EIGHT ARC MINUTES, THE TELESCOPE AND A DIALOGUE - KEPLER AND GALILEO

So far we have seen one man at a time wrestling with the problems of a cosmology which seemed to grow increasingly complex without changing of its very essence. Indeed it would be fair to ask if any real progress in understanding the world had been made between Aristotle and Tycho, for in the span of about 2000 years heliocentrism had come (Aristarchos) and gone (Ptolemy) because of the difficulties posed by a rotating Earth. Then again it had come (Copernicus) and gone again (Tycho) for the same reasons. Brahe had documented the celestial motions with unprecedented accuracy, yet the preferred world-system had reverted basically to that of Heraklides, Aristotle's pupil. It required entirely new thinking to break the cycle, and in this Chapter we shall see how two brilliant but dissimilar geniuses, working at the same time but not in concert, began finally to cross the elusive conceptual bridge towards a modern science of the Universe.

1. Kepler's Mystery

Johannes Kepler (Figure 1) was born on December 27, 1571, the son of an unstable mercenary adventurer and an innkeeper's daughter who Kepler himself described as "small, thin, swarthy, gossiping and quarrelsome, of a bad disposition". Johannes was sickly, myopic and unhappy as a child; a person less likely to galvanise the science of his time could hardly have been imagined.

In November 1577 a brilliant comet appeared. The determination of its motion among the stars was another of Tyge Brahe's great successes, for he showed that it lay beyond the Moon in the realm of the heavens, contrary to Aristotelian doctrine. Kepler, then six years old, was taken by his mother to a high place to see the comet, an outing for which science may be greatly indebted to this unpleasant-sounding woman, for the experience made some impression on the boy who was to inherit Brahe's incomparable data. Kepler entered a seminary where in addition to theology he learned mathematics, and later studied theology at the University of Tübingen. At Tübingen he encountered, became convinced by, and publicly defended Copernican heliocentrism. This may have had something to do with the fact that in 1593 the University recommended him for the post of lecturer in mathematics and astronomy at Graz in Austria,

Johannes Kepler. *(Yerkes Observatory.)*

Fig. 1

rather than for a position in the church. Soon after his arrival at Graz he conceived the basic idea of his first essay in cosmology, which was published in 1596 under the title "A Forerunner to Cosmographical Treatises, containing the Cosmic Mystery of the admirable proportions between the Heavenly Orbits and the true and proper reasons for their Numbers, Magnitudes and Periodic Motions".

Kepler's "Cosmic Mystery" was a geometrical conception of the planetary system that was breathtaking in its originality and which proved to be total in its irrelevance. Its importance to us today is that while Kepler was still a young man he experienced the euphoria of believing that he had discovered the deepest secrets of the Universe and became critical of Koppernigk's system and his data while remaining convinced that its underlying heliocentrism was correct. The experience gave Kepler the intellectual momentum to begin asking totally new questions about the underlying pattern in the celestial motions.

At the core of Kepler's "Mystery" lay the fact that there are only five regular solids--perfectly symmetrical three-dimensional figures all of whose faces are identical (Figure 2). No others exist which completely enclose three-dimensional space with identically-shaped and identically-sized faces. Each regular solid can be related to two spheres: the inscribed sphere exactly contained by its faces, touching each face precisely at its centre, and the circumscribed sphere exactly containing its corners, just touching each and every one. Thus the five regular solids can be nested into a set of six spheres, the circumscribed sphere of one solid being the inscribed sphere of the next largest, as in Figure 3. When this is done, there results a set of six spheres whose relative sizes are determined by a basic symmetry of three-dimensional space.

Kepler identified six such spheres with the six planetary orbits of the Copernican model. Computation showed him that the relative sizes of the Copernican planetary orbits approximated those of the six spheres if the solids were ordered as follows: cube (outermost), tetrahedron, dodecahedron, icosahedron, octahedron. He became so convinced that this coincidence would reveal the entire mystery of Creation that he declared the (approximately 5%) discrepancies between the Copernican data and his theory to be the result of poor observations of the planetary system, and he hoped that Brahe's then unpublished studies would prove him right. The significance of the regular solids was in fact to prove illusory, but Kepler's mistrust of Koppernigk's data and his faith in Brahe were to serve him well later.

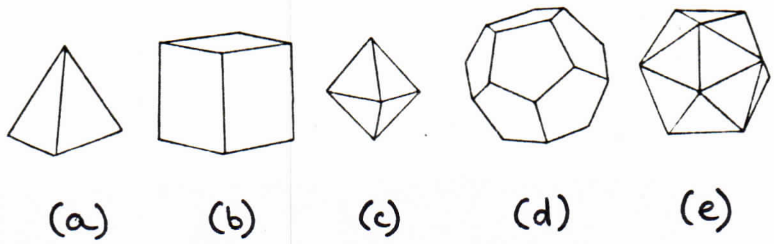
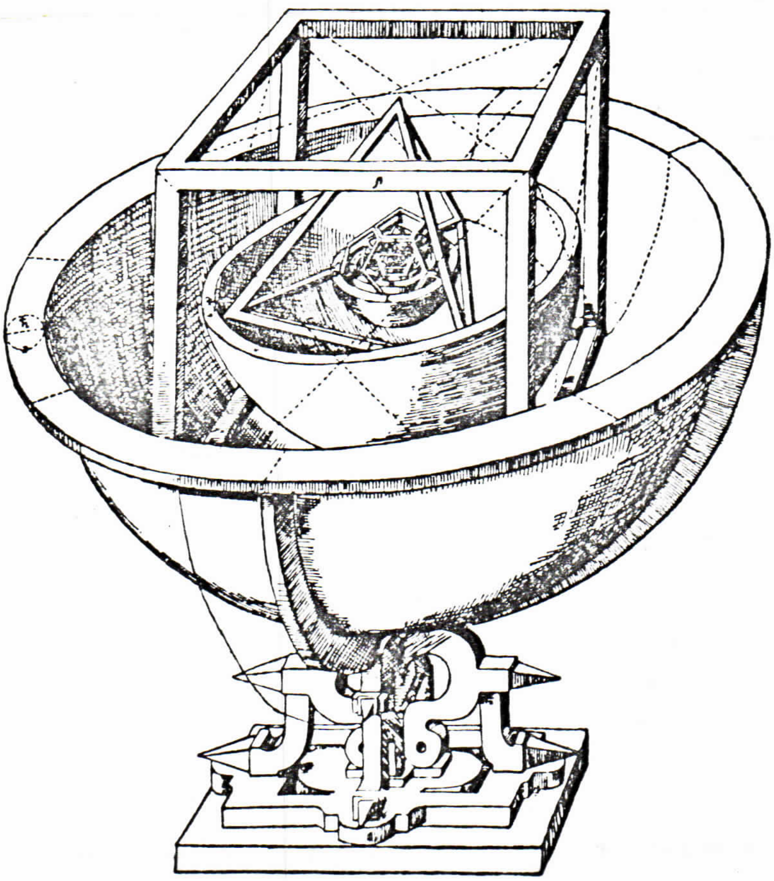
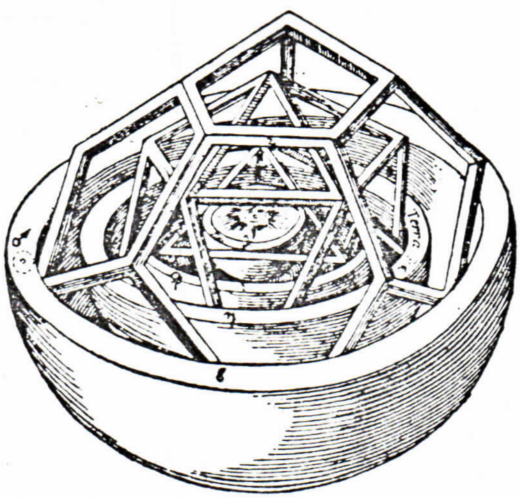


Fig. 2 The five regular solids

(a) Tetrahedron, 4 triangular sides
(b) Cube, 6 square sides
(c) Octahedron, 8 triangular sides
(d) Dodecahedron, 12 pentagonal sides
(e) Icosahedron, 20 triangular sides



Model of the universe; the outermost sphere is Saturn's.



Detail, showing the spheres of Mars, Earth, Venus and Mercury with the Sun in the centre.

Fig. 3

Diagrams from Kepler's "Cosmic Mystery"

Furthermore, the excitement of "explaining" both the *number* of then-known planets (there *could* be only six in his scheme) and the *relative sizes* of their orbits encouraged him to try to account for one more observation, namely the *time* taken by each planet to revolve around the Sun. The more distant a planet was from the Sun, the longer it took for its orbit; Koppernigk had noticed that. But Kepler read significance into the fact that the orbit times were not longer simply in proportion to the orbit sizes; the outer planets actually travelled at slower speeds in their orbits. He considered this to be consistent with the idea that the planets were moved by some influence which radiated outwards from the Sun, decreasing in effectiveness with distance just as the brightness from a lamp decreases with distance.

Here was Kepler's truly revolutionary conception. For the first time a cosmologist argued that something which had a counterpart in everyday experience *caused* the heavenly motions. Such an idea was quite alien to the Aristotelian world view, in which there could be no link between terrestrial phenomena and the cause of the motions in the skies. Kepler's embryonic idea of a solar influence which diminished with increasing distance from its source was still far from being a detailed theory of gravity--but it pointed in a conceptual direction from which there would, at last, be no return.

The prescience of Kepler's explanation for the orbit times can hardly be overestimated. Yet Kepler was still carrying a rich mix of concepts with him on his intellectual journey; the final section of the "Cosmic Mystery" provided a horoscope for the first day of Creation, estimated at Sunday, April 27, 4977 B.C. It was an optimistic horoscope.

2. Kepler and Tycho

Soon after the publication of his "Cosmic Mystery", Kepler's wish to have access to Tycho's data came true as an indirect result of some petty squabbling in Denmark and of religious persecution in Austria. Frederick II of Denmark, Brahe's original benefactor, had died in 1588. Tyge had done little to endear himself to the Danish royal house and nobility since then. He mistreated his tenants, neglected various small tasks associated with his grants, and became quarrelsome in his relations with various influential statesmen. Early in 1597 his royal grants were curtailed, and soon afterwards he moved most of his instruments and his household from Hveen, commenting that "an astronomer must be cosmopolitan, because ignorant statesmen cannot be expected to value his services." By 1599 he had secured the position of mathematician to Emperor Rudolf II and moved his instruments and retinue to Prague.

This was a happy accident, for he was then within range of Kepler, who was being pressured out of his job in Graz. Kepler had received a Lutheran theological education and Graz was in Catholic Styria, a province of Austria. In the Summer of 1598 all Lutheran preachers and teachers were ordered out of Styria within eight days on pain of death. An exception was made in Kepler's case because his ability had been recognised by his superiors, but he was understandably eager to move to a more benign environment. He had sent Brahe a copy of the "Cosmic Mystery" in 1598, and Tyge had replied, complimenting Kepler on his ingenuity but expressing doubts about the numerical accuracy of his work and about his adoption of Copernican heliocentrism. On 9th December 1599, Brahe wrote to Kepler from Prague hoping that they might soon meet. Kepler travelled to Prague during January 1600, to find Brahe much involved in getting his observations under way again. He was invited to stay as an assistant; he had hoped to be received as an equal. Neither Kepler nor Brahe being of a calm disposition, relations between them were fractious, and at one point Kepler actually left Tycho's castle contemplating permanent return to Graz. Sanity and Tycho's need for skilled colleagues prevailed however, and Kepler remained as one of Brahe's assistants until the latter's premature death on October 24, 1601.

On his deathbed, Tyge Brahe begged Kepler to finish the production of new planetary tables from the observations which had been the Dane's lifework, and to base their computation on Tycho's system rather than the Copernican model. On November 6, 1601, Kepler was appointed imperial mathematician as Brahe's successor --at half his dead patron's salary.

3. The Eight Arc Minutes

In 1609 Kepler published his second great treatise, under the revealing title: "A New Astronomy based on Causation, or a Physics of the Sky derived from Investigations of the Star Mars, founded on Observations of the noble Tycho Brahe". The "causation" of the planetary motions was now to Kepler a dominant factor worthy of its prominence in the title. The "Cosmic Mystery" of the regular solids was discarded--Brahe's observations had not confirmed it. Undaunted, Kepler had begun again, this time trying to construct an improved Copernican system, perfect circles, epicycles and all. The strictest test was to be the most obviously non-uniform planetary motion--that of Mars.

Kepler first cleaned out some absurd remnants of geocentrism in the Copernican model--for example, Koppernigk had ascribed to Mars' orbit a steady oscillation in space which depended on the position in space of the Earth. Kepler perceived that there should be no causal basis for this in a Sun-centred system, and that it had become necessary in the Copernican system because Koppernigk assumed that the planes of the planetary orbits intersected at Earth. As soon as Kepler made the more reasonable assumption that the orbital planes intersected in the Sun, the problem vanished. He then restored Ptolemy's Equants, at least for computational purposes, and proceeded to labour for five years over 900 pages of longhand calculations of the orbit of the one planet, Mars. A sample of his calculations is shown in Figure 4. By the time he had finished, his model could reproduce the observed motion of Mars to within 8 arc minutes (a little less than 1/7 of a degree) on the sky. Considering that the planetary tables of his time contained predictions which failed to match observations by as much as several degrees at worst, a lesser man might have settled for this result. But Kepler knew that Brahe's best planetary observations should be accurate to about 2 arc minutes (1/30 of a degree), so the discrepancy between his model and observation was as much as four times the expected uncertainty in the data. At this point, Kepler took the giant step from which all astronomers before him had shrunk. Perhaps it was easier for him because he had already discarded his own favourite theory--but what he did speaks for itself, for he abandoned the entire system of circular motions which had dominated the theories for over 2000 years.

