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Elementary Notes on

EINSTEIN'S THEORY OF GRAVITATION

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EINSTEIN'S LAW OF GRAVITATION

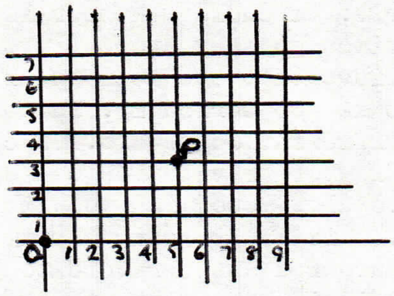
Before tackling Einstein's new law, it is as well to convince ourselves that Newton's law connot be quite right.

Newton said that between any two particles of matter there is a force which is proportional to the product of their masses and inversely proportional to the square of their distance. That is to say, ignoring for the present the question of mass if there is a certain attraction when the particles are a mile apart, there will be a quarter of this attraction when they are two miles apart, a ninth when they are three miles apart, and so on. The attraction diminishes much faster than the distance increases. Now, of course, Newton, when he spoke about distance, meant the distance at a given time: he thought there could be no ambiguity about time. But we have seen that this was a mistake. What one observer judges to be the same moment on the earth and the sun, another will kudge to be two different moments. "Distance at a given moment" is therefore a subjective conception, which can hardly enter into a cosmic law. Of course, we could make our law unambiguous by saying that we are going to use times as they are estimated by Greenwich Observatory. But we can hardly believe that the accidental circumstances of the Earth are deserving to be taken so seriously, and the estimate of distance, also, will vary for different observers. We cannot therefore allow that Newton's form of the law of gravitation can be quite correct, since it will give different results according to which of many equally legitimate conventions we adopt. This is as absurd as it would be if the question of whether one man had murdered another were to depend on whether they were described by their Christian names or their surnames. It is obvious that physical laws must be the same whether distances are measured in miles or in kilometres, and we are concerned with what is only an extension of the same principle.

Our measurements are conventional to an even greater extent than that which is admitted by the special theory of relativity. Moreover, every measurement is a physical process carried out with physical material; the result is bound to be an experimental datum, but may may not be susceptible of the simple interpretation we normally assign to it. We are therefore not going to assume to begin with that we know how to measure anything. We assume that there is a certain physical quantity called "interval", which is a relation between two events that are not widely separated - but we do not know in advance how to measure it, beyond taking for granted that it is given by some generalisation of the theorem of Pythagoras.

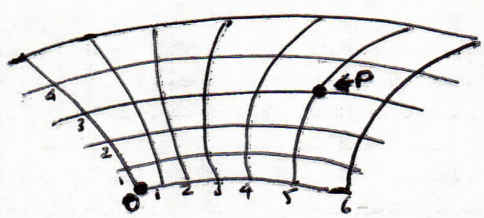
We do assume, however, that events have an ORDER, and that this order is four-dimensional. We assume, that is to say, that we know what we mean by saying that a certain event is nearer to another than a third, so that before making accurate measurements we can speak of the "neighbourhood" of an event; and we assume that, in order to assign the position of an event in space-time, four co-ordinates are necessary, but we assume nothing about the way in which these co-ordinates are assigned, except that neighbouring co-ordinates are assigned to neighbouring events.

They way in which these co-ordinates are to be assigned is neither wholly arbitrary nor a result of careful measurement - it lies in an intermediate region. While you are making any continuous journey, your co-ordinates must not alter by sudden jumps. In America one finds that the houses between 14th Street and 15th Street are likely to have numbers between 1400 and 1500, while those between 15th Street and 16th Street have numbers between 1500 and 1600, even if the 1400's were not used up. This would not do for our purposes, because there is a sudden jump from one block to the next. Or again, we might assign the time co-ordinate in the following way: take the time that elapses between two successive births of people called Smith; an event occurring between the births of the 3000th and 3001st Smith known to history shall have a co-ordinate lying between 3000 and 3001; the fractional part of its co-ordinate shall be the fraction of a year that has eleapsed since the birth of the 3000th Smith. This way of assigning time co-ordinates is perfectly definite, but is not admissible for our purposes, because there will be sudden jumps between events just before the birth of a Smith and events just after, so that in a continuous journey your time co-ordinate will not change continuously. It is assumed that, independently of measurement, we know what a continuous journey is, and when your position in space-time changes continuously, each of your four co-ordinates must change continuously. One, two, or three of them may not change at all; but whatever change does occur must be smooth, without sudden jumps. This explains what is NOT allowable in choosing co-ordinates.



