

April 15, 82

Dear Alan,

Here are my answers to your questions:

1. Equation (5) in the draft was miswritten, it should be

$$\frac{E_{\Delta}}{t_j} = \frac{I}{2R}$$

where I is supposed to be the radiation intensity on the jet axis. The error ~~is~~ is purely a "typo" ~~error~~ and it was not carried into later derivations. More accurately, I can be expressed as

$$I_{\Delta} = 2 \int_0^R \frac{E}{t_j} dr.$$

In deed, the relationship ~~betw~~ between R and the radio HWHM depends on the particle and field configurations. However, in the absence of detailed observational information about these factors, one has to make assumptions concerning the cross-sectional distributions of various quantities to simplify the problem. The assumption needed here is that a self-similar profile exists across the jet. Then the above equation can be written as

$$I_{\Delta} = \frac{2R}{t_j} \int_0^1 E_2(x) dx$$

where $x \equiv \frac{r}{R}$ is the independent variable for the self-similar profile. There is another constant ~~factor~~ ^{fudge} factor of order

unity required to write this equation into the form of Eq(5).
If you think that it is necessary to carry this ^{fudge}~~handy~~ factor
~~explicitly~~ explicitly in the derivation of later equations, it is
O.K. and easy.

2. I can be normalized (made dimensionless) by scaling
it with ~~some~~ ^{the} constant I at some fixed location on the
jet axis. Other quantities (e.g. R) can also be normalized
with the fixed values of these quantities at this location as well.
The observed I is of course resolution dependent. In
my draft, I assume that the resolution is much better than
the jet width ~~is~~ everywhere, then there is no problem.
However, if the resolution is comparable to the jet width, a
correction factor has to be put in to guess the theoretical
intensity from the observed value. If you want, this ~~is~~ geometric
factor can be estimated, but further assumption probably has
to be introduced.

Whether another ~~case~~ jet shows a similar correlation
between the polarization and the "entrainment" would be a
~~an~~ crucial test to the present approach. I hope to ~~be~~ hear
from you soon.

Phing

NATIONAL RADIO ASTRONOMY OBSERVATORY

P. O. BOX 0
SOCORRO, NEW MEXICO 87801
TELEPHONE 505-835-2924
VLA SITE 505-772-4011
TWX 910-988-1710

POST OFFICE BOX 2
GREEN BANK, WEST VIRGINIA 24944
TELEPHONE 304-456-2011
TWX 710-938-1530

EDGEMONT ROAD
CHARLOTTESVILLE, VIRGINIA 22901
TELEPHONE 804-296-0211
TWX 510-587-5482

SUITE 100
2010 NORTH FORBES BLVD.
TUCSON, ARIZONA 85705
TELEPHONE 602-882-8250

7 April 1982

Dear Kuniy -

I have two questions about your draft of the "entrainment" paper, regarding Eqn. (5) on p. 4

$$E_{\Delta} = \frac{1}{t_j} \left(\frac{I}{2R} \right)$$

1. Why do you normalize by the jet radius R ? The relation between R and the ratio HWHM depends on the particle distribution and field configuration.

2. How is I normalized? (It is resolution-dependent).

I have new NGC 6251 T_b -radius data but would like to feel happier about calculation of E_{Δ} before going ahead to try your analysis with that.



Enrichment and the jet environment — a case study of the northern jet of 3C31

p.1

I. Introduction

Recent observations (e.g. Burch 1974, Perley et al. 1979) on radio jets have accumulated evidence that in-situ particle acceleration is required to explain the slow decline of radio intensities along the jets. It is also commonly believed that the randomization of the large scale motion of the jet can supply energy to the relativistic electrons. Many theories have been proposed to describe the microphysics involved in the electron acceleration process (e.g. Pacholczyk and Scott 1976, Lacombe 1977, Bell 1978, Ferrari et al. 1979, Henriksen et al. 1981) and the in-situ production of magnetic fields (De Young 1980). In this paper we explore the macroscopic dynamics and energetics of radio jets by looking at their observable characteristics.