He wrote:

> "Since the divine goodness has given to us in Tycho Brahe a most careful observer, from whose observations the error of 8 arc minutes is shown in this calculation ... it is right that we should with gratitude recognise and make use of this gift of God. Let us certainly work it out, so that we finally show the true form of the celestial motions ... I myself shall lead the way for others ... according to my small abilities. For if I could have treated the 8 arc minutes as negligible I should already have corrected the hypothesis accordingly. But as they could not be neglected, these 8 arc minutes alone have led the way towards the complete reformation of astronomy and have been made the subject-matter of a great part of this work."

The same Johannes Kepler who in 1597 had asserted that if the data contradicted his hypothesis (of the regular solids) then the data must be wrong, in 1609 threw out all existing theories in the face of disagreement with data that he trusted. In so doing, he launched his science on a course that would unify

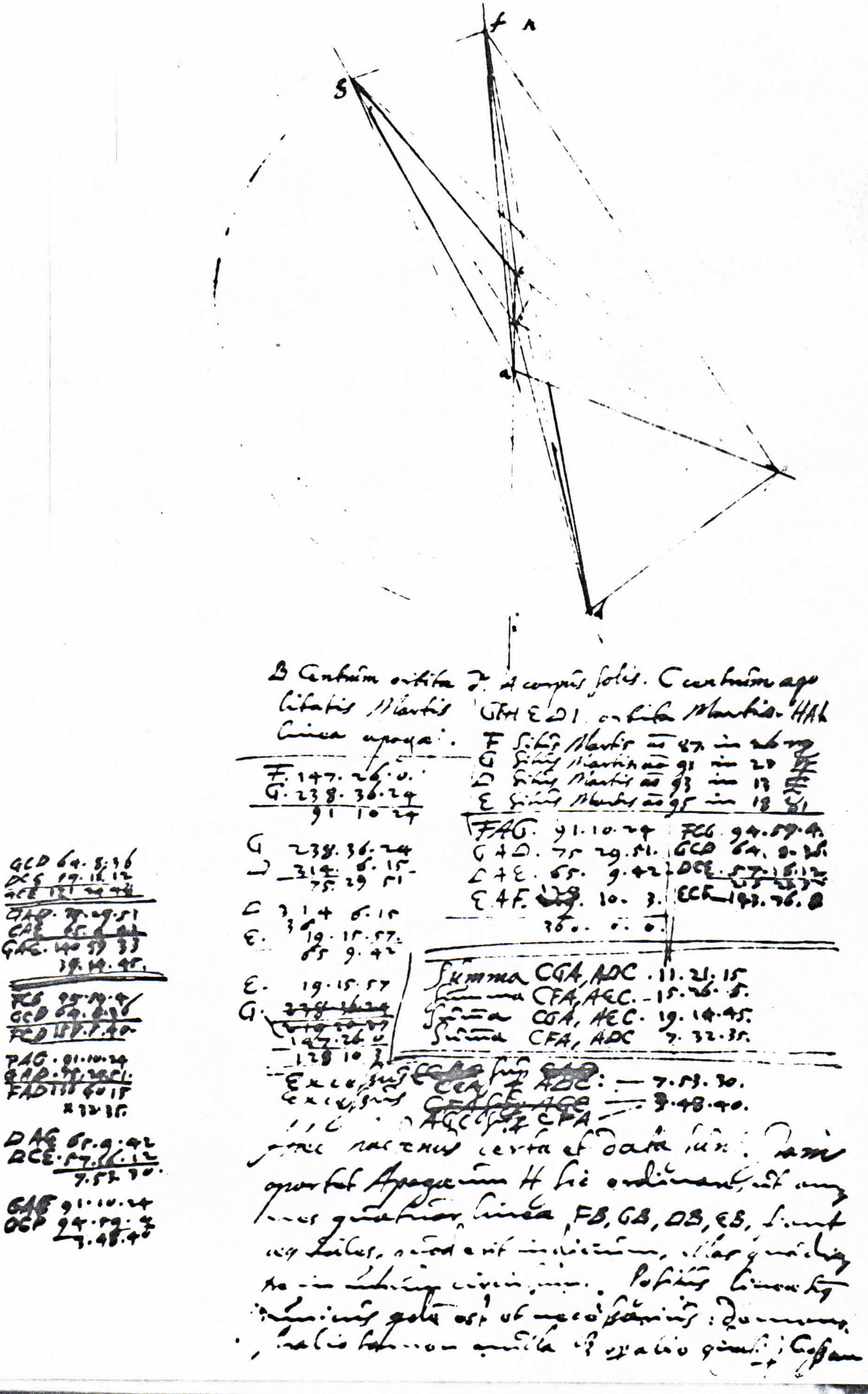


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Fig. 4 Reproduction of a page of Kepler's calculations theory and observation in physics and astronomy, through the framework of mathematical logic. Kepler was not to complete the task of formalising the new world-view--that privilege fell to Isaac Newton--but his reaction to the eight-arc-minute discrepancy marked the end of the Aristotelian dominance of astronomy.

4. Elliptical Orbits--Kepler's First and Second Laws

Kepler's "New Astronomy" went on to consider afresh the nature of the motion of Mars. Kepler realised that the shape of Mars' path around the Sun could be determined from Tycho's data by making use of observations made at times differing by exactly one Mars year--the time taken by Mars to perform one orbit. The first step was to discover what time interval that actually is. The method had been used by Copernicus, but we consider it here because this is the first occasion on which it played a crucial role in the analysis.

We observe Mars from Earth, which is itself in motion. By studying the path of the Sun across the stellar background we know that the Earth performs its orbit around the Sun once in 365 1/4 days, and call this--the Earth's orbital period--the "year". Because of the Earth's motion, we do not see Mars at the same place in the sky at the end of each Martian orbital period, or Mars year (Figure 5). Only if the Earth made an exact number of complete orbits in a Mars year would we see Mars against the same stellar background each time it reached the same place in its orbit. This means that the length of the Mars year cannot be determined directly by observing the motion of Mars against the stellar background. (The same problem exists for all the other planets too.) The trick is to measure the length of another period for each planet--the synodic period--and convert that to the orbital period by computation. The synodic period of a planet is the time interval between successive alignments such as in Figure 6a where Sun, Earth and planet are in-line (except for the different planes of the orbits, which are only a minor extra complication). Because it is the period between successive appearances of the planet opposite to the Sun in the sky (crossing the observer's meridian on Earth at "midnight") it can be measured fairly easily. For Mars it is 780 days, or two Earth years plus 49 1/2 days. Now look at Figure 6b. Suppose that observations are begun when Earth is at E_1 and Mars is at M_1. One synodic period of Mars later, Earth has made a little over two and one-seventh orbits and Mars a little over one, bringing the planets to E_2 and M_2 when the second measurement is made. The point is that, knowing the Earth's

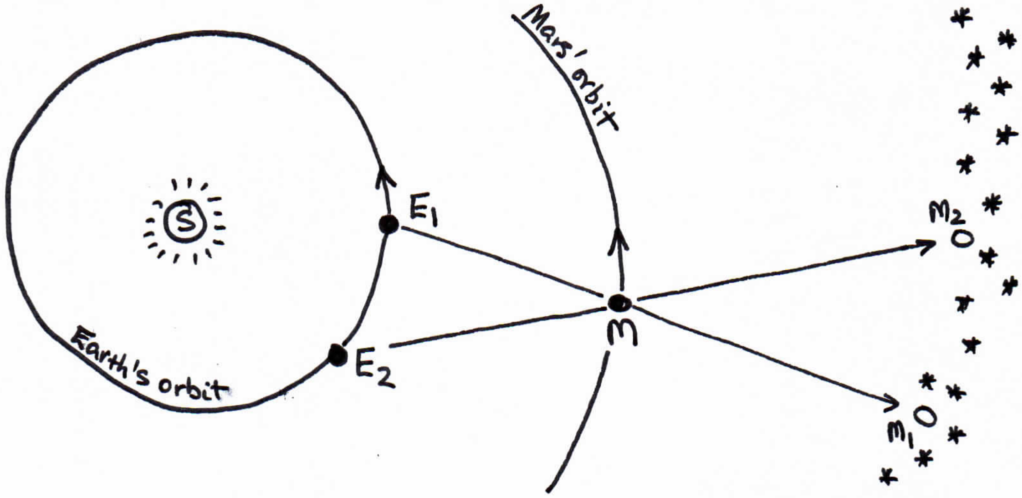
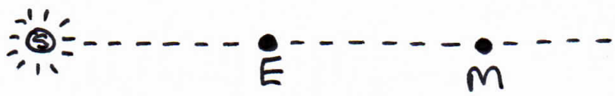


Figure 5 – Configuration of Sun (S), Earth (E) and Mars (M) at beginning and end of a "Mars year". Mars is at same place in its orbit at both times, but Earth is at E_1 at the beginning and E_2 at the end (having in fact made just under two complete orbits). Mars appears to be at the positions M_1 and M_2 among the stars on these two occasions

a) 

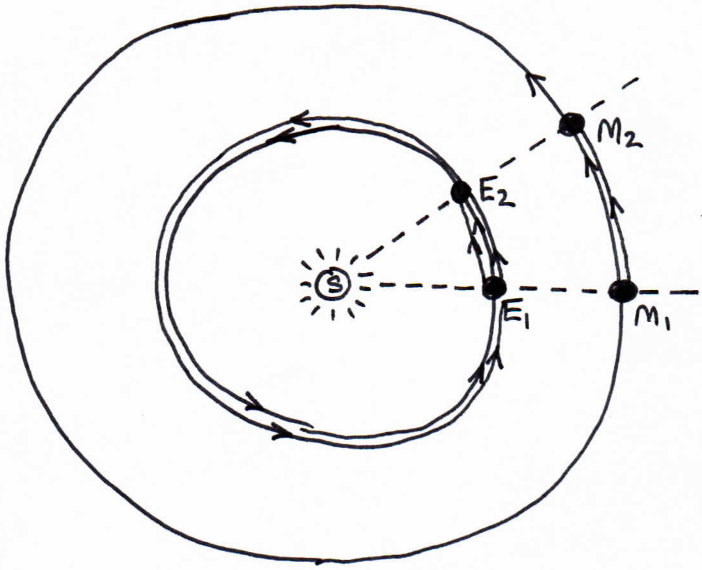
b) 

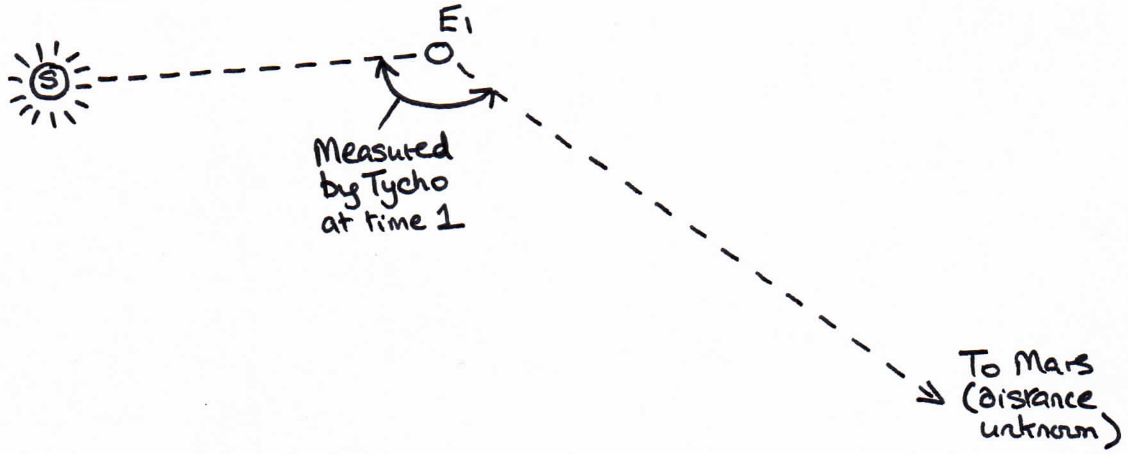
Figure 6 a) The configuration Sun – Earth – Mars marking the beginning and end of a synodic period of Mars

b) The motions of Earth and of Mars during a 780-day synodic period of Mars. Earth has travelled two complete orbits plus angle E_1SE_2. Mars has travelled one complete orbit plus angle M_1SM_2 ($= E_1SE_2$).

orbital period to be 365 1/4 days, we can calculate the size of the angle E_1SE_2 exactly: the Earth travels 360° in orbit in 365 1/4 days, so it travels (780 ÷ 365 1/4) x 360° = 768 1/2° in the 780-day synodic period of Mars. This is two complete orbits (720°) plus the angle E_1SE_2, which must therefore be 48 1/2°. In this time Mars has travelled one complete orbit (back to M_1) plus the extra path from M_1 to M_2. But M_1SM_2 is the <u>same</u> angle as E_1SE_2, so Mars has travelled 360° + 48 1/2° = 408 1/2° in the 780 days. This means that Mars would travel exactly one orbit (360°) in (360 ÷ 408 1/2) x 780 days, or 687 days. The orbital period of Mars is thus 687 days.

Kepler selected from Tycho's notes sets of observations of Mars that had been made on dates that were 687 days apart. Each such observation allowed him to calculate the <u>angle</u> between Sun, Earth and Mars at the time of the measurement. Consider Figure 7a. Any <u>one</u> such observation told Kepler that, on that date, Mars was somewhere in space in the direction of the arrowed line, but at an unknown distance from Earth which was at E_1. A measurement 687 days later would have been made while Earth was at a different position (E_2) in its orbit--the Earth would have travelled (687 ÷ 365 1/4) x 360°, or 677° around the orbit short of two full orbits by 43°. But because Kepler knew the length of Earth's year, he <u>knew</u> that the angle E_2SE_1 in Figure 7 would be 43°. He also knew that because one Mars year had elapsed between the two measurements, Mars was at the <u>same</u> place in <u>its</u> orbit at the second measurement as it was when originally seen from E_1. The second measurement told him that Mars was somewhere in space in the direction of the second arrow, also at an unknown distance: the intersection of the lines E_1M and E_2M (Figure 7b) must then define the unique position of Mars in space at the times when both measurements were made. It was then a simple exercise in trigonometry to deduce the exact location of that one point on Mars' orbit from a diagram like Figure 7b. Combination of several pairs of observations of Mars, each pair 687 days apart, thus allowed Kepler to determine a set of positions in space through which the orbit of Mars must pass--as in Figure 7c. What then remained was to deduce the mathematical nature of the oval curve which passed through the points. This last step was difficult and the shape of the curve gave Kepler much displeasure.