To explain all the changes that are legitimate in your co-ordinates, suppose you take a large piece of soft india-rubber. While it is in an unstretched condition, measure one-tenth inch squares on it. Put in pins at the corners of the squares. We can now take as co-ordinates the number of pins passed in going to the right from a given pin to just below the pin in question, and then the number of pins we pass on the way up to this pin. In the figure, let O be the pin we start from, and P the pin to which we are going to assign co-ordinates. P is in the 5th column and the third row, so its co-ordinates in the plane of the rubber are to be 5 and 3.

Now take the rubber and stretch and twist it as much as you like. Let the pins now be in the shape they have in the second figure:



The divisions no longer represent distances according to our usual notions, but they will still do just as well as co-ordinates. We may still take P as having co-ordinates 5 and 3 in the plane of the rubber, and we may still regard the rubber as being in a plane, even though we have twisted it out of what we should normally call a plane. Such CONTINUOUS distortions do not matter.

To take another illustration: instead of using a steel measuring-rod to fix our co-ordinates, let us use a live eel, which is wriggling all the time. The distance from the head to tail of the eel is to count as 1 from the point of view of co-ordinates, whatever shape the creature may be assuming at the moment. The eel is continuous, and its wriggles are continuous, so it may be taken as our unit of distance in measuring co-ordinates. Beyond the requirement of continuity, the method of assigning co-ordinates is purely conventional, and therefore a live eel is just as good as a steel rod.

We are apt to think that, for really careful measurements it is better to use a steel rod than a live eel. This is a mistake; not because the eel tells us what the steel rod was thought to do, but because the steel rod tells us no more than the eel obviously does. The point is, not that eels are really rigid, but that steel rods really wriggle. To an observer in just one possible state of motion, the eel would appear rigid, while the steel rod would appear to wriggle just as the eel does for us. For everybody moving differently from both this observer and ourselves, both the eel and the steel rod would appear to wriggle, and there is no saying that one observer is right and the others wrong. In such matters, what is seen does not belong solely to the physical process observed, but also to the standpoint of the observer. Measurements of distances and times do not directly reveal the things measured, but relations of the things to the measurer. What observation can tell us about the physical world is therefore more abstract than we have hitherto believed.

It is important to realise that geometry, as taught in schools since Greek times, ceases to exist as a separate science, and becomes merged into physics. The two fundamental notions of elementary geometry were the straight line and the circle. What appears to you as a straight road, whose parts all exist now, may appear to another observer as the flight of a rocket, some kind of curve whose parts come into existence successively. The circle depends on measurements of distances, since it consists of all the points at a given distance from its centre, and measurement of distances, as we have seen, is a subjective affair, depending on the way in which the observer is moving. The failure of the circle to have objective reality was demonstrated by the Michelson-Morley experiment, and is thus, in a sense, the starting-point of the whole theory of relativity. Rigid bodies, which we need for measurement, are only rigid for certain observers; for others they will be obstinately changing their dimensions. It is only our earth-bound imagination that makes us suppose a geometry separate from physics to be imaginable.

That is why we do not trouble to give physical significance to our co-ordinates from the start. Formerly, the co-ordinates used in physics were supposed to be carefully measured distances; now we realise that this care at the start is thrown away. It is at a later stage that care is required. Our co-ordinates now are hardly more than a systematic way of cataloguing events. But mathematics provides, in the method of tensors, such an immensely powerful technique that we can use co-ordinates assigned in this apparently careless way just as effectively as if we had applied the whole apparatus of minutely accurate measurement in arriving at them. The advantage of being haphazard at the start is that we avoid making surreptitious physical assumptions, which we can hardly help making if we suppose that our co-ordinates have initially some particular physical significance.

We need not try to proceed in ognorance of all observed physical phenomena. We know certain things. We know that the old Newtonian physics is very nearly accurate when our co-ordinates have been chosen in a certain way. We know that the special theory of relativity is still more nearly accurate for suitable co-ordinates. From such facts we can infer certain things about our co-ordinates, which, in a logical deduction, appear as postulates of the new theory.

As such postulates we take:

1. That the interval between neighbouring events takes a general form, like that used by Riemann for distances.

2. That every body travels on a geodesic in space-time, except in so far as non-gravitational forces act upon it.

3. That a light-ray travels on a geodesic which is such that the interval between any two parts of it is zero.

Each of these postulates requires some explanation.

