5. NEWTON’S LAWS OF MOTION

Newton began his discussion with a set of axioms and definitions which, as he gave them, were in fact tautological. The first was a definition of mass:

“The quantity of matter is the measure of the mass, arising from its density and bulk conjointly.”

This amounts to saying that mass is (density x volume). If we deal with material of given density, it is useful to recognise that the mass of a sample increases in proportion to the sample’s volume – but the only practical definition of density is “mass divided by volume” so we cannot independently relate the masses of different materials. While we may have an intuitive idea of what we mean by “quantity of matter”, as soon as we try to make it precise we find it necessary to invoke the concept of force, which we cannot properly do until we have defined “mass”. For the moment, we can presume, as Newton did, that we can eventually find a satisfactory independent definition of “quantity of matter”.

Newton’s second definition stemmed from the ideas of Descartes:

“The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly.”

This defines the quantity which we today call the linear momentum, or just “momentum” for short. It is usually given the symbol ‘p’. In algebraic language Newton is defining

$$p = mv$$

But wait! We have seen the importance of remembering that velocity has direction as well as magnitude, so I denoted velocity symbolically by v with an arrow over it. From here on I use the simpler bold-face notation **v** for this purpose. As momentum is directly proportional to velocity it also carries information about direction (i.e. velocity horizontal and to the right means that momentum is also horizontal and to the right). So we can write, more precisely:

$$\mathbf{p} = m\,\mathbf{v}$$

After making further definitions and clarifications, Newton enunciated three principles which became known as "Newton's Laws of Motion":

"<u>Law 1:</u> Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it."

"<u>Law 2:</u> The change of motion is proportional to the motive force impressed, and is made in the direction in which that force is impressed."

"<u>Law 3:</u> To every action there is always opposed an equal reaction, or the mutual actions of two bodies upon each other are always equal and directed to contrary parts."

The content of these three statements transformed physical science, so they merit careful examination. The "First Law" is based on Galileo's and Descartes' statements that matter in motion tends to remain in motion. It identifies a <u>state of motion</u>, not a <u>place</u>, as the preferred condition of matter. It also identifies the preferred motion as <u>uniform motion in a straight line</u>, and it stipulates how we are to recognise the entity to be called "force". It sets up the <u>language</u> of our description of motion by asserting that <u>any deviation from uniform straight-line motion is to be considered proof that a force has acted on the moving object.</u>

To write this using mathematical symbolism we need a new symbol for the new concept of the <u>force.</u> We will use the symbol **F** (the bold face again reminding us that this quantity has a <u>direction</u> as well as a <u>magnitude</u> F). Newton's First Law says that the velocity is constant in magnitude and direction if there is no force acting on the moving body. In symbols:

$$\mathbf{v} = \text{constant if } F = 0$$

$$\text{i.e., } \Delta\mathbf{v} = 0 \text{ if } F = 0 \text{ whatever the } \Delta t \text{ over which we measure } \Delta\mathbf{v},$$

$$\text{i.e., acceleration } \mathbf{a} = \Delta\mathbf{v}/\Delta t = 0 \text{ if } F = 0$$

How about momentum? As momentum $\mathbf{p} = m\mathbf{v}$, if both m and $\mathbf{v}$ are constant, it is obvious that:

$$\mathbf{p} = \text{constant if } F = 0,$$

$$\text{i.e., } \Delta\mathbf{p} = 0 \text{ if } F = 0 \text{ whatever the time interval } \Delta t \text{ over which we measure } \Delta\mathbf{p},$$

$$\text{i.e., } \Delta\mathbf{p}/\Delta t = 0 \text{ if } F = 0$$

You may be asking "how many more different ways are we going to invent to write down Newton's First Law, which was only a <u>definition</u> anyway?" The point of this is that all of these statements are <u>equivalent</u> because <u>any</u> of them embodies Newton's First Law. In different circumstances we may wish to invoke whichever of them is most helpful for analysing a motion. These statements also show the choices Newton had when he was deciding on his Second Law, which specifies <u>how to calculate a force from its effects.</u> Merely knowing that "F=0 makes $\Delta\mathbf{v}$, $\mathbf{a}$, and $\Delta\mathbf{p} = 0$" does not tell us all we need to know about how forces are to be related to changes in velocity, or in momentum. A host of different possibilities for the relationships between these quantities could be consistent with Newton's First Law. Newton's own proposal turned out to be so productive that nobody since has seriously criticised it as a working definition.

