(t_0) = 1 \text{ by convention}, \quad R(t) = e^{H(t-t_0)} \qquad (3.2)$$

$$\to ds^2 = dt^2 - \frac{e^{2H(t-t_0)}}{c^2}\left(dr^2 + r^2 d\Omega^2\right) \qquad (3.3)$$

"de Sitter metric"
∴ it's same as de Sitter model.

Space is Euclidean but space-time isn't.

$$r = \int \frac{c\,dt}{R(t)} = -\frac{c}{H}\left[e^{-H(t-t_0)} - e^{H(t_1-t_0)}\right]$$

$$= \frac{c}{H}\left(\frac{1}{R(t_1)} - 1\right) \qquad 1/R(t_1) = 1+z$$

$$= \frac{cz}{H} \tag{3.4}$$

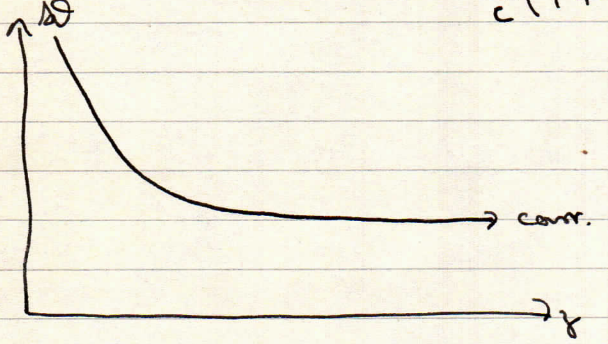
i.e. velocity $= Hr$ linear, note, in r.

<u>Angular Diameter.</u>

$$\Delta\theta = (1+z) . \frac{\text{Proper diameter}}{r}$$

$$= (1+z) \cdot \frac{H}{cz} \cdot \text{Proper diam.}$$

$$= \frac{H}{c}\left(1 + \frac{1}{z}\right) . \text{Proper diameter.} \tag{3.5}$$



← For given size of galaxy

As $z <$, $\Delta\theta \to$ native value of $\Delta\theta$ @ $r = c/H$.

<u>Bolometric Intensity.</u>

$$= \int_0^\infty L(\nu)\,d\nu \Big/ 4\pi\left(\frac{c}{H}\right)^2 z^2 (1+z)^2 \tag{3.6}$$

Will apply over an <u>identifiable</u> bit of spectrum. (Not given no. of c/s, but over a spectral line, or between 2 identifiable lines — remember that $\Delta\nu$'s are compressed by the redshift effect).

<u>Flux</u>

$$S = \frac{L(\nu_e)}{4\pi r^2 (1+z)} \longrightarrow \frac{L(\nu_0) H^2}{4\pi c^2 z^2 (1+z)^{1+\alpha}} \tag{3.8}$$

No. Density.

$$\Delta N = 4\pi\rho_n \frac{r^2 \Delta r}{(1+z)^3} = 4\pi\rho_n \frac{(c/H)^3 r^2 \Delta z}{(1+z)^3}.$$

"Relativistic" Cosmologies.

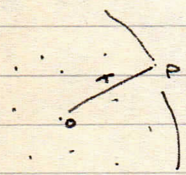
Distinguished by conservation of matter, not by Einstein Law of Gravitation.
H. just as "relativistic" as these in the old-fashioned sense.

Dynamics.

← Galaxies. How do they move?

Gravitation only really important force.
Must we use G.R. or Newton to → dynamics?

So long as $v \ll c$ can use Newton.
Assume local conservation laws hold.



$g = GM/r^2$ only $= \frac{4}{3}\pi G r^3 \rho / r^2 = 4/3\, \pi G \rho r$.
$g \propto r$, ∴ as big as we like @ P by choosing O far enough away in this system.
? ∴ accⁿ a fn. of which galaxy we take at centre?

		O	A	B
$t=0$	$v=$	0	Ha	Hb
Δt	Δv		$(H+\Delta H)(a+\Delta a) - Ha$	$(H+\Delta H)(b+\Delta b) - Hb$
	$\frac{\Delta v}{\Delta t}$		$\frac{\Delta a \Delta H + a\Delta H + H\Delta a}{\Delta t}$	$\frac{\Delta b \Delta H + b\Delta H + H\Delta b}{\Delta t}$

$$= \frac{r}{\Delta t}\left[H + \frac{H\Delta R}{R}\right]$$

But this is =ly ridiculous. Accⁿ depends on the mythical observer's distance, which could be anything.
This accⁿ cannot be "felt", ∵ it applies to all galaxies. It is undetectable. Not to worry then.

