

Physics 040 - Universal Gravitation

Application of Newton's laws to orbits.

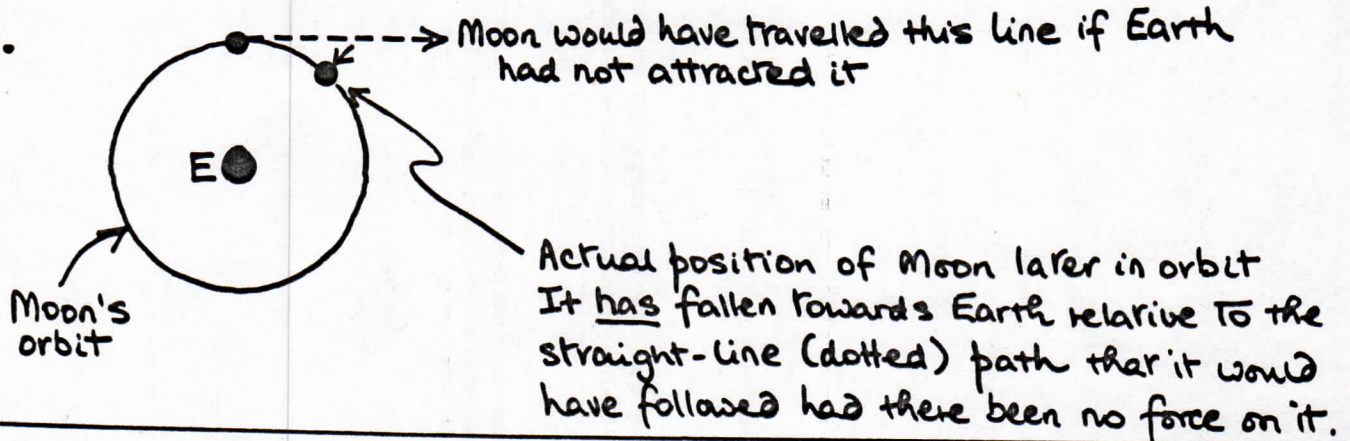
1.



If Earth exerted no force on Moon, Moon would fly off in a straight line. (LAW #1)

EARTH MUST EXERT ATTRACTIVE FORCE ON MOON TO MAKE MOON FALL TOWARDS EARTH.

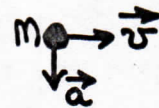
2.



3.

In every short section of orbit Moon has this motion:

velocity \vec{v} tangent to orbit
acceleration \vec{a} towards Earth



4.

Knowing size of Moon's orbit and time taken to complete it, Newton calculated acceleration of Moon towards earth. Compared with observed acceleration of bodies falling under gravity at Earth's surface, and found that

a falls off as $1/r^2$ (r = distance from Earth's centre)

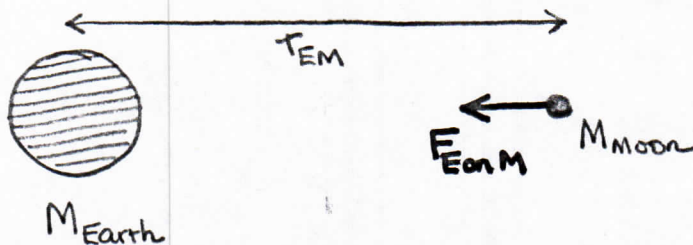
5.

Conclusion: Earth gravity acts on all bodies near it (heavenly or terrestrial) giving them an acceleration a that falls off with distance from earth centre r

6. Mathematically, this means that the relation between a and r is:

$$a = \frac{\text{CONSTANT FOR EARTH}}{r^2}$$

i.e. $a = \frac{C_{\text{EARTH}}}{r^2}$

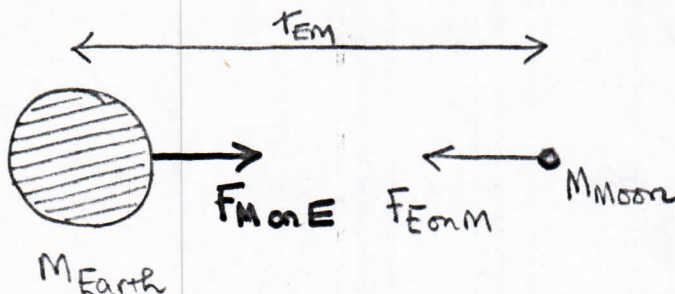


7. By Newton's second law, earth force on moon must be

$$\vec{F}_{E \text{ on } M} = M_{\text{Moon}} \times \vec{a}_{\text{Moon}}$$

i.e. $\vec{F}_{E \text{ on } M} = \frac{M_{\text{Moon}} \times C_{\text{Earth}}}{r_{EM}^2}$ **towards Earth**