Hinted by the observational results of 3C31 (Burch 1974, Fomalont et al. 1980, Bridle et al. 1980), we develop some methods to analyse the following two problems: (i) particle energy budget — What is the quantitative relationship between the "slow" decline of the radio intensity and the amount of energy

If (a) steady-state
(b) constant v_z .

JGC315
Willis et al.
CSI-Fomalont
et al.
JGC6251
numbers of
Willis
et al. 1982
3214319
Paci et al.
1982

replenishment required to sustain the radiating particles (§II)? (ii) dynamics of entrainment — if there is continuous dissipation of the kinetic energy of the large scale motion to replenish the relativistic electrons, what are the effects of on the dynamical development of the jet (§III)? In studying these problems we ~~try to~~ limit ourselves to ~~making only~~ simple and common assumptions.

~~When the above two aspects of the jet are~~ The above aspects of a jet are related. We ~~shall~~ show that when a model of energy conversion between the fluid and the particles is supplied, information about the density, pressure, and temperature distribution of the environment around the jet can be deduced.

II. Energy budget of the radiating particles

For a locally isotropic distribution of relativistic electrons which has a power spectrum with index $-n$, the evolution of the distribution function $N(x)$ is described by

$$\frac{d}{dt} N + \cancel{N \nabla \cdot u} = \mathcal{Q} - \delta \frac{\partial^2}{\partial x^2} N \quad (1)$$

where γ is the Lorentz factor, \underline{u} is the large scale velocity, g is the source term, and all other symbols have their standard meanings. The decay rate

? $\dot{\gamma}$ in energy space is composed of two parts: (i) adiabatic loss:

$$\dot{\gamma}_{ad} = -\gamma \nabla \cdot \underline{u} / 3, \text{ (ii) synchrotron loss } \dot{\gamma}_{sy} = -\gamma / t_{sy} \text{ where}$$

(Tucker 1975)

$$t_{sy} = 7.7 \times 10^8 \gamma^{-1} B^{-2} \text{ sec.} \quad (2)$$

B is the strength of the magnetic field. ^{units?} Let $E_{\Delta} \equiv \int \gamma m c^2 N d\gamma$ be the integrated relativistic energy contained within a small interval between γ and $\gamma + \Delta\gamma$ and $Q_{\Delta} \equiv \int \gamma m c^2 g d\gamma$ be the energy production rate in this same interval;

Eq (1) can be transformed into an equation for E_{Δ} .

$$\frac{d}{dt} E_{\Delta} + \left(1 + \frac{n}{3}\right) (\nabla \cdot \underline{u}) E_{\Delta} + \frac{n}{t_{sy}} E_{\Delta} = Q_{\Delta}. \quad (3)$$

An important fact is that E_{Δ} can be directly related to the radio intensity I observed in a frequency interval between ν and $\nu + \Delta\nu$ where

$$\nu = 4.2 \times 10^6 B \gamma^2 \text{ Hz.} \quad (4)$$

The relationship can be expressed as

$$E_{\Delta} = \frac{1}{t_j} \left(\frac{I}{2R} \right) \quad (5)$$

where R is the radius of the jet,

$$t_j = \frac{2}{27 a(m)} t_{sy} \quad (6)$$

and $a(m)$ is a constant of order 0.1 whose exact value depends on n (Tucker 1975).

Now, let us assume that the jet flow is stationary and the velocity u is approximately uniform across the flow, then Eqs(3) and(5) can be combined to yield a formula for Q_{Δ}

$$\frac{t_u}{t_j} \left(\frac{2R Q_{\Delta}}{I} \right) = \frac{d}{dz''} \ln \left[t_{sy} \left(\frac{I}{2R} \right) (R^2 u_z^{-1})^{1+\frac{n}{3}} \right] + n \frac{t_u}{t_{sy}} \quad (7)$$

in which $z'' = z/L$, $L_u = L/u_z$, z is the ^{projected} distance from the core (perpendicular to the line of sight), u_z is the projected velocity, and L is the projected distance of one arc second. The ~~the~~ evaluation of the ratio

$$\frac{t_u}{t_{sy}} = 0.11 L_{\text{kpc}}^{-1} U_{z8}^{-1} B_{-5}^{3/2} \lambda^{-1/2} \quad (8)$$

which describes the relative importance of synchrotron loss, requires a knowledge

of B and u_z . In later discussion, we ~~shall~~ approximate B by the equipartition value $[\propto \cancel{\text{I/2R}} (I/2R)^{2/7}]$. Furthermore, we ~~shall~~ take u_z to be approximately constant in the region of interest (see §II). Then, using Eq(7), one can find the rate of energy injection to E_Δ from observed development of I and R .

