The Integrated Extragalactic Radio Emission and its Relevance to Cosmological Models.

Introduction

The extragalactic radio emission integrated along a line of sight in the Universe is clearly a quantity of cosmological significance, for the measured noise power must contain contributions from the most distant regions accessible to observation. Knowledge of the magnitude of the integrated emission tells us little directly about the nature of these distant regions however, and it must be combined with other observational evidence (in particular the counts of radio sources at different flux levels) before any detailed conclusions may be drawn. The purpose of this account will be to investigate how different cosmological models predict different values for the integrated emission, and in particular to try to select a model which might be fitted to the existing observational data of radio astronomy.

The observational data will be taken as that presented by Ryle and Clarke (::1), Ryle and Neville (::2), and Turtle et al. (::3). The value for the "brightness temperature" of the integrated emission at 178 Mc/s will be taken as $25^{\circ} - 30^{\circ}K$, from reference (::3) ; preliminary results of an experiment designed to measure this quantity directly show no reason to suspect that the value lies outside this range.

A crude (but valuable) conclusion may be drawn immediately from the observation that the radio background is finite in intensity, however. It is easy to see that this implies that the Universe is "effectively" finite. By this is meant that either the distant regions of the Universe emit convergingly less radio noise the further they are from the Galaxy (so that eventually their emission becomes so little that they cannot be detected by a given radio telescope), or that the radiation they emit is systematically degraded by an amount depending in some manner on their distance away from us. In either case, the integration of the received power from all spherical shells out to infinity is made to converge ; if there were no effects of this kind in the Universe, successive spherical shells would contribute equal intensities of radio noise at our receivers and there would be no convergence. We should in fact be detecting a radio background of unimaginable intensity. This observation was first made in relation to the darkness of the night sky by Olbers in the Nineteenth Century - the condition of release from the dilemma of the infinite background is that either the Universe is young, or it is expanding, or both. It will appear below that these conditions provide a solution to the problem even in the curved spaces of modern cosmology ; the point is made here to show that the finite integrated emission commits us either to evolutionary cosmology or an expanding Universe with its associated red-shifting of photons, or both.

It may be noted that the possibility of the existence of absorbing material in intergalactic space does not affect this argument, for if such material had been allowed time to come to equilibrium with the radiation falling on it, it would re-emit as much as it absorbed and the dilemma would remain.

To proceed beyond this elementary point we shall need the apparatus of relativistic cosmology ; the necessary techniques and formulae will be developed here, but questions of principle will largely be passed over, except where they involve the interpretation of the observational data.

Attention will be largely confined to the Steady-State model of Bondi, Gold and Hoyle and to the Einstein - de Sitter model.

Model-independent formula for integrated emission.

We adopt the general model in which the Universe is represented by a homogeneous, isotropic fluid partaking of the general expansion regarded as established by the work of Hubble. The observable large-scale aggregations of matter are to be dismissed as statistically insignificant fluctuations about the mean represented by the fluid. We shall assign properties to the fluid derived from observation of these aggregations (radio sources, galaxies, etc.) without consideration of the validity of this approach. The radio-emissive properties of the Universe will be represented by an emissivity E, which will characterize the radio power in watts/hz/pc^3/ster/unit proper time emanating from the fluid. To take into account the actual mechanism of the radio emission found in the Universe, we shall make E a function of the frequency of emission ν_e, and to include possible evolutionary effects we shall make it a function of the proper time of emission, t_e. In the Steady-State model, E will be statistically independent of t_e. We adopt further a "co-moving" co-ordinate system in which the co-ordinates r, θ, ϕ of any particle moving with the fluid are constant, and the Robertson-Walker line element for a homogeneous isotropic model in the form (::4)

$$ds^2 = c^2dt^2 - R^2(t)\left\{\frac{dr^2 + r^2 d\Omega^2}{(1+kr^2/4)^2}\right\}, \quad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2 \qquad (1)$$

Here R(t) is a real function describing the expansion of the Universe, and k is a constant equal to 0 or ∓1 according as space is flat, spherical (closed, finite) or hyperbolic (open, infinite). This form of line element is common to all the cosmological models to be considered, these models being characterised initially by the form they assume for R(t) and the value taken for k, which will usually be 0.