For the eventual result was an orbit for Mars in the form of an <u>ellipse</u>, with the Sun not at its centre but offset to one side. Kepler initially regarded this result as ugly, describing it in a letter to the contemporary astronomer Longomontanus in 1604 as "a cartful of dung". It was indeed some years before he realised that the curve he had deduced <u>was</u> the classical elliptical form known to a)



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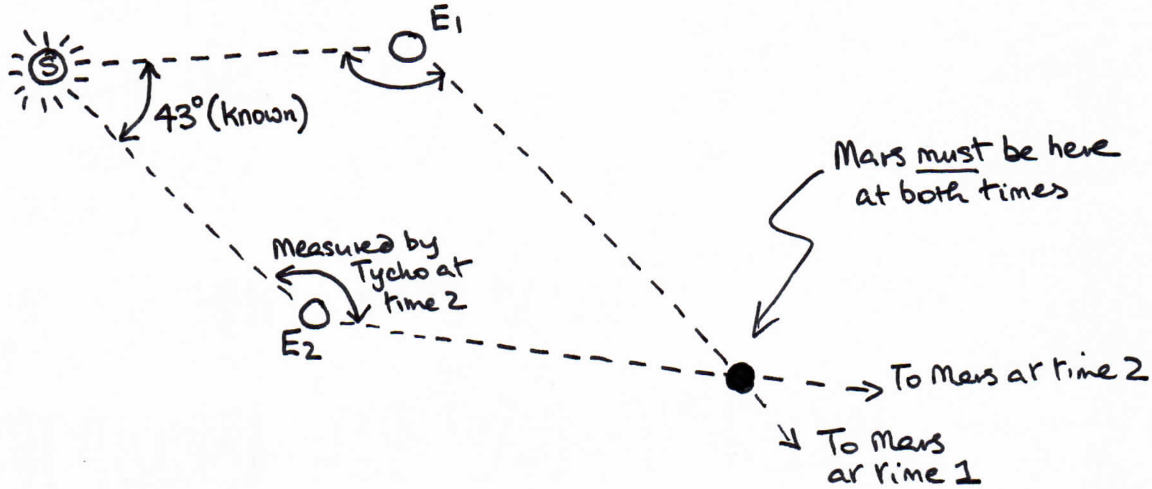

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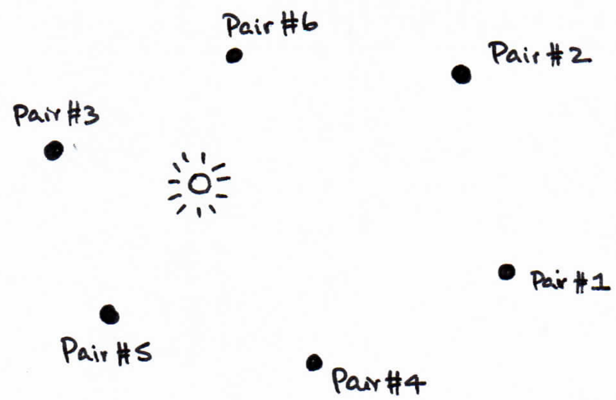


Figure 7 – illustrating Kepler's analysis of Tycho's data on the orbit of Mars.

a) One of Tycho's measurements, by itself, tells direction to Mars but not distance

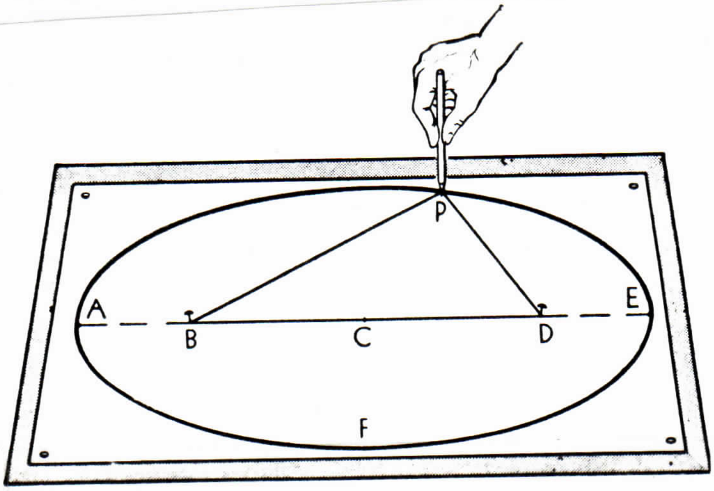
b) Pair of measurements one Mars year (687 days) apart locates Mars, knowing angle travelled by Earth in that time. Size of Earth orbit must be known to determine absolute distance scale (e.g. in km).

c) Data from a set of pairs of measurements of Mars, each pair being two times 687 days apart, defines set of positions of Mars around Sun. Orbit must go through all of these.

the Greek geometers. Figure 8 illustrates the definition of the elliptical curve. Whereas the circle has a single defining point (its centre) from which all points on the circle are equally distant, the ellipse has two defining points, the foci; all points on an ellipse are such that the sum of their distances to the two foci is the same. Thus an ellipse can be generated by attaching the two ends of a short cord to a drawing board with a pair of thumb tacks as in Figure 8 and moving a pencil on the drawing board to keep the cord taut. The points B and D are the two foci, and the total distance BP + PD is the same for any point P on the ellipse. This total distance is also equal to the length of the longest diameter or "major axis" of the ellipse, AE (Fig. 8). The separation of the foci (BD) divided by the major axis length (AE) is known as the "eccentricity". If folded about the major axis the two halves of the ellipse are identical. The shortest diameter of the ellipse is at right angles to the major axis, and is called the minor axis; the two halves of the curve on each side of the minor axis are also identical. The ellipse thus has considerable symmetry, perhaps less than Kepler desired but still far from mathematical anarchy.

Kepler further found that the Sun lay at one of the two foci of the orbit of Mars (the other focus being empty) and that the eccentricity of the orbit was about one part in 10. If Mars traced an elliptical orbit, then presumably so did Earth, and so Kepler proceeded to reanalyse Tycho's data until he obtained mutually consistent orbits for both planets. Earth's orbit he found to be an ellipse much closer in form to a circle than was the orbit of Mars--the eccentricity of Earth's orbit was only one part in 50. Again, the Sun lay at a focus of Earth's elliptical path. From these results Kepler inferred his First Law of Planetary Motion--the orbits of the planets are ellipses with the Sun lying at one focus of each ellipse. He later confirmed the result for planets other than the Earth and Mars.

As the elliptical path took each planet alternately closer to and further from, the Sun, Kepler expected to find a variation of orbital speed with distance, corresponding to his presumed solar "influence" varying with distance. He was not disappointed. The "New Astronomy" announced that Tycho's data revealed another aspect of the planetary motions--each planet travels fastest when closest to the Sun and slowest when furthest away. Kepler perceived further that the variation in orbital speed corresponded to an elegant symmetry: if an imaginary line joined each planet to the Sun, then the planet's orbital speed varied exactly so that the line swept out equal areas around the Sun in equal times (Figure 9). This "law of areas" became Kepler's Second Law of Planetary Motion. It and the distance variation combined to explain Hipparchos' observation of the unequal



The construction of the ellipse

Figure 8.

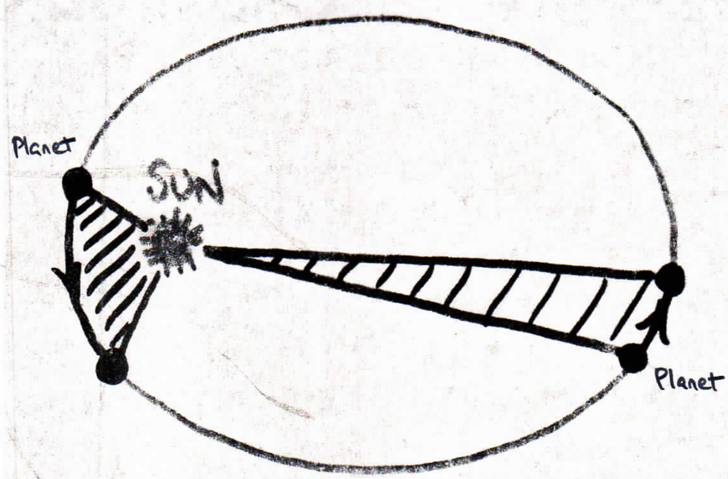


Figure 9

Kepler's Second Law of Planetary Motion

- the line joining the planet to the Sun sweeps out equal areas of the ellipse in equal times, so the planet travels fastest when closest to the Sun.

lengths of the four seasons.

Kepler's discovery of these regularities in Tycho's data was an analytical feat of the highest order, demanding both the courage to break with centuries of tradition and the mathematical skill to handle the new calculations. He elaborated further on his concept of a solar "influence"; impressed by the analysis of magnetic phenomena that had been made in 1600 by his English contemporary, William Gilbert, Kepler speculated that some magnetic force from the Sun drove the planets around their orbits. These ideas were wrong-headed, but we must remember that in considering any physical cause at all for the planetary motions, Kepler was treading completely new ground. His concept of a solar influence, though fuzzy compared with Newton's final description of gravity, was clear enough to dissuade him from considering Tycho's Earth-centred compromise any further, despite the fact that a Tychonic system with elliptical orbits would have been mathematically equivalent to Kepler's own.

5. Kepler's Celestial Music

When the "New Astronomy" was published Kepler was thirty-seven. His later years were not inactive. He published commentaries on Galileo's telescopic discoveries, a Biblical chronology, a treatise on the shape of wine-barrels, a textbook on optics, a discussion of the mathematics of logarithms, and several astronomical books. Despite the experiences of civil war and pestilence in Prague, of the fall from power of his patron Emperor Rudolph, of the deaths of his wife and favourite child, of his removal to Linz from war-torn Prague, and of the trial for witchcraft of his mother (whose legal defence he organised), he published in 1618 his "Harmony of the World". In this book Kepler attempted a synthesis of all that bore mathematical symmetry in geometry, music, astronomy and astrology. This work constructed a maze of speculations, including an interpretation of the planetary motions as "a continuous song for several voices ... a music which through discordant tensions, through sincopes and cadenzas ... progresses toward certain predesigned, quasi-six-voiced clausulas and thereby sets landmarks in the immeasurable flow of time". In this celestial choir Kepler found the Earth associated with the notes "Fa" and "Mi" in the scale--which in keeping with his times he interpreted as "Famine" and "Misery". Buried in this outpouring of mysticism was his Third Law of Planetary Motion--his final statement of the relation between the periods and the sizes of the planetary orbits which had first encouraged him to contemplate the presence of a regulating solar influence.

Kepler's Third Law was that the squares of the orbital periods of the planets increased in proportion to the cubes of the major axis lengths of the orbits. Denoting the orbital period of a planet by T and the length of the major axis of its orbital ellipse by a, then

$$T^2 = k\ a^3$$

where k was the same number for all the planets. The Law held to great accuracy (see Table 1) in Tycho's data. It was to be a crucial verification of Newton's theory of gravity more than fifty years later.

Table 1

KEPLER'S HARMONIC LAW
(data in units for which k = 1)

Planet	Distance (a) from Sun (Earth distance=1 unit)	Orbital Period (T) (Earth Years)	Distance (a^3) cubed	Period (T^2) Squared
Mercury	0.387	0.241	0.058	0.058
Venus	0.723	0.615	0.378	0.378
Earth	1.00	1.00	1.00	1.00
Mars	1.524	1.881	3.54	3.54
Jupiter	5.20	11.86	141	141
Saturn	9.54	29.46	868	868

The Thirty Years' War severely disrupted Kepler's life after the appearance of the "Harmony of the Worlds". In 1627 he finally produced the improved tables of planetary positions that the dying Tycho had entreated him to compute; Kepler's "Rudolphine Tables" were of course based on his own model of the Solar System, and were about 50 times more accurate in their predictions than any that preceded them. Astronomers could have had no better proof of the validity of Kepler's description of the Solar System. But for Kepler the years from 1618 to his death in 1630 were mainly a time of personal distress and confusion. His own view of his last years is illustrated by the final sentence of his letter, dated November 6, 1629, to Jakob Bartsch, a young mathematician and physician who assisted him in the production of planetary tables and married his daughter Susanna:

"When the storms are raging and the shipwreck of the state is frightening us, there is nothing nobler for us to do than to let down the anchor of our peaceful studies into the ground of eternity".

6. Galileo's Message from the Stars

The years of Kepler's life (1571-1630) were completely overlapped by those of the Italian Galileo Galilei (Figure 10), who was born in 1564 and died in 1642. Galileo's contributions to the downfall of the Aristotelian world-view and of geo-tric thinking were as irrevocable as those of Kepler, yet Galileo's insights have virtually nothing in common with Kepler's Laws. Each of these contemporaneous giants discovered the most profound subtleties of the world around him without being able to communicate the real essence of his work to the other. It is not that there was no correspondence between them; the facts are still more curious than that. Each seems to have been unable to take up and appreciate the ideas of the other--yet between them they had all the information from which the next generation of astronomers and physicists would distil the new world-picture.

Galileo as a young man was a rebel against authority who would believe nothing of the Aristotelian doctrine unless it was demonstrable to his own eyes. Never willing to accept any statement about the physical world on the evidence of ancient texts, and unable to conceal his contempt for those among his elders who were, Galileo became a temporary "drop-out" from the University of Pisa in 1585. Four years later, with several successful researches into mechanics and hydrostatics behind him, he was appointed to the same university as a lecturer in mathematics, and in 1592, he became Professor of Mathematics at the University of Padua.