8. Continue to suppose no distinction between "heavenly" and "earthly". Then must expect Moon to exert equal and opposite force on Earth.



9. Suppose force that Moon exerts on Earth follows same rules as force that Earth exerts on Moon:

i.e. $\vec{F}_{M \text{ on } E} = \frac{M_{\text{EARTH}} \times C_{\text{MOON}}}{r_{EM}^2}$ **Towards Moon**

Then if $F_{E \text{ on } M} = F_{M \text{ on } E}$ (from step # 8)

$$\frac{M_{\text{Moon}} \times C_{\text{Earth}}}{r_{EM}^2} = \frac{M_{\text{Earth}} \times C_{\text{Moon}}}{r_{EM}^2}$$

i.e. $M_{\text{Moon}} \times C_{\text{Earth}} = M_{\text{Earth}} \times C_{\text{Moon}}$

(r_{EM}^2 is same for both sides - cancels)

which means

$$\frac{C_{\text{Earth}}}{M_{\text{Earth}}} = \frac{C_{\text{Moon}}}{M_{\text{Moon}}}$$

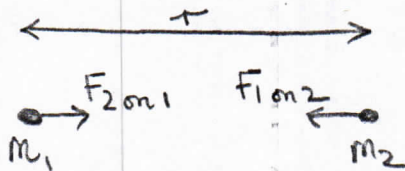
Earth parameters only

Moon parameters only

10. Newton supposed that $\frac{C_{\text{object}}}{M_{\text{object}}}$ is a universal constant G , characteristic of gravity itself, not of particular object.

Then $C_{\text{Earth}} = G \times M_{\text{Earth}}$, $C_{\text{Moon}} = G \times M_{\text{Moon}}$, etc.

Universal Gravitation



$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = \frac{G \times m_1 \times m_2}{r^2}$$

and both forces act along the line joining m_1 to m_2 , equal and opposite.

Newton showed that, with these laws:

1. Most general orbit of small mass m around big mass M is an ellipse with M at one focus.
2. If forces act on line joining m to M , then that line sweeps out equal areas in equal times.
3. Also could prove Kepler's Third Law ($\text{TIME}^2 \propto (\text{ORBIT SIZE})^3$)

Kepler's Empirical "Laws" all followed from Newton's one law of Universal Gravitation. Solar System motions explained if ALL MATTER GRAVITATES.

*Calculus was needed to prove these conclusions followed from the law of Universal Gravitation.