Firstly, we calculate the radio flux received by an observer O in an interval of proper time (t_o, $t_o + dt_o$) from a spherical shell around him of radius r, so that the radiation was emitted in an interval of proper time (t_e, $t_e + dt_e$). O will receive it in a bandwidth $d\nu_o$ centred on frequency ν_o, so it will have been emitted in bandwidth $d\nu_e$ centred on ν_e, where $\nu_e/\nu_o = 1 + z$, the red-shift relation. The physical process of emission will be included in so far as we assign a "spectral index" of β to the emission, so that

$$E(\nu_1, t) = \left(\frac{\nu_2}{\nu_1}\right)^{\beta} E(\nu_2, t) \qquad (2)$$

The assumption of a simple power law of this kind is broadly in keeping with observation (::4), and the best value for β is 0.7 ∓ 0.1 .

The energy emitted in the interval of proper time dt_e from an element of unit volume, in bandwidth $d\nu_e$ and over all solid angles is therefore

$$\mathcal{E} = 4\pi E(\nu_e, t_e)\, d\nu_e\, dt_e \tag{3}$$

If this energy now spreads out over an expanding spherical shell, and some is detected by an observer at proper time t_o, the area of the shell as measured by the observer is

$$A = \frac{4\pi r^2 R^2(t_0)}{(1 + kr^2/4)^2} \qquad (r = \text{radius of shell}) \tag{4}$$

The energy is however degraded by the cosmological red-shift, by a factor of $(1 + z)^2$. To show this, suppose that the source emits a photon of energy $h\nu_e$ every dt_e seconds. In an interval of proper time T_e it emits $n = T_e/dt_e$ photons, and the emitted power is $P_e = nh\nu_e/T_e$. These n photons arrive at the observer with energies $h\nu_o$ in a total time T_o. The received power is $nh\nu_o/T_o = (h\nu_o/h\nu_e).(T_e/T_o).P_e = P_e/(1+z)^2$

$$P_0 = \frac{1}{(1+z)^2} P_e \tag{5}$$

Therefore the received power/unit area/unit proper time/unit solid angle from unit volume of a spherical shell of radius r will be

$$P_r(\nu_0, t_0) = S_r(\nu_0, t_0)\, d\nu_0 = \frac{1}{4\pi} E(\nu_e, t_e)\, d\nu_e \cdot \frac{1}{(1+z)^2} \frac{(1+kr^2/4)^2}{r^2 R^2(t_0)}$$
$$= \frac{1}{4\pi} \frac{1}{(1+z)^\beta} E(\nu_0, t_e).(1+z)\, d\nu_0 . \frac{1}{(1+z)^2} \cdot \frac{(1+kr^2/4)^2}{r^2 R^2(t_0)}$$

$$S_r(\nu_0, t_0) = \frac{1}{(1+z)^{1+\beta}} E(\nu_0, t_e) \frac{(1+kr^2/4)^2}{4\pi r^2 R^2(t_0)} \tag{6}$$

The quantity S is the radio flux density received from unit volume of such a spherical shell. To calculate the flux density due to the whole shell we make use of the homogeneity of the world model taken, so that the geometry of the shell centred on the emitting region is identical with that of the shell of the same radius centred on the observer, as measured by the observer. The element of proper volume dV_e is given by

$$dV_e = \frac{4\pi r^2 dr\, R^3(t_e)}{(1 + kr^2/4)^3} \tag{7}$$

and the total flux density received from the shell is therefore

$$S_r(\nu_0, t_0) = \frac{1}{(1+z)^{1+\beta}} E(\nu_0, t_e) \frac{R^3(t_e)}{R^2(t_0)} \frac{dr}{(1+kr^2/4)}$$

To obtain the total integrated emission from the Universe, we must integrate this expression from r = 0 to a suitable distance. It will be convenient to transform the integral into one over the proper time co-ordinates of the emitters, however, and to do this we relate r to t_e through the equation of a light-path, which in this formalism is that of a null-geodesic, ds = 0, i.e.