Our first postulate requires that, if two events are close together, but not necessarily otherwise, there is an interval between them which can be calculated from the differences between their co-ordinates. We know, because Riemann's mathematics shows it to be so, that within any amall region of space-time we can choose the co-ordinates so that the interval has exactly the special form required in the special theory of relativity. It is not necessary for the application of the special theory to a limitedregion that there should be no gravitation in the region; it is enough if the intensity of gravitation is practically the same throughout the region. This enables us to apply the special theory within any small region. How small it will have to be depends on the neighbourhood : on the surface of the Earth, it would have to be small enough for the curvature of the Earth to be negligible. In the spaces between the planets, it need only be small enough for the attraction of the sun and planets to be sensibly constant throughout the region. In the spaces between the stars, it might be enormous - say half the distance from one star to the next - without introducing measurable inaccuracies.

Thus, at a great distance from gravitating matter, we can choose our co-ordinates as to obtain a very nearly Euclidean space. This is only really another way of saying that the special theory of relativity applies. In the neighbourhood of matter, although we can make our space very nearly Euclidean in a very small region, we cannot do so through any region within which gravitation varies sensibly - at least, if we do, we shall have to abandon the view expressed in the second postulate, that bodies moving under gravitational forces only move on geodesics.

We saw that a geodesic on a surface is the shortest line that can be drawn from one point to another. For example, on the Earth, the geodesics are Great Circles. When we come to space-time, the mathematics is the same, but the verbal explanations have to be rather different. In the general theory of relativity, it is only neighbouring events that have a definite interval, independently of the route by which we travel from one to another. The interval between distant events depends on the route pursued, and has to be calculated by dividing up the route into a number of elements and adding up the intervals for the elements. If the interval is space-like, a body cannot travel from one event to the other; therefore when we are considering the way bodies move, we are confined to time-like intervals. The interval between two neighbouring events when it is time-like will appear as the time between them for an observer who travels from one event to the other, and so the whole interval between two events will be judged by a person who travels from one to another to be what his clocks show to be the time that he has taken on the journey. For some routes this time will be longer, for others shorter. The more slowly the man travels, the longer he will think he has been on the journey. This must not be taken as a platitude. I am not saying that if you travel from London to Edinburgh you will take longer if you travel more slowly - I am saying something much more odd. I am saying that if you leave London at 10 a.m. and arrive in Edinburgh at 6.30 p.m. Greenwich Time, the more slowly you travel the longer you will take - if the time is judged by your watch. This is a very different statement. From the point of view of a person on the Earth, the journey takes eight hours and a half, but if you had been a ray of light travelling round the solar system, starting from London at 10 a.m., and reflected from Jupiter and Saturn, and so on, until at last you were reflected back to Edinburgh and arrived there at exactly 6.30 p.m., you would judge that the journey had taken you exactly no time. And if you had gone by any circuitous route, which enabled you to arrive on time by travelling fast, the longer your route the less time you would judge that you had taken; the diminution of time would be continual as you approached the speed of light. Now I say that when a body travels, if it is left to itself, it chooses the route which makes the time between two stages of the journey as long as possible; if it had travelled from one event to another by any other route, the time, as measured by its own clocks, would have been shorter. This is a way of saying that bodies left to themselves do their journeys as slowly as they can; it is a sort of cosmic laziness. Its mathematical expression is that they travel on geodesics, in which the total interval between any two events on the journey is GREATER than by any alternative route. (The fact that it is greater, not less, is due to the fact that the sort of interval we are considering is more analagous to time than to distance.) For example, if a person could leave the Earth and travel about for a time and then return, the time between his departure and return would be less by his clocks than by those on the Earth; the Earth, on its journey round the Sun, chooses the route which makes the time of any bit of its course by its clocks longer than the time as judged by clocks which move by a different route. This is what is meant by saying that bodies left to themselves move in geodesics in space-time.

It is important to remember that space-time is not supposed to be Euclidean. As far as the geodesics are concerned, this has the effect that space-time is like a hilly countryside. In the neighbourhood of a piece of matter, there is, as it were, a hill in space-time; this hill grows steeper and steeper as it gets to the top, like the neck of a champagne bottle. It ends in a sheer precipice. Now by the law of cosmic laziness which we mentioned earlier, a body coming into the neighboorhood of the hill will not attempt to go straight over the top, but will go round. This is the essence of Einstein's law of gravitation. What a body does, it does BECAUSE OF THE NATURE OF SPACE-TIME IN ITS OWN NEIGHBOURHOOD, not because of some mysterious force emanating from a distant body.