We employ the data ^{m 3C31} in Fomalont et.al. (1980) and Bridle et.al. (1980) to estimate the in-situ relativistic energy injection. I is taken to be the 6 cm ridge-line intensity at $2.5'' \times 5.0''$ resolution. ~~and R is taken from the~~
~~combined~~ The spectral index α is taken to be 0.5 so that $n=2$. The projected distance of $1''$ is taken to be 480 pc. Then,

$$\frac{t_h}{t_{sy}} = 0.02 u_{z8}^{-1} B_{-5}^{3/2} \quad (9)$$

which is proportional to $\cancel{\text{I/2R}} (I/2R)^{3/7}$ for equipartition field.

As the data always contain noise and is not smooth, the calculation of the derivative term in ~~Eq(7)~~ Eq(7) presents some difficulty. We use central differencing with a spacing of one ~~arc~~ arc second between data points

and then take averages over intervals of $5''$ to obtain smoothed values for this term. However, we note that this procedure is not vitally important to our later results. At it ~~turns~~ turns out, this ~~the~~ derivative term becomes negative in a region between $z'' = 25$ and 30 . This is caused by a fast drop in I and a later dip in R . The minimum value is -0.01 at $z'' = 27$. The existence of negative values in this term imposes restriction on the minimum size of the second term. Since the source term on the LHS of Eq(7) cannot be negative, the second term must be large enough to keep the sum on the RHS non-negative. This means that the synchrotron loss must be large enough to account for the fast drop of intensity. Using Eq(9), one obtains the following limitation on the factor $u_{z8}^{-1} B_{-5}^{3/2}$ at $z'' = 27$.

$$u_{z8}^{-1} B_{-5}^{3/2}(27) \geq 0.25. \quad (10)$$

Taking u_{z8} to be 1, $B_{-5}(27)$ must be ≥ 0.25 . We adopt a value of 1 for $B_{-5}(27)$ in our sample calculation; this yields an average value of ≈ 1.4 for B_{-5} in the region between $z'' = 5 - 35$. The resulting values of

the dimensionless source term $t_{\text{in}} 2RQ_2 / t_j I$ and the total production rate $\pi R^2 Q_2$ are plotted in Fig. 1 which also shows the percentage of polarization again in inverted direction. The existence of an anticorrelation between the energy injection and the polarization shows up clearly in the region with $z'' > 10$. For z'' below 10, the situation is complicated by the increased importance of the organized magnetic field whose strength does not follow the equipartition value. For example, at $z'' = 9$, the polarization goes through a minimum which corresponds to the transition of the organized field from parallel to perpendicular directions (w.r.t. the jet axis). To test the effect of the organized field on our results, we made several calculations with an added component of B proportional to $(1 + R(z)/R)/R$. We found that when the averaged size of the organized field is smaller or comparable to the equipartition field strength we have previously used, there is no qualitative difference between the results.

The existence of the anticorrelation between Q_2 and the polarization suggest:

??
 by
 several
 distances?

that the injection of energy is associated with some kind of "turbulent" process.