Popular mythology associates three famous events with the name of Galileo--the invention of the telescope, the disproof of Aristotelian mechanics by dropping balls of different weights from the leaning Tower of Pisa, and murmuring "eppur si muove" at the end of recanting his heliocentric teachings before the Inquisition. It is certain that the first event had nothing to do with Galileo, and it is probable that the other two are apocryphal. The myth does however point to the three dominant themes in Galileo's career--his use of new technology to observe the heavens, his use of experiments to refute Aristotelian concepts of motion, and his public conflict with the dogma of the Catholic Church.

Figure 10

Galileo Galilei

The principle of the telescope was discovered accidentally in 1607 by an apprentice in the shop of a Dutch maker of spectacles named Hans Lippershey. The realisation that a suitable combination of two convex lenses, as in Figure 11, can be used to produce a magnified image of a distant object was a technological advance which spread across Europe like wildfire, not the least of its applications being its use as a spyglass by the military. Galileo's contribution was to refine the instrument and to conceive of its use to explore the phenomena of the skies.

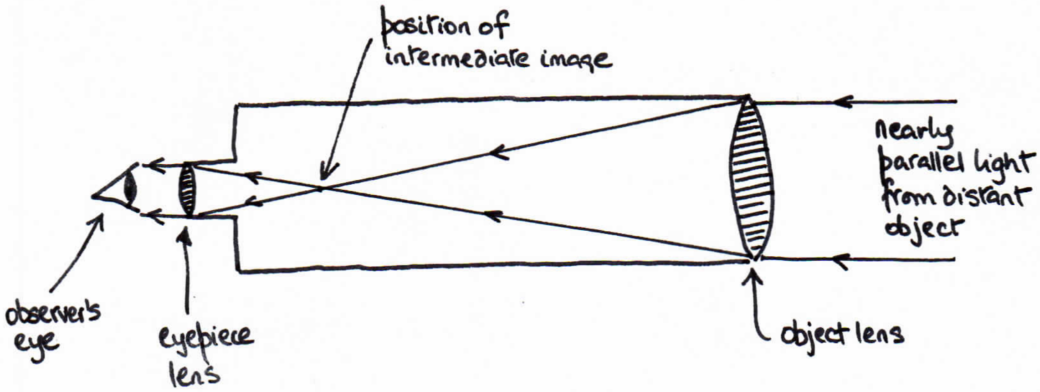
No program of observation of the heavens more capable of astounding the Seventeenth-Century world could have been attempted. By a strange quirk of human physiology, a vast array of phenomena in the Solar System lies just below the threshold of perception by the unaided eye but can be seen clearly with even the most rudimentary telescope--a reason for the great popularity of amateur astronomy with modest instruments even to this day. In just ten months Galileo's telescopic exploration of the heavens demolished almost every expectation of the Aristotelians concerning the nature of the celestial bodies.

In 1610, the year following Kepler's "New Astronomy" Galileo published his amazing discoveries in a book entitled

> "The Starry Messenger - Unfolding Great and Marvellous Sights, and proposing them to the Attention of Everyone, but especially Philosophers and Astronomers".

Unlike Kepler, whose rambling treatises mixed mathematical science with huge doses of mystical speculations, Galileo wrote briefly and clearly, not in Latin but in the Italian vernacular. There was no possibility of mistaking the nature of Galileo's messages from the stars:

> "About 10 months ago a report reached my ears that a Dutchman had constructed a telescope, by the aid of which visible objects, although at a great distance from the eye of the observer, were seen distinctly as if near ... A few days later I received confirmation of the report in a letter written from Paris by a noble Frenchman, Jacques Badovere, which finally determined me to give myself up first to inquire into the principle of the telescope, and then to consider the means by which I might compass the invention of a similar instrument, which a little while after I succeeded in doing ... At length, by sparing neither labour nor expense, I succeeded in constructing for myself an instrument so superior that objects seen through it appeared magnified nearly a thousand times, and more than thirty times nearer than if viewed by the natural powers of sight alone".



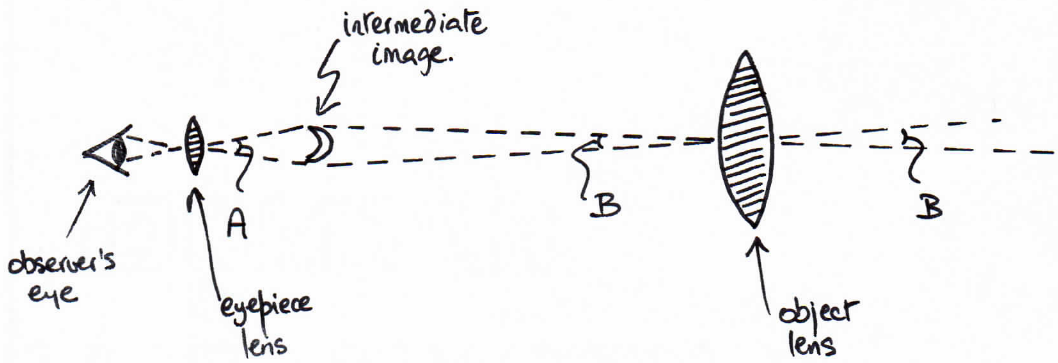


Fig. 11 A simple telescope arrangement, using two lenses (<u>refracting</u> telescope)

a) The arrangement of the lenses when the telescope is focussed on a distant object. The nearly-parallel light from the object is brought to a focus at a point a short distance in front of the eyepiece.

b) Magnification of the extended image of an object such as the Moon. The angle subtended by the intermediate image at the object lens (angle B) is equal to the angle subtended by the distant object at the telescope. The optical arrangement allows the eye to be placed so close to the intermediate image that this image subtends a <u>larger</u> angle A at the observer. The <u>magnification</u> is then $(A \div B)$.

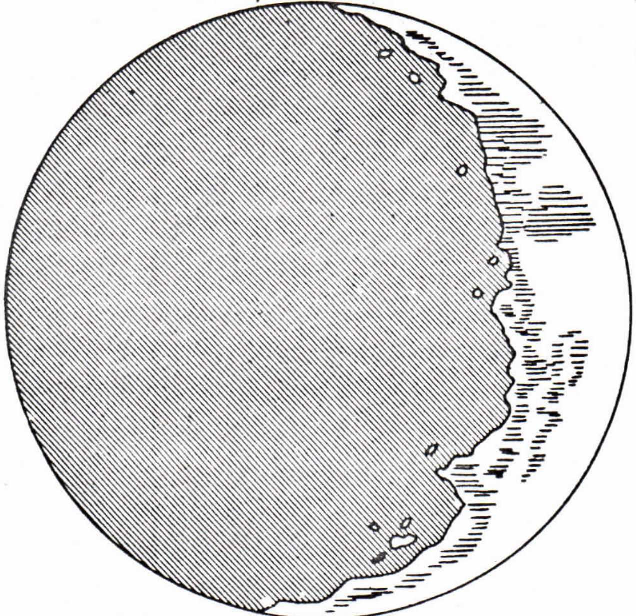
Galileo turned this instrument upon the Moon, and thus became the first Man to behold the rugged landscape of craters and mountains and the large dark flat areas which we now know to be boulder-strewn plains. His sketches showing the principal types of feature are reproduced in Figure 12. Galileo correctly interpreted the bright spots just inside the dark half of the Moon's sphere as the sunlit peaks of high mountains whose lower slopes were still in darkness before the lunar sunrise; he then used the phenomenon to estimate the heights of mountains on the Moon, concluding that some peaks were as much as 22,000 feet high. He thus saw that the Moon was not the perfect Aristotelian heavenly sphere, but a ruggedly landscaped world with features comparable to Earth's mountain ranges and valleys. A more powerful blow to Aristotelian concepts could hardly have been imagined, but many more followed in the next pages of the "Starry Messenger".

Galileo had found that his telescopes produced little or no increase in the apparent sizes of stars, yet magnified each known planet to a disc-like image. Even when brought "thirty times nearer" the stars appeared simply as brilliant points of light, whereas the faces of the planets became clearly resolved. This was consistent with Aristarchos' concept that the stars were vastly more remote than the Sun, so that even under the magnification of Galileo's telescope their true features could not be distinguished.

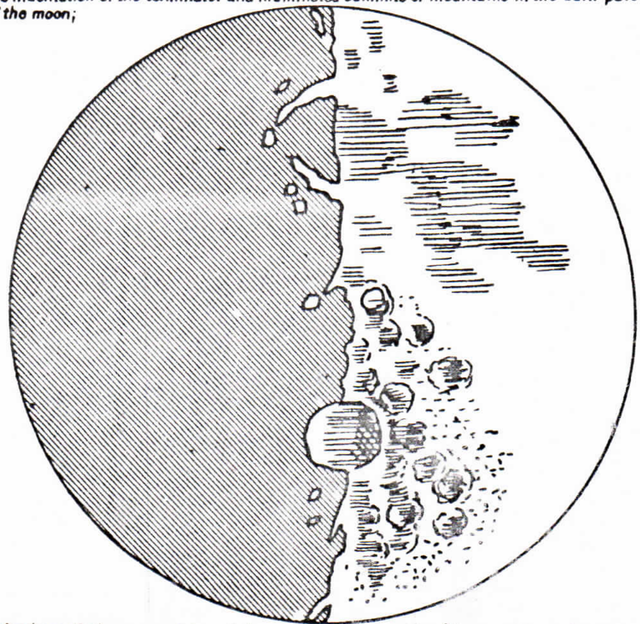
The telescope did however reveal one totally unexpected aspect of the stars--their vast number. The unaided eye forms an image using the light which passes through the open aperture of the eye-pupil--typically about one square millimetre in area; the telescopic image is formed from all the light which passes through its first (or "object") lens--in Galileo's instrument the area of the object lens was more than a thousand square millimetres. The sheer amount of extra light it concentrated into the image compared with that gathered by the eye made visible thousands of previously invisible faint stars. Galileo discovered that the band of pearly light across the sky known since ancient times as the "Milky Way" was actually:

> "A mass of innumerable stars planted together in clusters. Upon whatever part of it you direct the telescope straightaway a vast crowd of stars presents itself to view ... and whereas that milky brightness, like the brightness of a white cloud, is not only to be seen in the Milky Way, but several spots of a similar colour shine faintly here and there in the heavens, if you turn the telescope upon any of them you will find a cluster of stars packed close together".

Sketches by Galileo to shew:–



the indentation of the terminator and illuminated summits of mountains in the dark part of the moon;



the shape of a lunar mountain and of a walled plain. Galileo: 'Sidereus Nuncius', Venice 1610.

Figure 12 – Reproductions of two of Galileo's sketches from the "Starry Messenger", showing lunar craters and their mountain peaks just inside the unlit half of the Moon.

As long as the number of stars in the sky remained countably finite, the concept of a finite Universe had seemed preferable to the open-ended heliocentric view, and Aristarchos' concept of an infinite distance to the stars found little sympathy. But Galileo's discovery of uncountable numbers of faint stars opened the door to another infinity--to the notion that Man's experience of the Universe had hitherto been limited by the shortcomings of his sense-perceptions, and that an immense realm beyond that of the known stars remained to be explored.

Galileo turned next to Jupiter, and found around it four bright "stars" in a line. He soon realised that these "stars" were travelling across the sky with Jupiter, oscillating back and forth across the planet's bright disc. Every cloudless night for two months Galileo noted the positions of these "stars" around Jupiter--and what he saw (Figure 13) convinced him that here was a miniature Solar System on the Copernican model:

> "Since they are sometimes behind, sometimes before Jupiter, at like distances and withdraw from this planet toward the East and towards the West only within very narrow limits of divergence and since they accompany this planet when its motion is retrograde and direct, it can be a matter of doubt to no-one that they perform their revolutions about this planet ... Moreover it may be detected that the revolutions of the stars which describe the smallest circles around Jupiter are the most rapid ... We have a notable and splendid argument to remove the scruples of those who can tolerate the revolution of the planets around the Sun in a Copernican system yet are so disturbed by the motion of one Moon about the Earth ... that they consider that this theory of the constitution of the Universe must be upset as impossible".

If in later years Galileo had thought to test the validity of Kepler's Laws of Planetary Motion within the satellite system of Jupiter he would have confirmed that Jupiter exerted an influence on its satellites that was identical to that which Kepler said the Sun must exert on its family of planets. Indeed, the above passage shows that Galileo had already noticed that Jupiter's satellites moved as if in a miniature Copernican Solar System--the innermost travelling fastest--but the detailed implications of his discovery escaped him. It provided powerful evidence for multiple centres of motion in the Solar System, however, and thus helped the Copernican model (in which the Moon moved around Earth while Earth moved around the Sun) gain credence relative to the Ptolemaic model, in which all motions had the same centre.