$$c\,dt = \frac{R(t)\, dr}{(1 + kr^2/4)} \tag{8}$$

Therefore $$\frac{dt}{(1+kt^2/4)} = \frac{c\, d\tau_e}{R(t_e)}$$

and we may write the flux density due to the whole Universe of distant radio sources, in the standard units as the integral over proper time of emission :

$$S(\nu_0, t_0) = \int^{t_0} \frac{1}{(1+z)^{\beta+1}} E(\nu_0, t_e) \frac{R^2(t_e)}{R^2(t_0)} c\, d\tau_e \tag{9}$$

It is a standard result of relativistic cosmology that the cosmological red shift is given by the expression

$$\frac{1}{1+z} = \frac{R(t_e)}{R(t_0)} \tag{10}$$

so that the result can be expressed in the following rather simple form

$$S(\nu_0, t_0) = \int_T^{t_0} c\, E(\nu_0, t_e) \left[\frac{R(t_e)}{R(t_0)}\right]^{3+\beta} d\tau_e \tag{11}$$

The time T, which represents the earliest time back to which we carry the integration, is most plausibly assigned as the most recent epoch for which R(T) = 0. When this is done, we integrate back to the last singular state of the Universe and not beyond that. For the steady-state model, in which $R(t) = e^{Ht}$, T is negative infinity, whereas in the Einstein-de Sitter models with $R(t) = t^p$, p positive, we must take T = 0. A rigorous discussion at this point would need to examine carefully the precise significance to be attached to the condition R(t) = 0 ; if this is to be interpreted, at least vaguely, as "The Creation" one clearly cannot integrate past it meaningfully. If however one regards it as a phase through which the Universe has passed from one or more extended periods of contraction, then the integral will diverge in most models so far postulated, for both conditions of Olbers' Paradox, cosmic youth and cosmic expansion, fail. Here the view will be taken that R(t) = 0 does indicate a state beyond which we are not to integrate, but that the vanishing of the metric is to be regarded not as a sort of super-scientific phenomenon of Creation, but as a pointer to the inadequacy of our theory to cope with the world as it was in the close neighbourhood of R(t) = 0. It will be apparent that if we can extend the integral back to times for which $T \ll t_0$, the integral will converge rapidly, and so we can regard the carrying of it through to R(t) = 0 as a sort of asymptotic approximation.

The formula (11) is significantly independent of k, the constant in the metric (1) which specified the curvature of space-time. Accordingly, the integrated emission is determined solely by the expansion function R(t) and not the geometry, which is a somewhat surprising result ; the generality of Olbers' resolution of the background-intensity paradox becomes apparent. Whitrow and Yallop (::4) have in fact considered the problem of a closed Universe in which radiation can "re-circulate" many times in detail and shown that formula (11) is still applicable there.

Of the other quantities appearing in (11), $E(t_e)$ will present the greatest difficulty, except in the steady-state theory, in which it is statistically constant. The value of $E(t_0)$ will be a fundamental datum of both the steady-state and evolutionary-type calculations, however, and even this will subject to considerable uncertainty, owing to the great difficulty in obtaining luminosity data for radio sources. In the simplest form of approach available to us, we can assume that there is only one type of radio source, characterised by a spatial density $\rho(t_o)$, or ρ_o, and an absolute luminosity $P(t_o)$, or P_o, both at the present cosmic time t_o. Then the quantity $E(t_o)$, or E_o, $= \rho_o P_o$ simply. In fact a great dispersion in the absolute luminosities of radio sources is believed to exist (::1, ::5), and so ρ_o and P_o can only be taken as suitably weighted means of the observed distributions. It will be seen that this simple approach is probably good enough as far the integrated emission is concerned, but that a more realistic representation of the actual properties of sources is essential for an account of the number - flux density relationship in a given cosmology.