We shall discuss a turbulent entrainment model and its consequences in next section

III. Entrainment and the jet environment

Assuming that a ~~stationary~~ quasi-stationary jet flow has self-similar

cross-sectional profiles, its dynamics is described by the following conservation

laws:

$$\text{(mass)} \quad \frac{d}{dz}(m u) = \dot{m} \quad (11)$$

$$\text{(momentum)} \quad m u^2 = F \quad (12)$$

$$\text{(energy)} \quad \frac{d}{dz} \left(\frac{1}{2} m u^3 \right) = -\dot{E} \quad (13)$$

where $m = \pi R^2 \rho_j$, ρ_j is the mass density inside the jet, F (the momentum flux) is a constant (ignoring gravitational effects), \dot{m} is

the mass entrainment rate, and \dot{E} is the mechanical energy dissipation

rate (transformed into heat, relativistic particle energy ... etc.). For

Such a mechanical system, all variables can be related to the mass

entrainment rate :

$$u = \left(\frac{F}{m} \right)^{1/2}, \quad (14)$$

$$\frac{d}{dz} m = 2 F^{-1/2} m^{1/2} \dot{m}, \quad (15)$$

and
$$\dot{E} = \frac{1}{2} F \frac{\dot{m}}{m}. \quad (16)$$

Eq (10) directly links the dissipation of the mechanical energy to the entrainment of mass. A useful conclusion can then be made: if there is no entrainment, ~~bulk~~ no mechanical energy can be transformed into relativistic energy.

If \dot{m} is known, all other variables can be determined. We need a model for \dot{m} . We shall assume that it has the form

$$\dot{m} = \rho_e \pi R u' \quad (17)$$

where ρ_e is the density of the external medium and u' is some kind of velocity. For a turbulent jet, u' is proportional to the turbulent velocity. For jets under laboratory conditions (uniform gas), the turbulent velocity is about 10% of the mean velocity of the jet.

However, when both the external and the jet densities are non-uniform,

the exact relationship is completely unknown. Even so, we make the assumption that u' is proportional to u ; this is consistent with the assumption of self-similarity in the cross-sectional profile.

Now, let us look at some of the consequences of such a simple model of entrainment. Suppose that ρ_e is proportional to z^{-K} and

$R/z \approx$ a constant $\ll 1$, Eq (15) gives

$$m = m_0 + \left[\frac{2u'}{u} \right] (\rho_{e0} \pi R_0^2) \times \begin{cases} \frac{1}{2-K} \left[\left(\frac{z^2 \rho_e}{z_0^2 \rho_{e0}} \right) - 1 \right] & (K \neq 2) \\ \ln \left(\frac{z}{z_0} \right) & (K = 2) \end{cases} \quad (1)$$

where in which the subscript 0 indicates ~~that~~ ~~the quantities at some reference~~ ~~location the reference location z_0~~ ~~are evaluated~~ the original values of the quantities.

If ρ_e drops faster than z^{-2} ,

both m and u approach a finite value. On the other hand, if $K \leq 2$,

m increases without limit and u approaches 0, implying that the jet

can only travel a limited distance from the core. Furthermore,

entrainment and dissipation of mechanical energy is important only in

regions where ρ_e drops slower than z^{-2} .

evidence? ~~Observationally~~, The pc scale jets (VLBI) are believed to have a speed closed to ~~relativistic speed~~; on the other hand, the Kpc scale jets ~~are believed to~~ have speeds around

$10^{-2} c$. Therefore, m ~~must~~ must have increased 10^4 times between the

two scales (total mass flux increased by 10^2 times). This suggests the

following scenario for the "cascade" of the jet between the two scales.

When the jet is in relativistic speed, its density is low compared to that of the external medium so that m increases very quickly near the core.

ρ_j can then catch up with ρ_e very quickly. However, as the jet ~~continues~~ continues outward, its density may become higher than that of

the environment. Then, the speed of the jet would not drop as quickly

and the jet can travel a long distance.

I shall add a discussion on why jets start radiating away from the core here.

Our next step is to introduce a relationship between the mechanical energy dissipated by the fluid and the relativistic energy received by the radiating electrons. Then the variation of the jet intensity will be able to provide information about the jet environment. We shall

adopt the simplest model here, namely

$$\cancel{\dot{M}} \pi R^2 Q_{\Delta} \propto \dot{E} \quad (19)$$

Then, given Q_{Δ} , the distribution of P_e can be found by Eqs. (16) and (17).