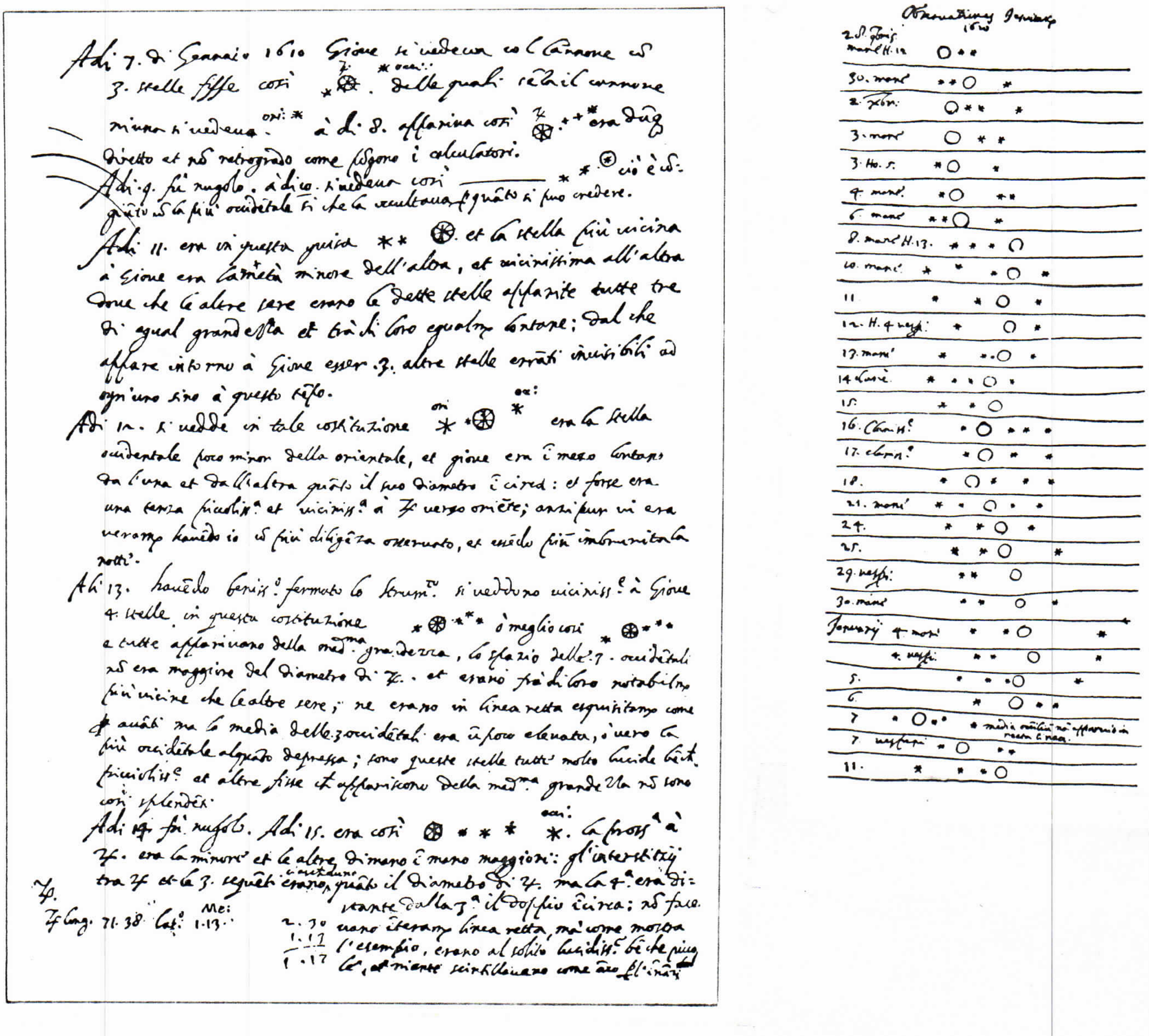


Figure 13 — Notes and sketches by Galileo recording his early observations of the satellites of Jupiter he discovered in 1610.

7. Galileo's Anagrams

Galileo's discoveries had an immediate impact that Kepler's could not have had. You did not have to be a skilled mathematician or an erudite scholar to read and understand Galileo. He spoke in relatively simple terms of phenomena which he could demonstrate, if necessary, to the senses of anyone who would carefully view the skies through his telescope. Needless to say, evidence from beyond the realm of human sense-perceptions was not immediately welcomed by all. Some of Galileo's contemporaries regarded his telescopes with profound suspicion --even considering that they might be the work of the devil because they could delude the human senses into seeing phenomena that contradicted the Aristotelian geocentric world-view that was supported by the Church. Yet Galileo could use a telescope to view the flag of a distant ship sailing into port and demonstrate that it indeed identified the arriving vessel correctly--giving the telescope's user an advantage in knowing which merchant to bargain with at the dockside for the incoming cargo. There was an undeniable immediacy about Galileo's work, both in the practicality of the instrument he used and in his manner of reporting it.

Galileo's astronomical instrument was unwieldly however and its magnification resulted in a severely limited field of view on the sky. Not everyone who looked through it or similar devices saw the satellites of Jupiter as clearly as he did. Their existence became a matter of controversy, into which Kepler entered on Galileo's side, thereby adding his prestige as Imperial Astronomer to Galileo's discovery. In fact Kepler had few grounds for so doing, as he had only rudimentary telescopes at Prague and thus no comparable observations of his own. Galileo might have been grateful to Kepler, given that Kepler was then in a position more exalted than Galileo's own. Yet Galileo made up his mind to accept Kepler's support without paying any real attention to Kepler's own ideas. Kepler had sent him copies of both the "Cosmic Mystery" and the "New Astronomy" as they were published, hoping for Galileo's comments in return. Galileo answered that he had read only the introduction to the "Mystery" and never answered the second letter. He also ignored a letter from Kepler enclosing the latter's own pamphlet supporting the contents of the "Starry Messenger". It was only when Kepler wrote yet again politely requesting to know who else had confirmed the telescopic discoveries that Galileo deigned to reply, saying that future publications of his would elaborate on the matter and promising that he would soon send telescopes of his own making to his "friends". Evidently he did not count Kepler among his "friends", for he never wrote to him again and never sent him a telescope.

He did send, not to Kepler, but to the Emperor in Prague, a communication which read: "SMAISMRMILMEPOETALEUMIBUNENUGTTAURIAS", with instructions that it might be conveyed to Kepler. Thus Kepler received Galileo's next telescopic discovery as an anagram which he was required to decipher! Kepler solved it as "Salve umbistineum geminatum Martia proles"--"Hail, burning twin, offspring of Mars". He concluded that Galileo had found two satellites of Mars. This fitted Kepler's ideas of celestial symmetries--if Earth had one satellite (2^0) and Jupiter four (2^2), then two (2^1) was an appropriate number to accompany Mars. Only three months later did he learn, via his Emperor, that the solution was "Altissimum planetam tergeminum observavi"--"I have seen the highest planet in triplet form". Galileo had seen Saturn's disc accompanied on each side by objects whose form he could not quite make out, but definitely not a pair of moving satellites. The phenomenon he had glimpsed was the "rings" of Saturn--yet another departure from the perfect Aristotelian spheres and thus a further blow to the Aristotelian world-view. Kepler was mortified that Galileo withheld his discoveries from him in this fasion, and on receiving a second anagram wrote to Galileo: "not to withhold from us the solution for long. You must see that you are dealing with honest Germans ... consider what embarrassment your silence causes me."

Galileo's second anagram, whose solution Kepler received only indirectly, announced his discovery of the phases of Venus. This was a strong argument against the Ptolemaic system, for Venus showed the "full" phase when apparently smallest (and hence most remote from Earth) and the "crescent" phase when apparently biggest (and hence closest to Earth). This was consistent with the Copernican picture (see Figure 14) but not with Ptolemy's, and so provided further evidence against the ancient views. The fact that both the Venus phases and Jupiter's satellites could easily be accommodated in Tycho's system was conveniently overlooked by Galileo.

By the end of 1610 Kepler had given up trying to persuade Galileo to share his discoveries with him, and the two men pursued their careers without any further direct interaction. At first sight Galileo's treatment of Kepler is despicable--yet we must remember that Galileo was an articulate scholar who published his discoveries with the minimum of excess verbal baggage required to

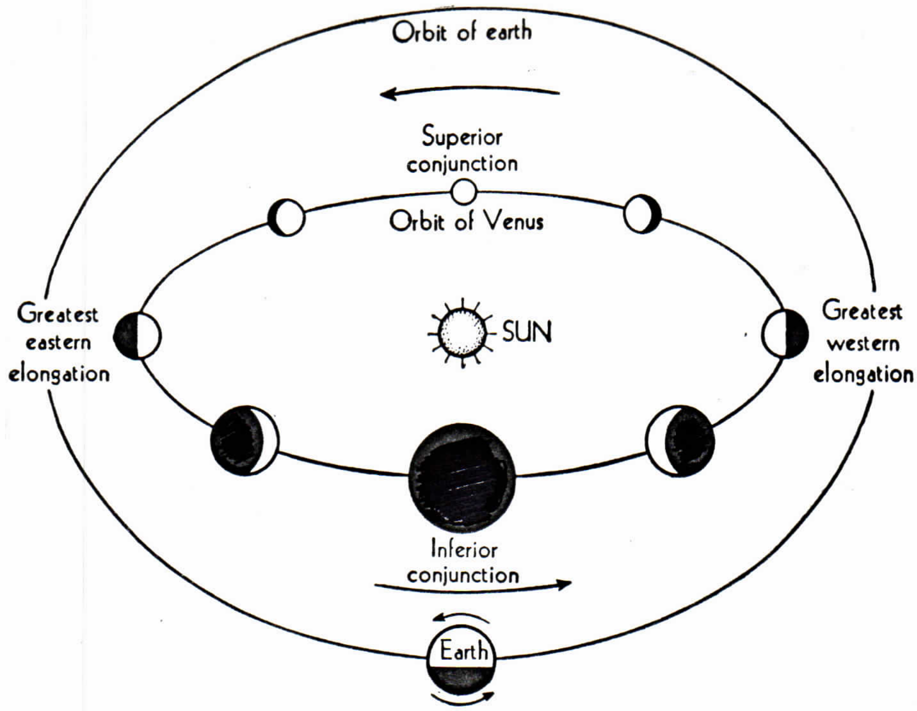


Fig. 14. Phases of Venus in perspective

The variation in apparent size of Venus, accompanied by the changing phases, first observed by Galileo. The diagram also illustrates the rationale for this cycle in the Copernican system.

flatter his patrons, while Kepler's books were rambling mystical mazes in which his Laws of Planetary Motion were deeply embedded. Galileo's reaction to Kepler's published work must have been one of total alienation. Galileo's guiding principle was to believe only that which he had seen before his own eyes, and Kepler's willingness to endorse Galileo's discovery of Jupiter's satellites without having seen them for himself may have earned him Galileo's scorn, however useful his support at the time. Kepler's pleas for a Galilean telescope may have reinforced this. There is no evidence that Galileo ever took Kepler's ellipses seriously--throughout his career he defended the Copernican system as Koppernigk himself had left it, epicycles and all. To Kepler this must have seemed incredibly obtuse, and contrary to Galileo's own principles--it was, after all, Tycho's observations which had persuaded Kepler to discard the original Copernican system. After the incident of the anagrams, Kepler resigned himself to proceeding without special contact with Galileo.

Galileo's telescopic discoveries created an atmosphere of disbelief in Aristotelian geocentrism throughout Europe which undoubtedly helped Kepler's new view of the Solar System to gain acceptance in scholarly circles. To this extent the one man's work assisted the other's. But a closer association between the two might have led them to combine Galileo's insights into the nature of motion with Kepler's solution to the planetary orbits. This combination was finally to clear the fog from the bridge so that Isaac Newton could walk confidently across to produce a totally reformed science of physics and astronomy.

8. Galileo and the Motion of Falling Bodies

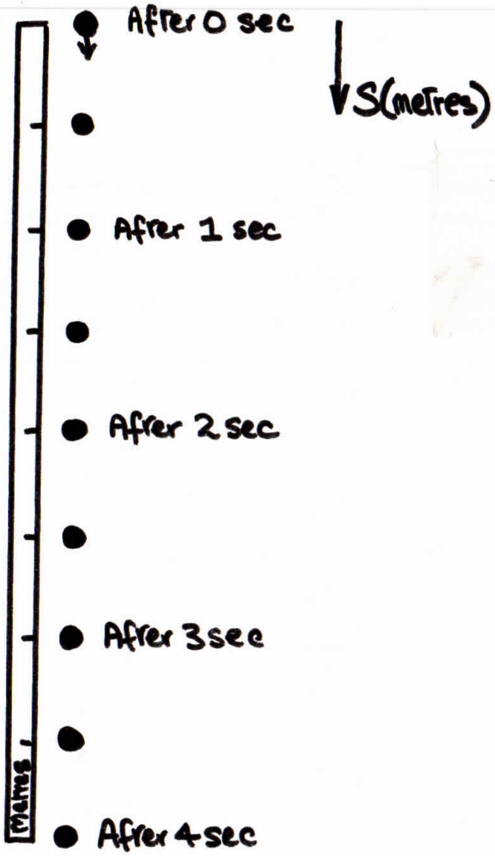
Galileo's attack on Aristotelian beliefs was not confined to his observations of the heavenly objects; he also demonstrated by experiments that the Aristotelian description of motions near Earth's surface left much to be desired. His work in this area began the reformation of concepts of motion which was to be an essential ingredient of the new world-view.

The difficulties encountered by the Aristotelians in explaining the motion of a projectile, such as a javelin or an arrow, were mentioned briefly earlier when we discussed the rejection of Aristarchos' heliocentric views. The Greeks had classified motions under two headings--"natural" and "violent". "Natural" motions resulted from the desire of heavy matter (the elements Earth and Water in the Aristotelian scheme) to reach its "proper place"--the free fall of a stone dropped over a cliff was an example of "natural" motion. "Violent" motions resulted from the expenditure of effort, or force, by Man or another animal. The motion of a cart pulled by an ox was a violent motion; the material of the cart, prevented by the road from getting any closer to its "proper place", would remain at rest unless forced to roll along the road by a pull from the ox. Early axle bearings had sufficient friction that the motion produced by the ox would soon cease if the ox stopped pulling. Thus a push or a pull--a force--was considered necessary to maintain "violent" motion.