At this stage, we shall anticipate later discussion a little and mention that source-count data (::1) provide an estimate for the quantity $\rho_o P_o^{3/2}$ as 1.4×10^{14} with ρ_o in sources/pc^3 and P_o in watts/hz/ster at 178 Mc/s, which will be taken as the standard frequency of observation ρ_o throughout all the subsequent discussion. In order to know ρ_o and P_o separately we need to determine another quantity involving one or both in a different combination. It is the usual practice in cosmological discussion of radio astronomical results to use the value of the integrated emission due to observation, together with an assumed cosmology, as providing an estimate of $\rho_o P_o$. Here this practice will be departed from in order to present the integrated emission in the role of a criterion for the models. The mean luminosity of the sources, which is the best number for P_o alone, can be estimated if the distribution of luminosity is known over a sufficiently large sample not afflicted with appreciable selection effects. There is a great deal to be said about this, but it seems reasonable at present to accept a value for P_o alone from all possible sources of information about the absolute luminosity of the radio emitters of 8×10^{25} watts/hz/ster at 178 Mc/s. The luminosity distributions to be derived from different methods, e.g. optical identifications and angular diameters, are subject to mutual discrepancies, and the methods of obtaining them inevitably involve selection effects. With this in mind, we shall note here that P_o may be regarded as uncertain perhaps to within a power of ten. Combining the above value for P_o with that of $\rho_o P_o^{3/2}$ we can derive a value of $\rho_o P_o = E_o = 25$ watts/hz/ster pc^3. The uncertainty in P_o then represents apossible variation of E_o in the range 8 - 75 watts/hz/pc^3.

<u>Steady-State theory</u>

Using the fact that E is independent of t, and that $R(t) = e^{Ht}$, performance of the integration in (11) gives the result

$$S(178, t_0) = \frac{cE_0}{(3+\beta)H} = \frac{1}{3+\beta} \cdot cE_0/H \qquad (12)$$

Converting to an equivalent brightness temperature through $T_b = \lambda^2 S/2k$, where $\lambda = 170$ cm., $k = 1.4 \times 10^{-23}$ cgs, and assuming the value 3×10^{17} secs for the Hubble time, the factor cE_o/H goes over to a temperature of 8^oK, so that putting $\beta = 0.7$ in the above gives $T_b(e-g) = 0.27\ cE_o/H$, or 2.2^oK. The lower limit on P_o would only permit raising this to about 7^oK.

The magnitude of the integrated emission from a steady-state model, evaluated for any values of the parameters ρ_0 and P_0 in the permissible range, is therefore too small, being deficient by a power of ten for the most probable values. We must therefore conclude that there is little hope of fitting the observational data with a steady-state cosmology : this is of course the conclusion of Ryle and Clarke (::1), who adopted the approach of choosing ρ_0 and P_0 to fit the observed values of $\rho_0 P_0^{3/2}$ and the integrated emission, and then showed that the N(S) relationship predicted by the theory was in conflict with experiment. A qualitative disagreement between the steady-state model and the observed N(S) relation is inescapable, for in the absence of evolutionary effects the steady-state model cannot provide a rising N(S) curve at all. Taking the integrated emission as a cosmological criterion therefore verifies the conclusions of Ryle and Clarke, the extent of disagreement depending on the estimate made of P_0. Even with P_0 as low as 4×10^{24} watts/hz/ster at 178 Mc/s, there is still a factor of four between the model and observation, and so this method of criticism seems to afford an equally unfavourable view of the steady-state theory.

Evolutionary Cosmology - General Properties

We shall henceforth be concerned with evolutionary models of the Universe, and especially those for which the emissivity is a function of the cosmic moment of emission t_e. The discussion is now of course more complicated than for the steady-state model, and an element of arbitrariness arises as we do not know enough about the detailed mechanism (or mechanisms) of emission of the radio sources to assign a functional dependence E(t) from observational astrophysics. The method of attack will be to introduce a general form for E(t) involving arbitrary parameters, and then to attempt to narrow the acceptable range of these parameters by demanding some form of agreement with data such as the N(S) curve. The integrated emission from the narrowed ranges of models will then be evaluated, and a picture of the general types of evolutionary model which will withstand observational criticism thereby obtained.