¶ Using the values of Q_{Δ} found in the previous section, we calculated the

distribution of P_e for 3C31; the result is shown in Fig. 2. If we

further take ~~P_e~~ the external pressure P_e to be approximately

equal to the pressure of the relativistic particles inside the jet ~~which~~

~~is proportional to~~ $\left[\propto (I/2R)^{4/7} \right]$, all thermodynamic variables of the galactic

environment can be determined. The distribution of P_e is also shown in Fig 2.

Notice that even though the distribution of P_e is very bumpy beyond $z'' > 20$,

the average slopes of both curves are clearly flatter than -2 , indicating

that the observed jet is in a plateau of both P_e and P_e . In fact,

by looking ~~at~~ more closely at these curves, one can notice that the

curves between $z'' = 10$ to 20 are a little flatter than other portions.

These distributions agree qualitatively with the pressure confinement model used

by Bridle et. al. (1980) to explain the increased collimation of 3C31.

The apparant dip of p_e between $z'' = 20-35$ has to be interpreted carefully. The exact depth of this dip sensitively depends on the size of the second term ~~on the RHS of Eq(7)~~ on the RHS of Eq(7) which in turn depends on the chosen value of $u_{28}^{-1} B_{-5}^{3/2}$. Even though it is safe to say that some dip exists, we cannot be certain how deep it really is. If it is really as deep as indicated, the matter distribution in this region of the galaxy is very clumpy (~~either~~ ^{either} filled with hot bubbles or cold clouds). Another feature demonstrated in Fig. 2 is that p_e in general declines faster than p_e . This implies that the temperature of the environment increases outward. For the distributions shown in Fig. 2, the temperature in the region below $z'' = 20$ is about 3 times cooler than the region above it. Such a behavior is surprisiny similiar to that of M87 found by X-ray observations (Fabricant, Lecar, and Gorenstein 198

IV Summary and discussion

By assuming that the radiating electrons have an isotropic power law distribution, we have derived a rather general equation which relates the evolution of I and R to ~~be~~ the rate of in-situ relativistic energy production. The application of this equation to interpret observational results can yield useful information on the physical conditions of a jet. The fact that the percentage of polarization is anti-correlated with the rate of energy production tends to support the view that the production ~~is~~ of relativistic energy is associated with some kind of violent process.

Assuming that the jet has ~~a~~ self-similar profiles in the ~~cross-sectional~~ cross stream direction, we studied the entrainment dynamics of the jet and arrived at the conclusion that entrainment ~~is~~ ^{is} the necessary condition for converting the fluid energy to ~~the~~ electron energy. This conclusion is independent of the detail process of conversion.

When we ~~used~~ ^{use} the word "turbulent" in this paper,

We take its most liberal meaning. A "turbulent" jet in a non-uniform medium can be very different from those in uniform medium. For example, if the jet is very ~~close~~ dense compared to the environment, its rectilinear motion may not be affected substantially by entraining. Its ^{expansion} ~~cone~~ angle ($2dR/dz$) can be more sensitive to the pressure balance between itself and the environment, ~~and~~ rather than the lateral spreading effect of the weak turbulence. This serves as a good explanation to why expansion angles of radio jets can be very small compared to those of turbulent jets in laboratory ($\frac{\text{half width}}{\text{full width}} / \text{length} \approx \frac{0.1}{\text{length}}$)

If the radio jets are really turbulent as described above, the pressure confinement model of Chan and Henriksen (1980) and Bridle et al. (1981) needs to be revised. First, the notion of a sonic point is misleading. For the ~~entrainment~~ entrained material, even though they have to go through the process of being accelerated from subsonic to supersonic speed, the transition is not at a single location. Furthermore

the speed of the ~~jet~~^{jet} is not accelerating as in the nozzle model; instead, it is decelerating. Secondly, the effect of turbulence on the expansion angle should be included in the ~~equation for the deve~~ dynamical equations. In view of the lack of knowledge in the behavior of turbulent jets in nonuniform medium, this is not going to be ~~easy~~ a easy task.

Better ways to handle the derivatives of R and I required by Eq(7) are useful ~~to~~ for obtaining more definite informations. This problem can be approached from two sides. From the observational side, results with better resolution and less noise will improve the ~~base~~ quality of the basic data. From the technical side, employing more sophisticated technique to find the derivatives of ~~smooth~~ noisy data can also help.