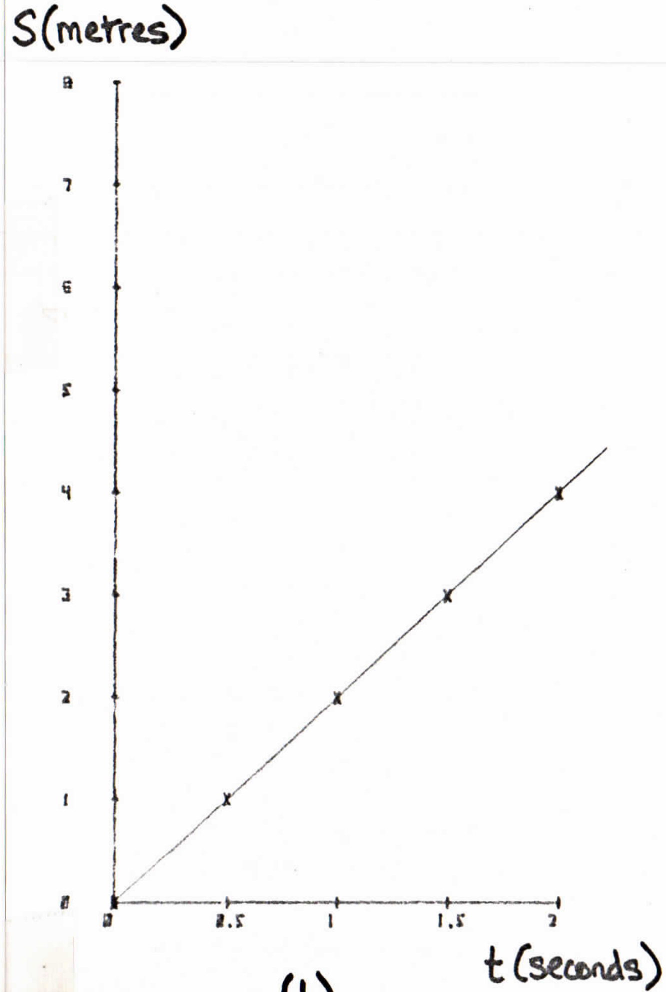
The flight of a projectile was considered to be a combination of "natural" and "violent" motions. The problem was to understand the nature of the force which supposedly maintained the sideways motion of the projectile once it had been launched. The eventual fall to ground required no force: this was the "natural" motion arising from the wish of the projectile's material contents to be close to the Earth. Aristotelians supposed that some movement of the air through which the projectile moved supplied the force which maintained its "violent" sideways motion.

Galileo recognized the weakness of the Aristotelian doctrine at this point. The different rates of fall of, for example, a heavy metal ball and a leaf were interpreted by Aristotelians as resulting from the greater desire of the ball's mass to seek its proper place. Well before Galileo became famous Simon Stevin of Bruges had performed experiments in which balls of different weights were let fall together through distances of some ten metres onto a plank--the sounds of the impacts on the plank were used to mark the relative times of arrival of the different balls. By 1605 it was well-known that such experiments showed the time of fall not to depend significantly on the weight of the ball, as Aristotelian theory said it should. The famous story of Galileo dropping balls of differing weights from a parapet on the Leaning Tower of Pisa in order to demonstrate this is not substantiated by any reliable contemporary accounts, or even by any of Galileo's own writings. The important point however is that Galileo was well aware of the discrepancy between the Aristotelian statement and direct observation. He also (correctly) surmised that a metal ball fell faster than, e.g. a leaf because of the greater effect of air resistance on the fall of a light flat body than on the fall of a heavy compact one. This deficiency of the Aristotelian description encouraged Galileo to consider the nature of motion more fully.

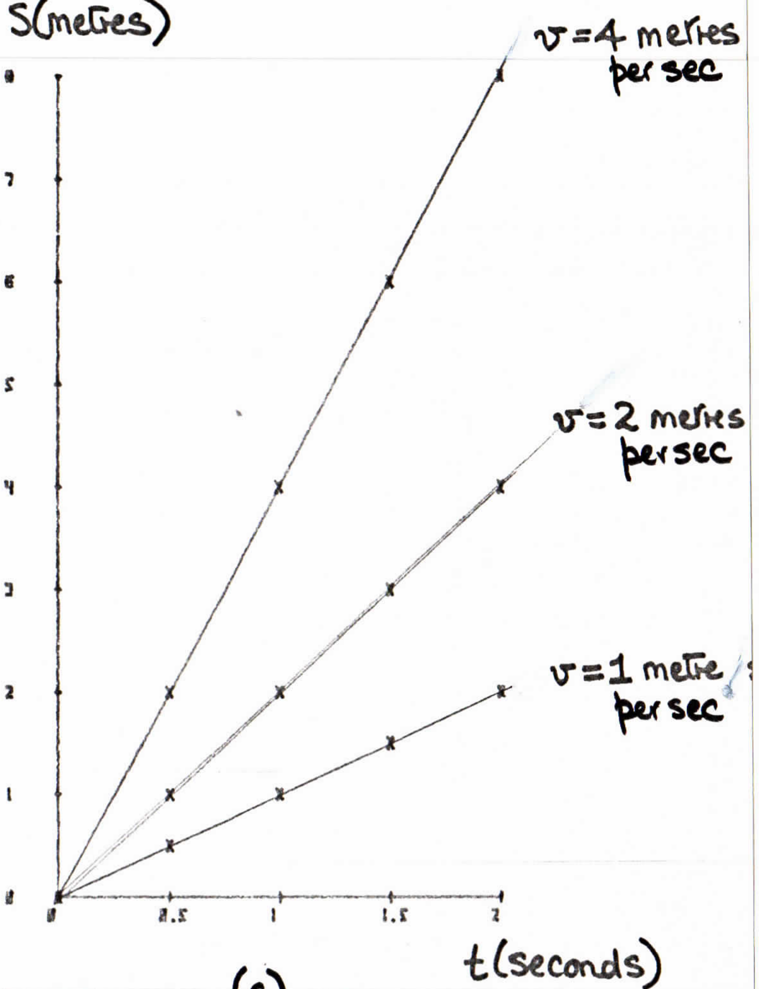
The Aristotelians had also supposed that a body descending by "natural" motion fell at constant speed, as in Figure 15a, which represents descent at a


After 0 sec
S(metres)
After 1 sec
After 2 sec
After 3 sec
After 4 sec
Metres

(a)


S(metres)
t (seconds)

(b)


S(metres)
v = 4 metres per sec
v = 2 metres per sec
v = 1 metre per sec
t (seconds)

(c)

steady speed of two metres per second. It will be important in what follows to make very careful statements about bodies in motion, so we will begin here to introduce the precise language which we will ultimately use for the description of motion. Suppose we describe the descent of a vertically falling body by noting its position in space (s) at a variety of times (t) during its fall. The quantity s could be measured in metres vertically downwards from the point at which the body was released, and t could be measured in seconds from the instant at which it was released. The motion in Figure 15a could then be represented either as a graph of s against t, which would be a straight line (Figure 15b), or by the algebraic equation

$$s = 2t$$

which specifies the arithmetical rule for calculating s (in metres) at any given time t (in seconds) when the steady rate of descent is 2 metres per second. Figure 15b and the equation are exactly equivalent; the equation could be used to calculate a series of points along the graph, each of which would correspond to a different stage in the fall of the body. According to the Aristotelians, bodies with different weights should have fallen at different steady speeds--as represented by the set of graphs in Figure 15c. All possible graphs of this kind can be summarised in the single equation

$$s = vt$$

where v symbolically represents the steady speed in metres per second (we use 'v' in anticipation of the definition of velocity, which is to follow). The graphs in Figure 15c correspond to $v = 1$ metre per second, $v = 2$ metres per second and $v = 4$ metres per second.

Scholars such as Nicolas Oresme at the University of Paris in 1330 had concluded that the distance dropped by freely falling bodies does not in fact increase linearly (i.e. as in a straight-line graph) with time--that the equation $s = vt$ does not describe the observed motions. Galileo performed some ingenious experiments to show the correct relationship. He argued that a ball rolling in a straight groove down an inclined ramp descends vertically as if falling freely, but with an added horizontal motion which slows down the phenomena and thus makes them easier to measure than those of vertical free fall. By repeatedly letting balls roll from rest down long ramps and using his own pulse as a "stopwatch", Galileo established that the distance rolled, s, increased as the square of the elapsed time, t, i.e. that

$$s = kt^2$$

where k was a constant which depended only on the angle of the ramp, not on the weight of the ball. For a horizontal ramp (on which a ball initially at rest would not move at all), k = 0. Galileo found that the relation $s = kt^2$ held for all non-horizontal ramps, the value of k being greater the larger the angle of the ramp above the horizontal (Fig. 16).

Now consider the graph of the relationship $s = kt^2$, shown with k = 2 in Figure 17a. It is not a straight line, but an upward-turning curve. After travelling only 2 metres in the first second, the body travels a further 6 metres in the second one, so the total distance travelled after two seconds is 8 metres. Figure 17b compares the graph of this motion with that corresponding to s = 2t. The graphs cross at the value s = 2 metres, t = 1 second. Before t = 1 second, the body moving according to the equation $s = 2t^2$ travels less distance in any given time--i.e. it travels at a slower speed--than the body moving according to s = 2t. After t = 1 second, the body moving as $s = 2t^2$ travels faster than the other. Obviously the result that Galileo demonstrated amounts to the statement that balls rolling down ramps (and by extrapolation freely falling bodies also) do not descend at constant speed as expected by the Aristotelians, but that their speed increases as they move--i.e. they are accelerating.

To help us describe Galileo's observation still more closely, we will now become more precise in our use of the terms speed and acceleration. If we simply defined

$$\text{Speed } v = \frac{\text{Distance Travelled}}{\text{Time Taken}} = \frac{s}{t}$$

then the "speed" would not give unique information about the way in which the balls rolled during Galileo's experiment. For example, after one second the ball rolling so that $s = 2t^2$ would travel the same total distance (2 metres) as one rolling so that s = 2t. Are their "speeds" both 2 metres per second? In a sense they are, because both arrive at the same place at the same time, but the one following $s = 2t^2$ travels much further in the next second. All they have in common is their average speed in the first second. If we compare their average speeds using the distances travelled in the first half second, we find that the ball following $s = 2t^2$ travelled only 1/2 metre in that 1/2 second, for an average speed of 1 metre per second: a ball following s = 2t

(continued) ...

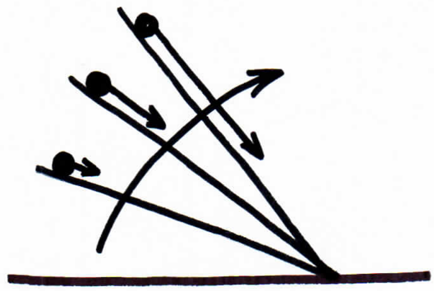
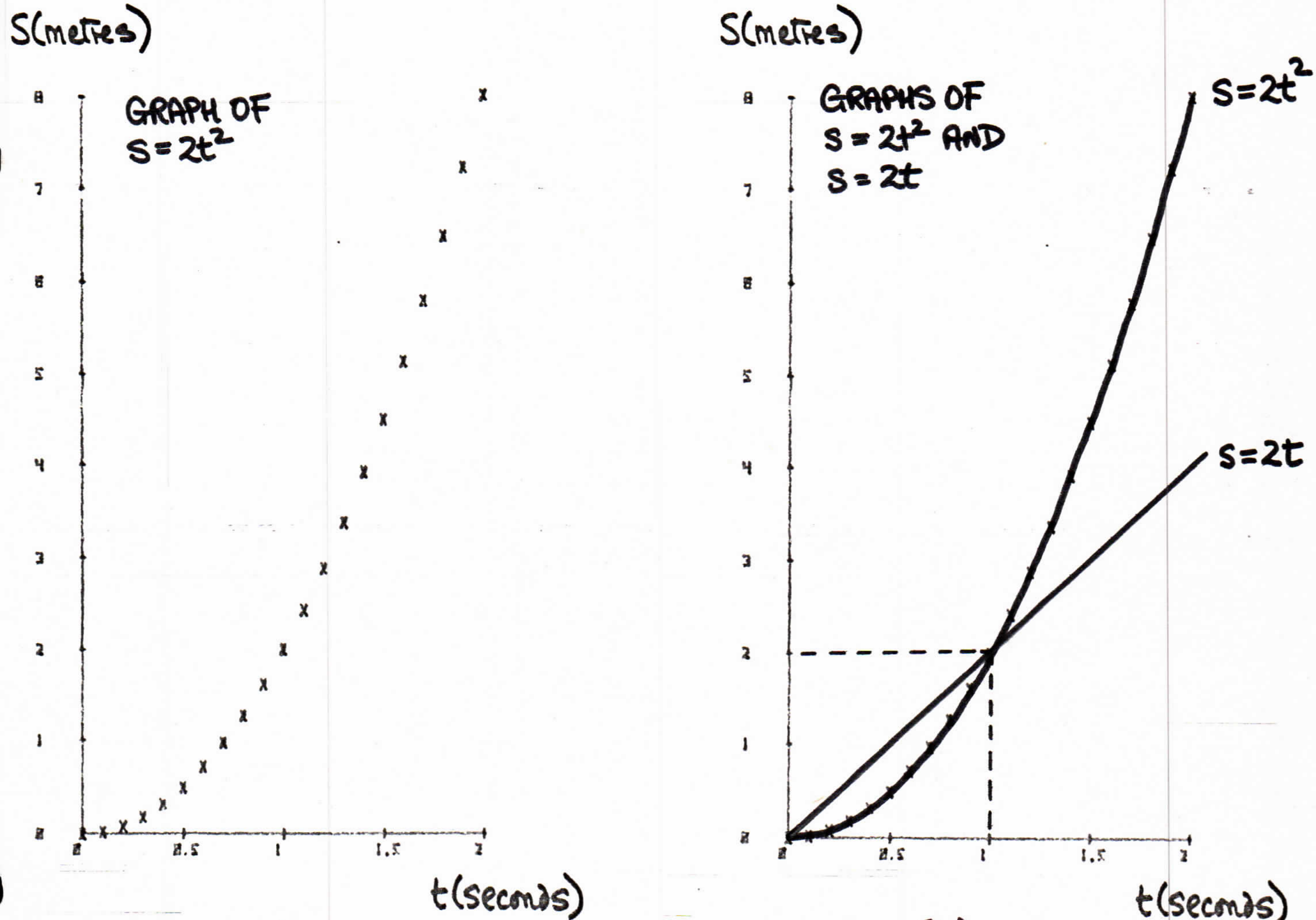


Figure 16
Arrows indicate distance travelled in unit time increases (k increases, see text) as ramp is made more nearly vertical.