An analogy will serve to make the point clear. Suppose that on a dark night a number of men with lanterns were walking in various directions across a huge plain, and suppose that in one part of the plain there was a hill with a flaring beacon on the top. Our hill is to be such as I have described, growing steeper as it goes up and ending in a precipice. I shall suppose that there are villages dotted about the plain and the men with lanterns are walking to and from the various villages. Paths have been made showing the easiest way from one village to another. These paths will all be more or less curved., to avoid going too far up the hill; they will be more sharply curved when they pass near the top of the hill than when they keep some way off it. Now supposing that you are observing all this as best you can, from a place high up in a balloon, so that you cannot see the ground, but only the lanterns and the beacon. You will not know that there is a hill, or that the beacon is at the top of it. You will see that people turn out of their straight course when they approach the beacon, and that the nearer they come the more they turn aside. You will naturally attribute this to an effect of the beacon; you may think that it is very hot and the people are afraid of getting burnt. But if you wait for daylight and see the hill, you will find that the beacon merely marks the top of the hill and does not influence the people with lanterns in any way.

Now in this analogy the beacon corresponds to the sun, the people with lanterns to the planets and comets, the paths correspond to their orbits, and the coming of daylight corresponds to the coming of Einstein. Einstein says that the sun is at the top of a hill, only the hill is in space-time, not in space. Each body, at each moment, adopts the easiest course open to it, but owing to the hill the easiest course is not a straight line. Each particle of matter is at the top of its own hill. What we call a large piece of matter is a piece which is at the top of a large hill - the hill is what we know about, the bit of matter is assumed for convenience. Perhaps there is really no need to assume it, and we could do with the hill alone, for we can never get to the top of anyone else's hill, any more than the pugnacious cock can fight the peculiarly irritating bird that he sees in the looking-glass.

I have given only a qualitative description of Einstein's law of gravitation; to give its exact mathematical formulation is impossible without more mathematics than I am permitting myself. The most interesting point is that it makes the law no longer a result of action at a distance; the sun exerts no force on the planets whatsoever. Just as geometry has become physics, so, in a sense, has physics become geometry. The law of gravitation has become the geometrical law that every body pursues the easiest course from place to place, but this course is affected by the hills and valleys that are encountered on the road.

We have been assuming that the body is acted upon only by gravitational forces. We are concerned at present with the law of gravitation, not with the effects of electromagnetic forces or the forces between sub-atomic particles. There have been many attempts to bring all these forces into the framework of general relativity, by Einstein himself, and by Weyl, Kaluza, and Klein, to mention only a few of the others, but none of these attempts has been entirely satisfactory. For the present, we may ignore this work, because the planets are not subject, as wholes, to appreciable electromagnetic or sub-atomic forces; it is only gravitation that has to be considered in accounting for their motions, with which we are concerned in this chapter.

Our third postulate, that a light-ray travels so that the interval between two parts of it is zero, has the advantage that it does not have to be stated only for small distances. If each little bit of interval is zero, then the sum of all of them is zero, and so even distant parts of the same light-ray have zero interval.The course of a light-ray is also a geodesic according to this postulate. Thus we now have two empirical ways of discovering what are geodesics in space-time, namely light-rays and bodies moving freely. Among freely-moving bodies are included all which are not subject as wholes to appreciable electromagnetic or sub-atomic forces, that is to say, the sun, stars, planets and satellites, and also falling bodies on the earth, at least when they are falling in a vacuum. When you are standing on the Earth, you are subject to electromagnetic forces. The electrons and protons in the neighbourhood of your feet exert a repulsion on your feet which is just enough to overcome the force of gravitation. This is what prevents you from falling through the earth, which is mostly empty space.

PROOFS OF EINSTEIN'S LAW

The reasons for accepting Einstein's law of gravitation are partly empirical, partly logical. We will begin with the former.

Einstein's law of gravitation gives very nearly the same results as Newton's, when applied to calculation of the orbits of the planets and their satellites. If it did not, it would not be true, since the consequences deduced from Newton's law have been found to be almost exactly verified by observation. When, in 1915, Einstein first published the law, there was only one empirical fact to which he could point to show that his theory was better than Newton's. This was what is called the motion of the perihelion of Mercury.