By (2.19), $H = \dot{R}/R$ Accⁿ av. $= r\left(\frac{d}{dt}\left(\dot{R}/R\right) + \dot{R}/R \cdot \dot{R}/R\right) = r\left(\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} + \dot{R}^2/R^2\right)$

$$= r\ddot{R}/R$$

By comparing terms,
$$\ddot{R} = -\tfrac{4}{3}\pi G \rho R \qquad (3.10)$$

$$\rho(t) \sim R^3(t_0)/R^3(t) \qquad \rho = \rho_0/R^3(t) \qquad (3.11)$$

$G(t)$ also? (by Mach). Not very popular.

$$\ddot{R} = -\tfrac{4}{3}\pi G \rho_0/R^2 \qquad (3.12)$$

If ∃ λ-term
$$\ddot{R} = -\tfrac{4}{3}\pi G \rho_0/R^2 + \tfrac{1}{3}\lambda R \qquad (3.12\lambda)$$

$$\dot{R}\, d\dot{R} = \left(-4/3\,\pi G \rho_0/R^2 + \tfrac{1}{3}\lambda R\right) dR$$

$$\dot{R}^2 = \tfrac{8}{3}\pi G \rho_0/R + \tfrac{1}{3}\lambda R^2 - \text{constant.}$$

Const. of ∫ation?
Initial conds.

$$(\dot{R}/R)^2 = \tfrac{8}{3}\pi G\rho_0/R^3 + \tfrac{1}{3}\lambda - c$$

$[R_0 = 1]$ $\rightarrow$ $$H_0^2 = \tfrac{8}{3}\pi G\rho_0 + \tfrac{1}{3}\lambda - c \qquad (3.14)$$

$$8\pi G\rho = \frac{8\pi G\rho_0}{R^3} = \frac{3\dot{R}^2 + c}{R^2}\ (-\lambda)$$

cf. Bondi C.U.P., P.103 from G.Rel. His "R" = R/Ac here.

V. good. But Newton cannot get us to → metrical properties of Universe a priori'd by Newton.

G.R. field Eqn $R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = -\frac{8\pi G}{c^2}T_{\mu\nu}\ (-\lambda g_{\mu\nu})$

$$\rightarrow \quad 8\pi G\rho = -\lambda + \frac{3}{R^2}(\dot{R}^2 + A^2c^2)$$

$$\frac{8\pi G p}{c^2} = \lambda - \frac{2R\ddot{R} + \dot{R}^2 + A^2c^2}{R^2}$$

Can put p fairly safely = 0 if we aren't in a relativistic gas.

$$\lambda R^2 = 2R\ddot{R} + \dot{R}^2 + A^2c^2$$

$$\tfrac{8}{3}\pi G\rho R^2 + \tfrac{1}{3}\lambda R^2 = \dot{R}^2 + A^2c^2$$

$$-\tfrac{8}{3}\pi G\rho R^2 + \tfrac{2}{3}\lambda R^2 = 2R\ddot{R}$$

$$\ddot{R} = -\tfrac{4}{3}\pi G\rho R + \tfrac{1}{3}\lambda R$$ Hooray!

and stuff about A.

$$\dot{R}^2 = \tfrac{8}{3}\pi G\rho R^2 + \tfrac{1}{3}\lambda R - A^2c^2 \qquad (3.17)$$

Same as Newton but with definite value of const. of integ.

$$A^2c^2 = \tfrac{8}{3}\pi G\rho_0 - H^2 + \tfrac{1}{3}\lambda \qquad (3.18)$$

From 3.17 $$dt = dR\Big/\sqrt{\tfrac{8}{3}\pi G\rho_0/R + \tfrac{1}{3}\lambda R^2 - A^2c^2}$$

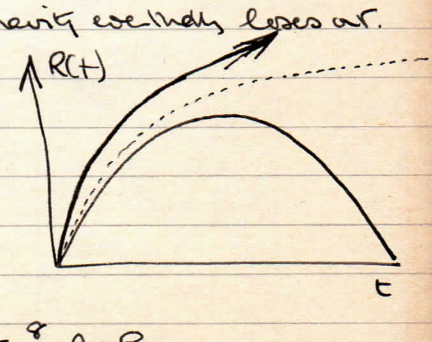
Not very nice.
Elliptic fn. in general.
Horrible even with $\lambda = 0$.

Take $\lambda = 0$, $\dot{R}^2 = \frac{8}{3}\pi G\rho_0 \cdot \frac{1}{R} - A^2c^2$

If original H is big enough, A^2c^2 must be −ve $\because$ $H^2 = \frac{8}{3}\pi G\rho_0 - A^2c^2$
("original" = now, here).

Then $\dot{R}$ always > 0, however big R gets. Gravity eventually loses out.

Limiting value $\dot{R} = A^2c^2$. $\dot{R} \rightarrow \sqrt{-A^2c^2}$



If H is small, Gravitation will win.

$\dot{R} = 0$ when $\frac{8}{3}\pi G\rho_0/R = A^2c^2$, $R = \frac{8}{3}\frac{\pi G\rho_0}{A^2c^2}$

Limiting case of course. $H^2 = \frac{8}{3}\pi G\rho$, $A^2c^2 = 0$, $\dot{R}^2 = \frac{8}{3}\pi G\rho_0 R$
[Einstein – de Sitter]