In order to investigate what are the effects of evolution on the theory in themselves, an undispersed model will first be considered , i.e. one in which we consider all radio sources to have the same luminosity, P_0, which will correspond fairly closely with the P_0 mean luminosity used above, and which will be taken as 4×10^{25} l.u. henceforth (l.u. are "luminosity units", viz. w/hz/ster at 178 Mc/s). The further modifications made by inserting dispersion of the luminosity (which will be seen to be essential) will be considered later.

We make the assumption that the number of galaxies in the Universe is statistically constant for epochs t sufficiently removed from a state R(t) = 0. Then the number density of galaxies, ρ_g, is a function of time $\rho_g(t) \propto R(t)^{-3}$ owing to the expansion of the universe. It seems reasonable to suppose that the time-dependence of the number density of radio sources is essentially similar : if the source mechanism were to be largely the conversion of kinetic energy of galactic objects in collisions the number density of radio sources might be proportional to the square of the number density of galaxies, but as the modern view is that such a mechanism is unlikely to account for more than a small fraction of the total radio emission from the universe this case will not be considered explicitly. We shall let the number density of radio sources be of the form

$$\rho(t) \propto R(t)^{-3\phi} \tag{13}$$

where ϕ is a parameter probably very close to unity. We suppose also that the source power decreases as cosmic time advances, so that sources of energy are on average used up, and put

$$P(t) \propto R(t)^{-\theta} \tag{14}$$

where θ is a similarly arbitrary parameter. This form is taken primarily for mathematical convenience in what follows.

Substitution for $E(t)$ as $\rho(t)P(t)$ in (11) then gives

$$S(\nu_0, t_0) = cE(\nu_0, t_0)\int_T^{t_0}\left[\frac{R(t_e)}{R(t_0)}\right]^{3+\beta-3\phi-\theta} dt_e \tag{15}$$

With the de Sitter metric, $R(t) = e^{Ht}$, this gives

$$S(\nu_0, t_0) = cE(\nu_0, t_0)\int_{-\infty}^{t_0} e^{\mu H(t_e - t_0)}\, dt_e = \frac{1}{\mu}\cdot\frac{cE_0}{H} \tag{16}$$

where the substitution $\mu = 3 + \beta - 3\phi - \theta$ is made.

With the Einstein - de Sitter metric, $R(t) = t^p$, we find

$$S(\nu_0, t_0) = cE(\nu_0, t_0)\int_0^{t_0}\left(t_e/t_0\right)^{\mu p} dt_e = \frac{1}{\mu p + 1}\, cE_0 t_0 \tag{17}$$

The basic problem is now to assign a value to μ, through β, ϕ and θ. β we have from observation as 0.7 ∓ 0.1 as before ; a constraint on ϕ and θ can be established by the argument of Davidson (::6). By an approach similar to that used in deriving (11) we may deduce that the flux received from a source of luminosity $P(\nu, t)$ at a co-ordinate distance r is

$$S(\nu_0, t_0) = P(\nu_0, t_0)\,\frac{R(t_e)^{1+\beta}}{R(t_0)^{3+\beta}}\,\frac{(1 + kr^2/4)^2}{r^2} \tag{18}$$

where r and t_e are related through (8), the equation to a null-geodesic.

This may be expressed as a series expansion in the red-shift parameter z (::7)

$$S(\nu_0, t_0) = \frac{P(\nu_0, t_0)}{c^2 z^2}\,\frac{\dot{R}(t_0)^2}{R(t_0)^2}\left\{1 - \left[\frac{\ddot{R}(t_0)R(t_0)}{\dot{R}(t_0)^2} + \beta - \frac{R(t_0)}{\dot{R}(t_0)}\,\frac{\dot{P}(\nu_0, t_0)}{P(\nu_0, t_0)}\right]z + O(z^2)\right\} \tag{19}$$