The planet Mercury, like the other planets, moves round the sun in an ellipse, with the sun in one of the foci. At some points of its orbit it is nearer the sun than at some other points. The point where it is nearest the Sun is called the "perihelion". Now it was found by observation that, from one occasion when Mercury is nearest the sun until the next, Mercury does not go exactly once round the sun, but a little bit more. The discrepancy is very small: it does not amount to more than an angle of forty-two seconds in a century. Since Mercury goes round the sun rather more than four hundred times a century, it must move about one-tenth of a second of angle from one perihelion to the next. This very minute discrepancy from Newton had puzzled astronomers. There was a calculated effect due to perturbations caused by other planets, but this small discrepancy was the residue after allowing for these perturbations. Einstein's theory accounted exactly for this residue. There is a similar effect for the other planets, but it is much smaller and more difficult to observe. Since Einstein published his new law, the effect has been observed for the earth, and with a fair degree of certainty for Mars. This perihelion effect was, at first, Einstein's only empirical advantage over Newton.

His second success was more sensational. According to orthodox opinion, light in a vacuum ought always to travel in straight lines. Not being composed of material particles, it ought to be unaffected by gravitation. However, it was possible without any serious breach with old ideas to admit that, in passing near the sun, light might be deflected out of the straight path as much as if it were composed of material particles. Einstein, however, maintained, as a deduction from his law of gravitation, that light should be deflected twice as much as this. That is to say, if the light of a star passed very near the sun, Einstein maintained that the ray from the star would be turned through an angle of just under $1\frac{3}{4}$ sec. His opponents were willing to concede half this amount. Now it is not every day that a star just in line with the sun can be seen. This is only possible during a total eclipse, and not always then, because there may not be any bright stars in just the right position. Eddington points out that, from this point of view, the best day of the year is May 29, because then there are a number of bright stars close to the sun. It happened by incredibly good fortune that there was a total eclipse of the sun on May 29, 1919. Two British expeditions photographed the stars near the sun during the eclipse, and the results confirmed Einstein's prediction. Some astronomers who remained doubtful whether sufficient precautions had been taken to ensure accuracy were convinced when their own observations of a subsequent eclipse gave exactly the same result.

Results of observations at subsequent eclipses have again confirmed Einstein's estimate, which is therefore now almost universally accepted.

The third experimental test is on the whole favourable to Einstein, but the quantities concerned are so small that it is only just possible to measure them, and the result is therefore not decisive. Einstein deduces from his law of gravitation that any periodic process which takes place in an atom in the sun (whose gravitation is very intense) must, as measured by our clocks, take place at a slightly slower rate than it would in a similar atom on the earth. The "interval" involved will be the same in the sun and on the earth, but the same interval in different regions does not correspond to exactly the same t time. This is due to the "hilly" character of space-time which constitutes gravitation. Consequently, any given line in the spectrum ought, when the light comes from the sun, to seem to us a little nearer the red end of the spectrum than if the light came from a source on the earth. The effect is expected to be very small - so small that there is still uncertainty as to whether it exists or not. Einstein's theory predicts a similar effect for every star, but the technical difficulties of measuring it are so great that after forty years of observations we cannot be sure that it exists.

No other measurable differences between the consequences of Einstein's law and those of Newton's have hitherto been discovered, at least so far as the solar system is concerned, but the above experimental tests are quite sufficient to convince astronomers that, where Newton and Einstein differ as to the motions of the heavenly bodies, it is Einstein's law that gives the right results. Even if the empirical grounds for Einstein's law stood alone, they would be conclusive. Whether his law represents the exact truth or not, it is certainly more nearly exact than Newton's, though the inaccuracies in Newton's were all exceedingly minute.

But the considerations which originally led Einstein to his law were not of this detailed kind. Even the consequence about the perihelion of Mercury, which could be verified at once from previous observations, could only be deduced after the theory was complete, and could not form any part of the original grounds for inventing such a theory. These grounds were of a more abstract logical character. I do not mean that they were not based upon observed facts, and I do not mean that they were a priori fantasies such as philosophers indulged in previously. What I mean is that they were derived from certain general characteristics of physical experience, which showed that Newton MUST be wrong and that something like Einstein's MUST be right.

The arguments in favour of the relativity of motion are quite conclusive. In daily life, when we say that something moves, we mean that it moves relative to the earth. In dealings with the motions of the planets, we consider them as moving relative to the sun, or to the centre of mass of the solar system. When we say that the solar system itself is moving, we mean that it is moving relative to the stars. There is no physical occurence which can be called 'absolute motion'; consequently, the laws of physics must be concerned with relative motions, since these are the only kind which occur.