Why not do all
this with NEC315
NEC6251?

Smoothed data

FIG. 1

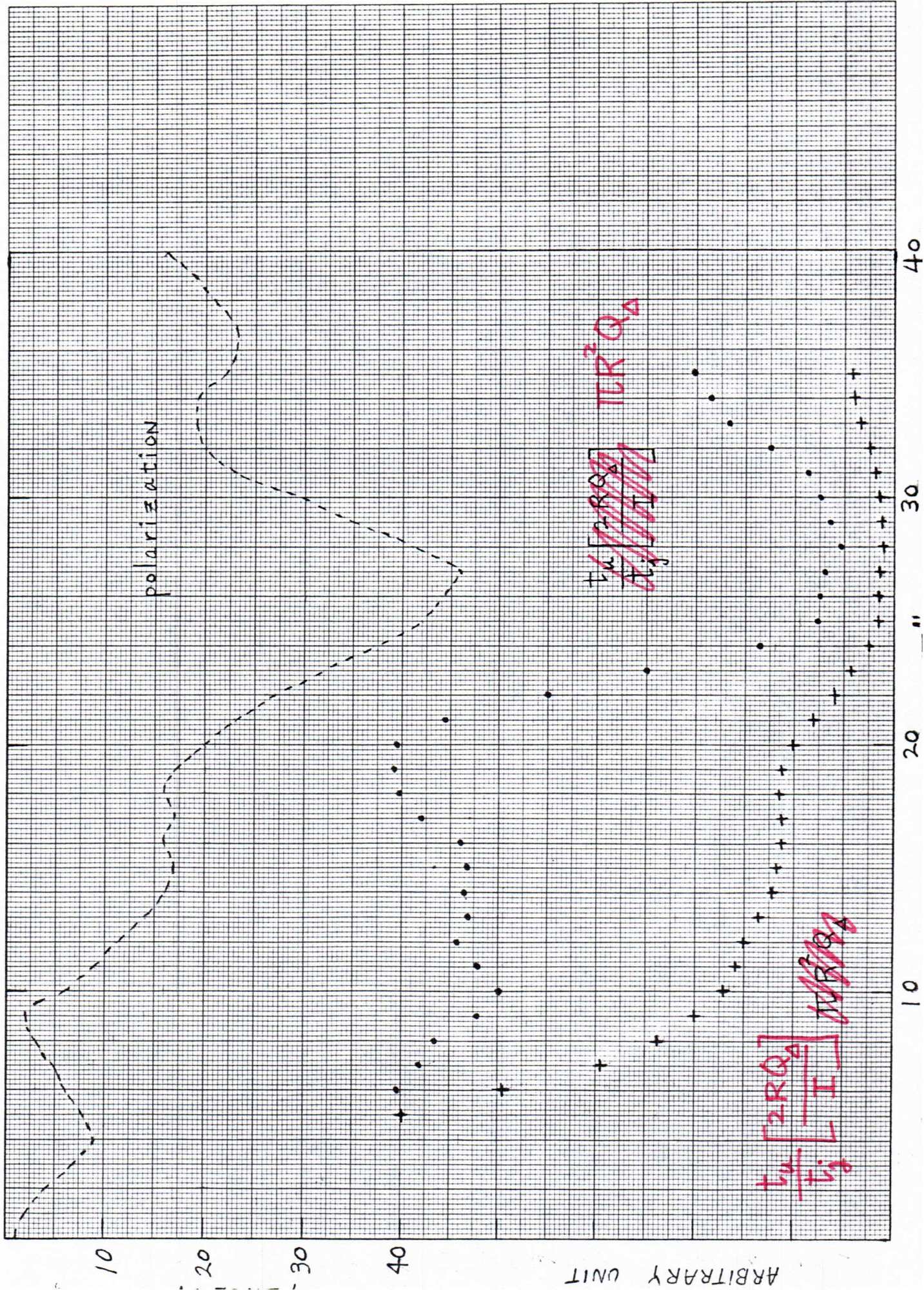


FIG. 2