Similarly, the number of sources with co-ordinate less than a given r, $N(<r)$ is

$$N(<r) = 4\pi\int_0^r \frac{\rho(t_e)R^3(t_e)\, r^2\, dr}{(1 + kr^2/4)^3} \tag{20}$$

with r and t_e again related through (8). We can then expand N as a series in z (::7) :

$$N(<z) = \frac{4}{3}\pi\rho(t_0)c^3 z^3\,\frac{R^3(t_0)}{\dot{R}^3(t_0)}\left\{1 - \frac{3}{4}\left[\frac{2\ddot{R}(t_0)R(t_0)}{\dot{R}^2(t_0)} - \frac{R(t_0)}{\dot{R}(t_0)}\,\frac{\dot{\rho}(t_0)}{\rho(t_0)} - 5\right]z + O(z^2)\right\} \tag{21}$$

It is then possible to eliminate z to give N as a series in S, thus :

$$N(>S) = \frac{4}{3}\pi\rho_0 P_0^{3/2} S^{-3/2}\left\{1 - \frac{3}{4}\left[5 + 2\beta + \frac{R(t_0)}{\dot R(t_0)}\frac{\dot\rho(t_0)}{\rho(t_0)} + \frac{2R(t_0)}{\dot R(t_0)}\frac{\dot P}{P}\right]\frac{\dot R(t_0)}{R(t_0)} P^{1/2} S^{-1/2} + O(S^{-1})\right\} \qquad (22)$$

The first term gives N(>S) depending on $(P/S)^{3/2}$, which is the simple result for a static Euclidean Universe. The higher terms in the expansion include the effects of red-shift and evolution, and so the deviations from the simple 3/2 power law provide information on the combined effects of expansion and evolution in the Universe we observe. Now for flux densities greater than 1 f.u., the N(>S) curve is observed to rise relative to that for a static Euclidean Universe. Taking only the first term into the expansion for N(>S) after the basic term, we find the condition for this rising N(>S) behaviour is the negative sign of the term

$$\left[5 + 2\beta + \frac{R_0}{\dot R_0}\frac{\dot\rho_0}{\rho_0} + \frac{2R_0}{\dot R_0}\frac{\dot P}{P}\right]$$

which implies that, for qualitative agreement with the observed form for N(>S) we must have

$$\frac{R_0}{\dot R_0}\left[\frac{\partial}{\partial t}\left(\log \rho P^2\right)\right]_{t=t_0} < -(5+2\beta) \qquad (23)$$

Thus for any evolutionary model with the conditions (13) and (14), the source-count data impose the restriction (23) on θ and ∅ for a given β.

de Sitter Model

As the metric of the de Sitter model is determined by H, the Hubble constant, regarded as observationally established through all this work, we can straight away place limits on the integrated emission from such a model using the above result. If ∅ = 1, we must have 3∅ + 2θ > 6.4, taking β = 0.7, so that θ > 1.7 is required. Now with ∅ = 1, μ = β - θ, and must therefore be negative, which will give a negative result for the integrated emission, which is clearly absurd.

For the integrated emission to be positive, with β = 0.7, we must have 3∅ + θ < 3.7 ; from this we must have 3∅ + 2θ < 3.7 + θ in turn > 6.4 These conditions are only satiable if θ > 2.7, which necessitates ∅ < 1/3. Now the condition on ∅ means a very slow decrease in source density and a very rapid decrease in source power. The decrease in source density is in terms of unit co-ordinate volume however, and means, as ∅ < 1, that the proportion of radio sources to normal galaxies is increasing as time goes on, as $R(t)^2$, or e^{2Ht}. Now this could be reasonable if it were to be supposed that it is the fate of a normal galaxy to end its life as a radio galaxy in the majority of cases, with a characteristic time of transition to the radio-emissive state from normality being of order half the Hubble time. This seems rather a large step to take within the scope of the present general discussion, and we shall pass on at this point to another type of cosmology which does not involve us so immediately in assumptions of detailed astrophysical behaviour. The conclusion to be drawn here is that the de Sitter Universe could perhaps be fitted with a "highly" evolutionary picture, in which there was a rapid transition from normal galaxies to radio galaxies and a rapid falling-off of source power.