In modern language Newton's Second Law reads "<u>The rate of change of momentum of a moving object is proportional to the force acting on the object, and the direction of the change produced by a given force is in the direction of the force itself."</u> In symbols, this is:

$$\mathbf{F} = \Delta\mathbf{p} / \Delta t.$$

In fact this proportion is more general in its applications than we will usually need. Because $\mathbf{p} = m\,\mathbf{v}$, a change $\Delta\mathbf{p}$ in $\mathbf{p}$ could involve either a change in velocity $\mathbf{v}$ or a change in the mass m of the moving object. For example, a rocket burning fuel and ejecting hot gases, or discarding empty fuel tanks, may change its mass during flight. Newton's Second law is flexible enough to handle that case, but such generality is often more than we need. The changes in momentum that we most often need to analyse come from changes in velocity alone. If a moving object with fixed mass m has momentum $\mathbf{p} = m\mathbf{v}$ now and momentum $\mathbf{p}' = m\mathbf{v}'$ later, the change in momentum would be:

$$\Delta\mathbf{p} = \mathbf{p}' - \mathbf{p} = m\mathbf{v}' - m\mathbf{v} = m(\mathbf{v}' - \mathbf{v}) = \mathbf{m}\,\Delta\mathbf{v}$$

Then:

$$\mathbf{F} = \Delta\mathbf{p}/\Delta t = m\,\Delta\mathbf{v}/\Delta t = m\mathbf{a}$$

i.e., <u>Force = (mass x acceleration)</u>, and <u>the force is in the direction of the acceleration that it causes</u>. This is the hard core of Newton's concept which soon became the most productive tool in the science of motion (later known as "Newtonian mechanics" in recognition of Newton's role in clarifying the basic concepts).

Be sure to recognise Newton's Second Law for what it is: a <u>definition</u> of what will be meant by "force" from now on. It says nothing about what a force <u>is</u>, nor why there should be such things as forces in Nature. It simply says "when we see a moving object of mass m change its motion so that there was an acceleration $\mathbf{a}$, we will say that this acceleration was produced by a force $\mathbf{F} = m\,\mathbf{a}$". It is a matter of language, really, but "force" is now so precisely defined that the concept can be handled mathematically to make quantitative statements. It is the goal of experimental science to provide quantitative data about Nature, and the goal of theoretical science to make quantitative predictions to compare with future (or other) data. To reach these goals, we need unambiguous definitions and measures of our concepts, and those are the essential elements provided by Newton's Second Law.

The definition **F** = m **a** is not arbitrary. It would not have been useful if it did not correspond to practical realities. Consider a situation in which you are trying to push a stalled car down a road. To get it moving with given velocity you will try to provide a given acceleration **a** for a given time, say ten seconds. You know that a massive limousine will require a heftier push than a tiny sub-compact, i.e. that to give "more m" a given **a** will require "more F". You could establish that twice the car (i.e. twice the m) requires twice the F (one person pushing twice as hard or two similar people pushing instead of one). You also know that you must push in the direction of the acceleration that you want to achieve.

F = m**a** is evidently an intuitively reasonable definition based on this sort of experience. Its power comes from its utility as a tool for analysing much less intuitive situations, such as guiding a space capsule launched from the Earth's surface towards a "soft" landing on the Moon or Mars by making detailed calculations with the same basic recipe for "force". In detail, the proof of a pudding is in the eating, and this one has now eaten well for several centuries.

Newton was not alone in thinking along these lines. Robert Hooke's lecture to the Royal Society in 1666 had argued that all bodies move in straight lines unless deflected by some external force; and in 1679 Hooke wrote to Newton about a force towards the Sun, a solar pull, keeping the planets in their orbits - but if Hooke had come to a concept of force that was as clearly formulated as Newton's, he kept it to himself.

Newton's Third Law also embodies some familiar experiences. Suppose you stand with a friend of about your own height and build (i.e., of about your mass) on an ice rink and you give him or her a steady push. If your footings are the same, you both move, about equally and in opposite directions. While pushing on your friend you feel a push back on you: a reaction. If you push on a mass much larger than your own, the effects of the reaction may be much larger than those of your own force, as you can learn by trying to push a massive limousine in an icy parking lot: the limo may move forward a little but the force it exerts back on you can give you an acceleration much larger than the one you gave it. You try to push it, but you end up being pushed backwards. Another example: you bang your hand on a table, exerting a force and possibly crumpling something on the table, but the reaction force can make your hand hurt afterwards. Your force can accelerate things on the table, but the reaction force accelerates bone and tissue in your hand, possibly making it hurt for a while.

Newton's Third Law says that when bodies exert forces on each other, these forces come in equal and opposite pairs – action and reaction. When you push on the limousine, you exert a force **F** giving the limousine an acceleration

$$\mathbf{a}(\text{limo}) = \mathbf{F} \;/\; m(\text{limo})$$

but by Newton's Third Law the limousine exerts an equal and opposite force **-F** on you, so, again from **F** = m**a** you can conclude that your acceleration will be

$$\mathbf{a}(\text{you}) = -\mathbf{F} \;/\; m(\text{you})$$

so if your mass is much less than the limousine's, your acceleration will be much greater in inverse proportion. This is what makes it so difficult to push a heavy car out of an icy parking lot unless you wear cleated shoes. Good cleats anchor your feet firmly to the Earth, so m(you) is effectively replaced by m(you+Earth) which is huge compared to m(limo) whereon the car moves and you (almost) do not.

Newton's Third Law implies that a force is not something that one piece of matter does to another in isolation, but is an interaction between two pieces of matter. The concept of a force as a mutual interaction between two bodies reads more deeply into everyday experience than is immediately obvious, but it was a very significant concept in the development of our description of the world, first demonstrated brilliantly by Newton's mathematical description of gravitation in the Solar System.