(a) (b)

Figure 17

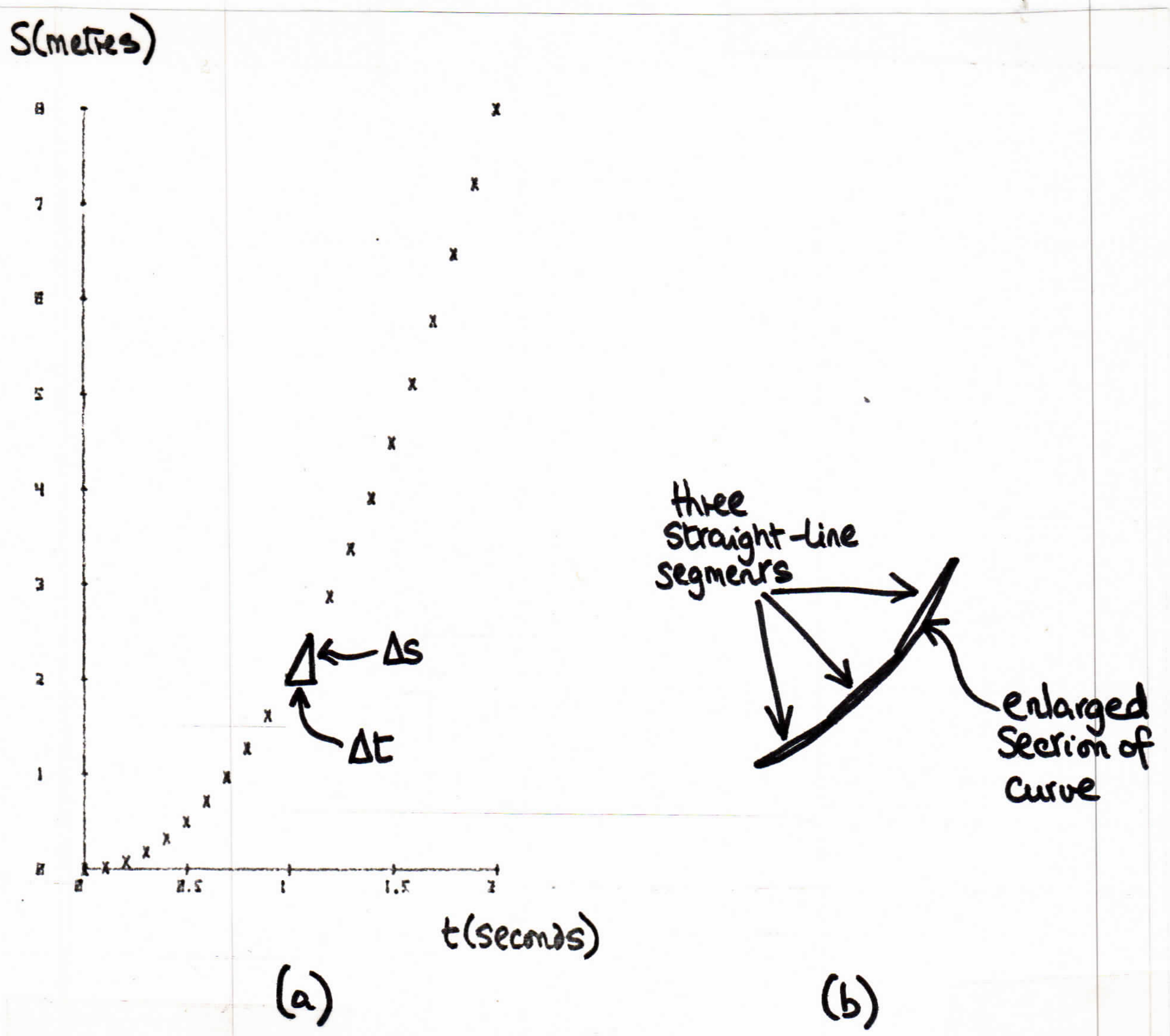
would travel a full metre in that 1/2 second, for an average speed of 2 metres/second. Similarly, over the first two seconds, the average speed for "$s = 2t^2$" is 8 metres in 2 seconds, or 4 metres per second, while that for "$s = 2t$" is 4 metres in 2 seconds, or 2 metres per second. Obviously the "average speed" does not completely describe the $s = 2t^2$ motion, precisely because that motion _is_ accelerated. In a real descent the average speed depends on the time interval over which we measure it. It is obvious though that the ball rolling down a ramp (or falling freely) so that $s = 2t^2$ has some definite speed _at any given instant_. The problem is that this speed changes (here, increases) from one instant to the next. A useful concept in this case might be a set of average speeds measured over a series of time intervals each so short that the speed does not change significantly during it. Consider Figure 18a, where we again show the graph of $s = 2t^2$. Suppose that at $t = 1$ second we measure the distance Δs (metres) travelled by the ball in a very short time interval Δt (seconds). (We will always use the Greek symbol Δ--"delta" --to denote a small _change_ in a measured quantity.) Then if _Δt is sufficiently small_, the distance Δs travelled during Δt by the ball rolling so that $s = 2t^2$ will be the same as that travelled in the same time by a ball rolling with constant average speed:

$$v = \frac{\Delta s}{\Delta t}$$

Indeed, if we make the time interval Δt small enough, then the curve of $s = 2t^2$ could be closely approximated (Figure 18b) by a series of short straight-line segments, on each of which the ball's speed was constant. The smaller we take Δt, the smaller would be the short distance Δs described by each segment, and the closer the sequence of linear segments would approximate the actual continuous graph of s against t; hence the better the description of the actual motion. If we carry this to the limit of Δt approaching zero (symbolically, $\Delta t \to 0$), and the number of segments becoming very great, _the approximation can be as good as we wish_: This concept is the basis of the precise definition of the "instantaneous velocity" at any given moment as the value of the average speed during a time interval Δt around that moment, in the limit $\Delta t \to 0$. Symbolically, we write this as

$$\text{Instantaneous velocity } v = \lim_{\Delta t \to 0} \left(\frac{\Delta s}{\Delta t} \right)$$

The mathematical technique of _calculus_ is a set of rules for assigning definite



S(metres)
8
7
6
5
4
3
2
1
0
0
0.5
1
1.5
2
t(seconds)
Δs
Δt
(a)
three
straight-line
segments
enlarged
section of
curve
(b)

Figure 18

numerical values to such limits, once the relation between s and t is specified. The fact that the instantaneous velocity at any instant is the slope (gradient) of a graph of s against t at that value of t should be evident from Figure 18b.

Having made these definitions we can now state Galileo's deduction more clearly--the instantaneous velocity of a falling body increases with time. But according to what rule? Suppose at some instant t the instantaneous velocity is v, and during the next short time interval Δt the instantaneous velocity increases by an amount Δv. Then by exact analogy to the procedure used to define instantaneous velocity we can define an instantaneous acceleration, or rate of change of velocity:

$$a = \underset{\Delta t \to 0}{\text{Limit}} \left(\frac{\Delta v}{\Delta t} \right)$$

From now on we will make an abbreviation: we will consider the word "instantaneous" to be implied whenever we speak of velocity and acceleration. We will also consider the taking of limiting values appropriate to very small time intervals to be implied, so that we can write, for example, the shorthand forms:

$$v = \frac{\Delta s}{\Delta t} \quad \text{and} \quad a = \frac{\Delta v}{\Delta t}$$

Some mathematical analysis convinced Galileo that his measurements could be summarised in the following very simple way: the acceleration a of all balls rolling down a ramp at a given angle was a constant, independent of the weight of the ball. The numerical value of a depended only on the angle between the ramp and the horizontal. (For a discussion of the mathematical basis of Galileo's conclusion see Appendix 1.) From this, he extrapolated that bodies falling freely to Earth--the "natural" motion of the Aristotelians--do not fall at constant velocity, but with constant acceleration, the numerical value of the acceleration being independent of the weight of the body. He stated this conclusion, together with some further mathematical deductions which will not concern us, in a treatise entitled "Discourses and Mathematical Demonstrations concerning Two New Sciences Pertaining to Mechanics and Local Motions" which was published in 1638. The description of this work as "new science" was no overstatement, for Galileo was indeed in the process of setting the science of motion onto the new path, combining careful observation and mathematical interpretation, which would lead to deeper understanding both of the Solar System and of terrestrial phenomena.

9. Galileo and the Motion of Projectiles

Galileo attacked the vexed question of the motion of projectiles on two fronts--experiment and theory. He carried out experiments to provide data on the trajectory of an object which moved sideways and fell to the ground at the same time. He knew, as a consequence of his analysis of motion under constant acceleration (see Appendix 1), that a ball rolling down a ramp acquired a final velocity proportional to the square root of the distance it had rolled. This provided him with a means of launching projectiles at controlled velocities; his experimental arrangement is shown schematically in Figure 19. A small bronze ball was rolled down the ramp, and deflected at the end so that it left the ramp moving as horizontally as possible. Galileo then measured the horizontal distance from the end of the ramp to the point of the ball's impact on the floor (he smeared the ball with ink to mark this) for a number of different starting positions of the ball on the ramp--corresponding to a number of different "launch velocities". These measurements allowed him to check a theoretical calculation which he first made in 1609 (the year of his telescope and Kepler's "New Astronomy"), predicting that the path of such a projectile should be a section of the curve known as a parabola.

As early as the Sixth Century A.D. the philosopher John Philiponus had argued that a thrown body had an attribute called "impetus" which carried it along sideways (without the need for any continuing "push" or "pull") while it fell to the ground. This concept--that a piece of matter, once set in motion, tends to remain in motion--was a cornerstone of Galileo's discussion of the parabolic path. His experiments with rolling bodies had indicated that the constant acceleration in their motions was intimately associated with the vertical part of their path. Not only had he observed that the acceleration was greater the more nearly vertical the ramp, but also that the acceleration became zero (i.e. the velocity was constant) if the ramp was horizontal and the ball was set in motion with a push. He correctly interpreted the slight slowing down of a real ball in a real horizontal ramp as resulting from friction between the ball and the ramp; experience with balls and ramps of different materials and of different roughness had led him to recognise that some loss of velocity was due to imperfections in the experiments and to distinguish the effects of these imperfections from the more fundamental phenomena which he sought to understand. He therefore made a theoretical analysis of the trajectory of a projectile by assuming its motion to be the combination of two independent motions: a vertical