We now take the relativity of motion in conjunction with the experimental fact that the velocity of light is the same relatively to one body as relatively to another, however the two may be moving. This leads us to the relativity of distances and times. This in turn shows that there is no objective physical fact which can be called "the distance between two bodies at a given time", since both the time and the distance will depend on the observer. Therefore Newton's law of gravitation is logically untenable, as it makes use of "distance at a given time".

This shows that we cannot rest content with Newton, but it does not show us what we are to put in his place. Here several considerations enter in. We have in the first place what is called the equality of gravitational and inertial mass. What this means is as follows: when you apply a given force (Although force is no longer one of the fundamental concepts of dynamics, it may still be employed as a convenient way of speaking, like "sunrise" and "sunset") to a heavy body, you do not give it as much acceleration as you do a light body. What is called the "inertial" mass of a body is measured by the amount of force required to produce a given acceleration. At a given point of the Earth's surface, the "mass" is proportional to the "weight". What is measured by scales is rather the mass than the weight: the weight is defined as the force with which the earth attracts the body. Nowthis force is greater at the poles than at the equator, because at the equator the Earth produces a "centrifugal force" which partly counteracts gravitation. The force of the earth's attraction is also greater on the surface of the earth than it is at a great height or at the bottom of a very deep mine. None of these variations are shown by scales, because they affect the weights used just as much as the body weighed; but they are shown if we use a spring balance. The mass does not vary in the course of these changes of weight.

The "gravitational" mass is differently defined. It is capable of two meanings. We may mean (1) the way a body responds in a situation where the gravtiational field has a known intensity, for example, on the surface of the earth, or on the surface of the sun; or (2), the intensity of the gravitational force produced by the body, as, for example, the sun produces stronger gravitational forces than the earth does. Newton says that the force of gravitation between two bodies is the product of their masses. Now let us consider the attraction of different bodies to one and the same body, say, the sun. Then different bodies are attracted by forces which are proportional to their masses, and therefore produce exactly the same acceleration in all of them. Thus, if we mean gravitational mass in sense (1), that is to say, theway a body responds to gravitation, we find that "the equality of gravitational and inertial mass" reduces to this: in a given gravitational situation, all bodies behave exactly alike. As regards the surface of the earth, this was one of the first discoveries of Galileo. Aristotle thought that heavy bodies fall faster than lighter ones; Galileo showed that this is not the case, when the resistance of air is eliminated. In a vacuum, a feather falls as fast as a lump of lead. As regards the planets, Newton established the corresponding facts. At a given distance from the sun, a comet, which has a very small mass, experiences exactly the same acceleration towards the sun as a planet experiences at the same distance. Thus the way in which gravitation affects a body depends only upon where the body is, and is in no degree dependent upon the nature of the body. This suggests that the gravitational effect is a characteristic of the locality, which is what Einstein makes it.

As for the gravitational mass in sense (2), i.e. the intensity of the force produced by a body, this is no longer EXACTLY proportional to its inertial mass. The question involves some rather complicated mathematics and I shall not go into it.

We have another indidcation as to what sort of thing the law of gravitatio MUST be, if it is to be a characteristic of a neighbourhood, as we have reason to suppose it is. It must beexpressed in some law which is unchanged when we adopt a different system of co-ordinates. We saw that we must not, to begin with, regard our co-ordinatesas having any physical significance: they are merely systematic ways of naming different parts of space-time. Being conventional, they cannot enter into physical laws. That means to say that, if we have expressed a law correctly in terms of one set of co-ordinates, it must be expressed by the same formula in terms of another set of co-ordinates. Or, more exactly, it must be possible to find a formula which expresses the law, and which is unchanged by change of co-ordinates. It is the business of the theory of tensors to deal with these formulae, and the theory of tensors shows that there os one formula which obviously suggests itself as being possibly the law of gravitation. When this possibility is examined, it is found to give the right results; it is here that the empirical confirmations come in. But if Einstein's law had not been found to agree with experience, we could not have gone back to Newton's law. We should have been compelled by logic to seek some new law expressed in terms of tensors, and therefore independent of our choice of co-ordinates. It is impossible without mathematics to explain the theory of tensors; the non-mathematician must be content to know that it is the technical method by which we eliminate the conventional element from our measurements and laws, and thus arrive at the physical laws which are independent of the observer's point of view. Of this method, Einstein's law of gravitation is the most splendid example.