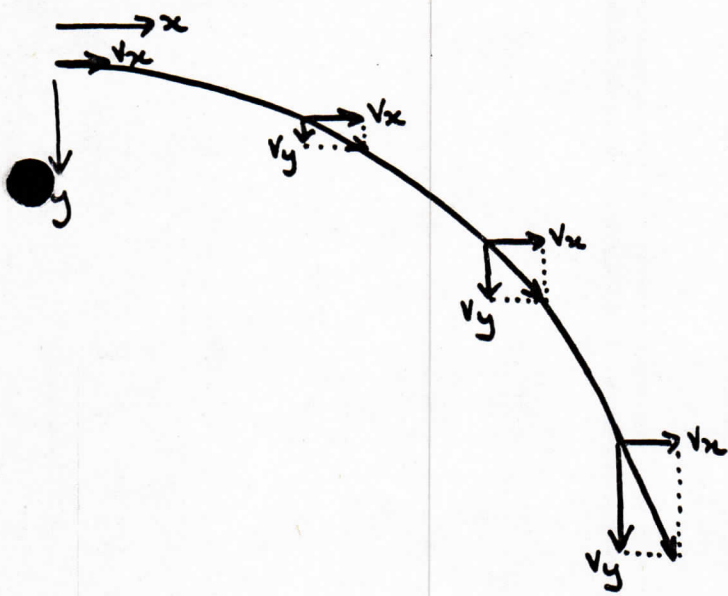
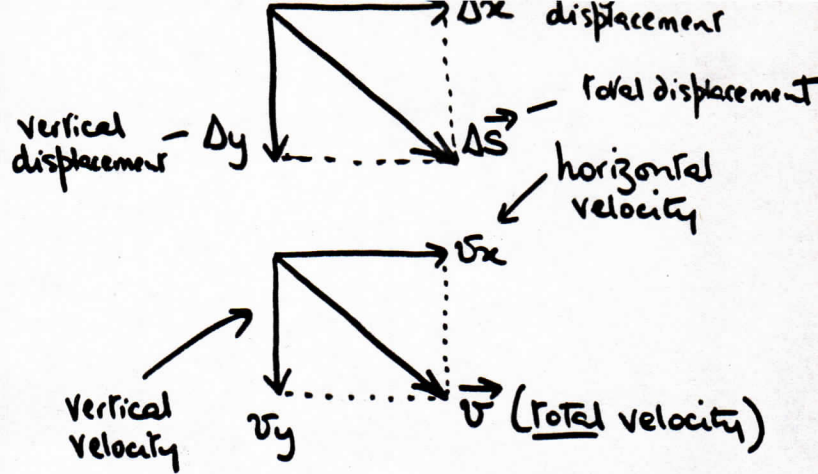


Figure 2a



2b

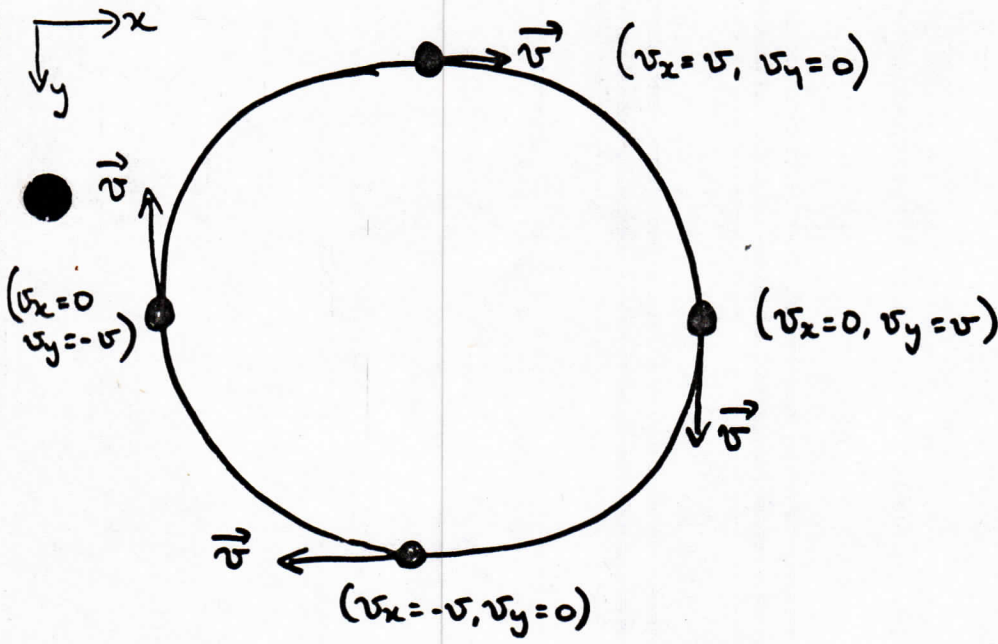


Fig 3

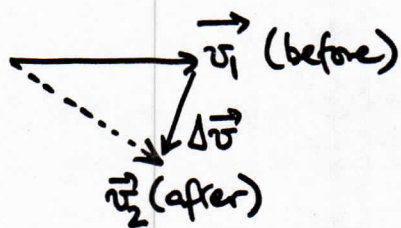


Fig. 4



Fig 5