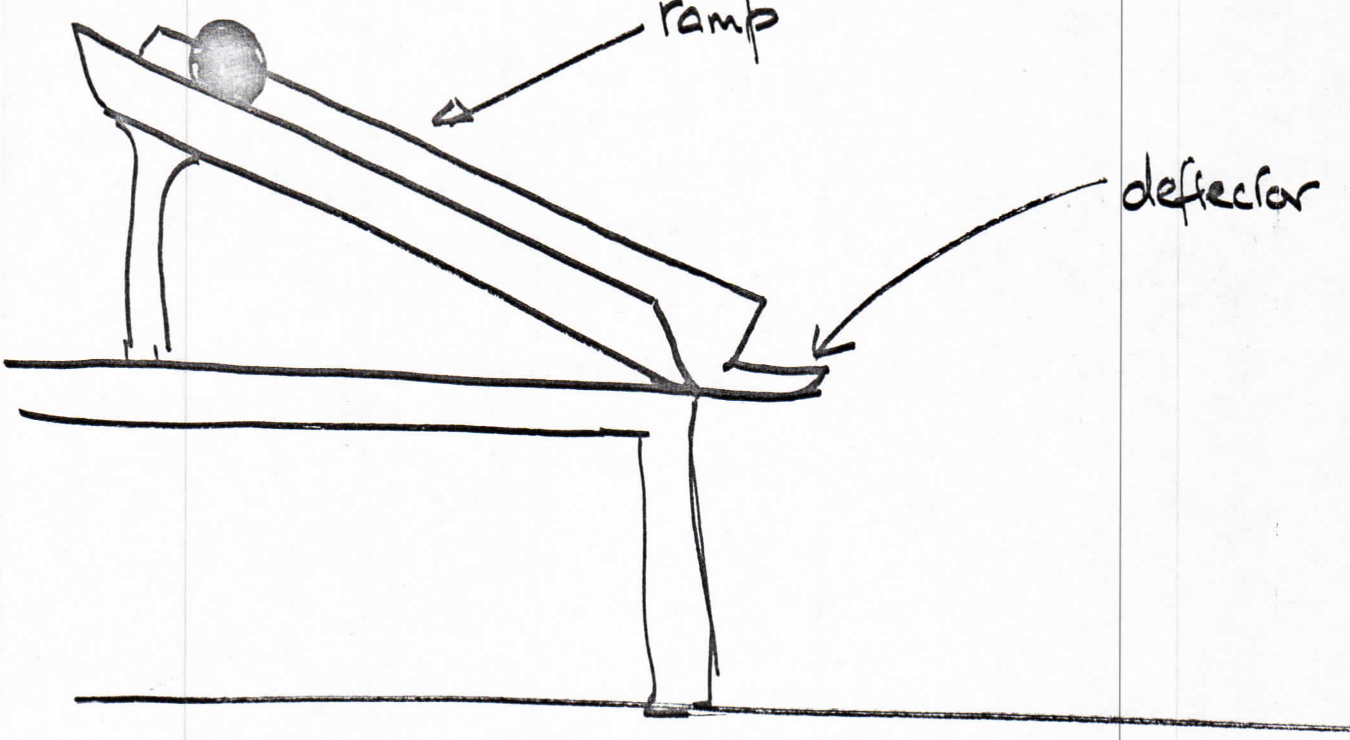


Fig 19

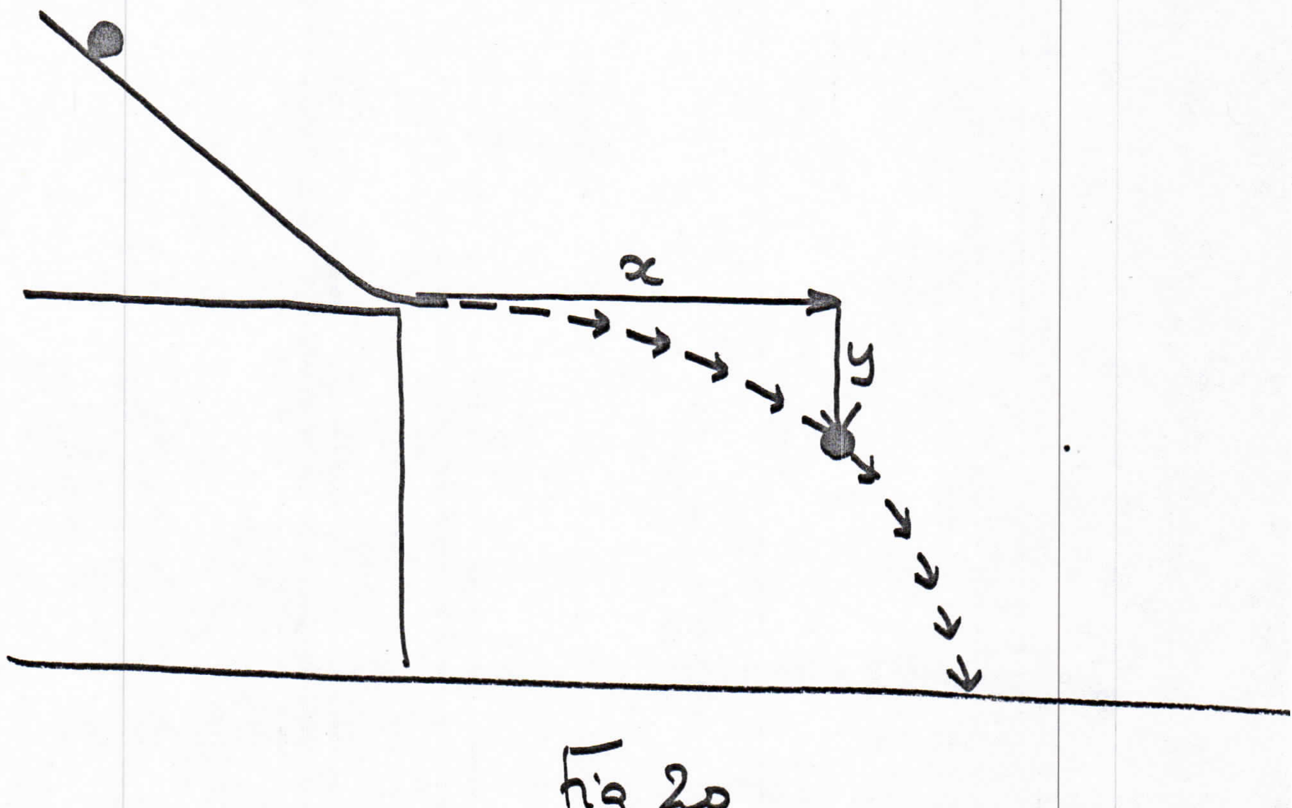


Fig 20

fall in which the acceleration was constant, and a horizontal motion in which the velocity was constant.

We can reproduce Galileo's analysis using Figure 20 to see how the problem has to be described mathematically. In his experiment, he had arranged that the ball left the deflector at the bottom of the ramp travelling horizontally, and with a known velocity, v. His interpretation of Philoponus' concept of "impetus" was that the ball's subsequent motion in the horizontal direction would be such that this horizontal velocity remained constant. If the horizontal distance travelled by the ball in time t seconds after leaving the ramp was x metres (Fig. 20) then the horizontal part of the motion should be described by the equation:

$$x = vt$$

with v constant. He argued that the motion in the vertical direction would be a free fall from rest (if the ball indeed had no downwards motion as it left the deflector). He therefore expected that in the same time t the ball would have fallen a distance y metres vertically (Fig. 20) where

$$y = kt^2$$

and k was the constant appropriate for a pure vertical fall with constant acceleration (see Section 8 above and Appendix 1). It was then a simple matter to express the horizontal distance travelled in terms of the corresponding vertical drop, eliminating the time of fall (which could not be measured accurately using Galileo's techniques). To do this, first rearrange

$$x = vt \quad \text{to the equivalent form} \quad t = \frac{x}{v}$$

and then substitute (x/v) for t in the equation describing the vertical motion:

$$y = kt^2 = k\left(\frac{x}{v}\right)^2 = \frac{kx^2}{v^2}$$

Thus it was possible to predict that the vertical drop should increase as the square of the horizontal distance travelled--which is the form of a parabola whose apex was at the end of the deflector. The equation also involves v^2 --the square of the velocity of the ball as it leaves the deflector--which Galileo could relate to the distance the ball had rolled down the ramp before reaching the deflector. He could therefore make a detailed comparison of his theory with his experiments, and was satisfied that, to within the uncertainties associated with imperfections in the experiment, the theoretical analysis matched what he observed to occur in the real world.

This agreement provided powerful confirmation of Galileo's two starting assumptions--1) that the vertical motion of a projectile has constant acceleration, and 2) that the horizontal motion of a projectile has constant velocity. The second assumption amounted to a new conception of what the Greeks had termed "violent" motion: it implied that a body, once set in motion, preserved its velocity in the absence of obvious external pushes and pulls. This attribute of matter, its tendency to remain in motion at constant velocity, had been obscured in Aristotle's time by the complications of friction. It was soon to be known as the property of "inertia".

10. Galileo's "Crime"

Galileo had made his critical studies of matter in motion during the first decade of the Seventeenth Century--the same decade that had witnessed the death of Tycho, Kepler's "New Astronomy" and Galileo's own telescopic discoveries. Yet the "Discourses and Mathematical Demonstrations" in which he set down his conclusions fully were not written until almost thirty years later. The intervening years were not exactly idle however; they were spent in a spirited advocacy of Copernican principles which had led to Galileo's arrest and confinement at his farm near Arcetri, where he wrote the "Discourses" during his last years.

Galileo's telescopic discoveries had convinced him of the falsity of Aristotelian cosmology and, perhaps too readily, of the value of the Copernican hypothesis which made the Earth a planet and, conversely, made the planets other "worlds". His studies of motion convinced him--again perhaps too quickly--that the Aristotelian objections to Earth's motion were invalid. Having found that matter when set in horizontal motion preserved its velocity without need of a continuing push or pull, he extrapolated (this time incorrectly) to the statement that uniform circular motion might similarly be preserved. If this were true, then matter near the Earth's surface, once given the Earth's daily rotation would retain that motion as it rose or fell; this would avoid the disruptive effects and Westward drifts predicted by the Aristotelians if the Earth rotated from West to East. Thus Galileo concluded, in effect, that Koppernigk had been correct when he argued that circular motion required no forces.

At this point Galileo unknowingly shut himself off from the final explanation of the phenomena of the Solar System. His false conclusion made him so ardent an advocate of strict Copernicanism, with all its perfect circles and epicycles, that he failed to appreciate the importance of Kepler's ellipses; even more important, he never became aware of Kepler's Third Law--the result, buried deeply in the "Harmony of the Worlds", which was the key that could unlock the secret of gravitation.

Galileo's belief in the Copernican system as a physical reality rather than as a computational method was not concealed from his church. Indeed he tried to persuade theologians to reassess their discussions of certain Biblical texts, so that interpretations of "God's word" would not conflict with observable facts. He was a pious man who wished to restrain his church from unnecessary conflict with the emerging sciences. But he was not a tactful man, nor did he suffer the opinions of others gladly if they conflicted with his own. By 1616 he was in collision with influential Jesuits who denounced him before the Inquisition for advocating heretical views. The issues in dispute are summarised in the following extract from a letter dated 12 April 1615 from Cardinal (later Saint) Roberto Bellarmino, a powerful Jesuit, to Paolo Foscarini, a Carmelite monk who had written a discourse favouring the Copernican system:

> "If there were a real proof that the Sun is in the centre of the Universe, that the Earth is in the third heaven, and that the Sun does not go around the Earth, then we should have to proceed with great circumspection in explaining passages of Scripture which appear to teach the contrary, and rather admit that we did not understand them than declare an opinion to be false which is proved to be true. But, as for myself, I shall not believe such proofs until they are shown to me. Nor is it a proof that, if the Sun be supposed at the centre of the Universe and the Earth in the third heaven, everything works out the same as if it were the other way around. In case of doubt we ought not to abandon the interpretation of the sacred text as given by the holy Fathers.
>
> I may add that the man who wrote: 'The Earth abideth for ever; the Sun also riseth, and the Sun goeth down, and hasteth to the place whence he arose, was Solomon, who not only spoke by divine inspiration but was wise and learned above all others in human sciences and in the knowledge of created things. As he had all this wisdom from God Himself, it is not likely that he would have made a statement contrary to a truth, either proven or capable of proof. If you tell me that Solomon speaks according to the appearances, inasmuch as though the Sun seems to us to revolve, it is really the Earth that does so, just as when the poet says: 'The shore is now receding from us', I can answer that, though it may appear to a voyager as if the shore were receding from the vessel on which

> he stands rather than the vessel from the shore, yet he knows this to be an illusion and is able to correct it because he understands clearly that it is the ship that is in movement. But as to the Sun and the Earth, a wise man has no need to correct his judgement, for his experience tells him plainly that the Earth is standing still and that his eyes are not deceived when they report that the Sun, Moon and stars are in motion."

What impact could lunar mountain ranges, Jupiter's satellites, and conclusions drawn from the motions of projectiles have against such a web of argument? Bellarmino states that if there be alternative descriptions which will fit the appearances, then the alternative which is consistent with Scripture and with the divine inspiration of such as Solomon must be accepted. He also refers to the relativity of motion, and hence to a possibility of alternative descriptions which Galileo could not deny. Indeed Galileo's own analysis of the parabolic path depended on the notion that relative to its uniform horizontal motion the projectile drops vertically as if from rest. And the astronomers themselves had offered alternative descriptions of the planetary motions, e.g. Koppernigk's alternative sets of epicycles, and Brahe's Earth-centred system (which with Kepler's ellipses replacing the Copernican epicycles would have been an excellent representation of the planetary motions). Unless Galileo could have proven that only a Sun-centred description could be valid, men such as Roberto Bellarmino would unhesitatingly abide by the wisdom of Solomon--and, both in their eyes and retrospectively in ours, Galileo had no such proof. Kepler's "solar influence", his Third Law, and Galileo's own concept of matter's tendency to preserve its velocity could have been combined to furnish proof (as we shall see below). But for all his other insights Galileo neither recognised the value of Kepler's work, nor the correct conclusions to be drawn from his own observations of projectiles.

Without "proof" Galileo was perceived as basing his conclusions on the "visions" he had had while viewing the skies through telescopes and on unwarranted extrapolation from his terrestrial experiments--extrapolation which did not even prove that Koppernigk was correct, merely that he might not be blatantly incorrect! Viewed in these terms, Galileo's failure to convince the Jesuits to reinterpret the Scriptures is hardly surprising.

In any event, on February 24, 1616, the "Qualifiers" of the Congregation of the Holy Office, a committee of the Inquisition, declared the proposition that the Sun is the centre of the Universe to be "foolish and absurd, philosophically and formally heretical, inasmuch as it expressly contradicts the doctrine of the Holy Scripture in many passages, both in their literal meaning and according to the general interpretation of the Fathers and Doctors". A similar declaration was made concerning the proposition that the Earth moved or rotated. According to the files of the Inquisition, Galileo was called before a small committee on February 26 at Bellarmino's palace in Rome and

> "enjoined in the name of His Holiness the Pope and the whole Congregation of the Holy Office to relinquish altogether the said opinion that the Sun is in the centre of the Universe and immovable and that the Earth moves; nor further to hold, teach, or defend it in any way whatsoever, verbally or in writing; otherwise proceedings would be taken against him by the Holy Office."

One week later, almost seventy-three years after Koppernigk's death, his "Book of Revolutions" was placed on the Index of forbidden books "until corrected". The heavy-handedness of these decisions created much discontent among Catholic intellectuals away from Rome, and undoubtedly stimulated interest in the confrontation which was to follow. One can imagine some copies of the Copernican treatise being reread for the first time in many years in search of what they contained that had been accepted for so long but which was now heretical.

Galileo "lay low" until August, 1623, when an intellectual friend of his named Maffeo Barberini was elected Pope. Then he rejoiced in the belief that he would at last be free to argue in favour of the Sun-centred system and to convince the theologians to accommodate the new discoveries in their interpretation of the Scriptures. Early in 1624 he travelled to Rome and had numerous audiences with his former friend, now Pope Urban VIII. But although Barberini in 1616 had defended Galileo, as Urban VIII he urged him to consider that God could organise the Universe to present the observed appearances while leaving the Earth unmoved, and that to assert otherwise would be tantamount to constraining God's infinite power within the limits of Galileo's own ideas.

These conversations told Galileo that Maffeo Barberini was a changed man. Yet on December 24, 1629 he announced what proved to be his most famous book, the "Dialogue on the Great World Systems". Cast in the form of a four-day-long debate among three intellectuals--Simplicio (an Aristotelian), Salviati (a surrogate Galileo) and Sagredo (an unbiassed Venetian nobleman)--the "Dialogue" voiced all the arguments against the Copernican system through the inept character Simplicio, to be refuted time and again by Salviati while Sagredo became more impressed. Yet at the very end of the book, in his last words Simplicio says in effect that God could manufacture a given appearance in many ways beyond the comprehension of the human intellect, and Salviati quickly agrees that perhaps Man is not capable of understanding God's infinite wisdom. Thus Galileo paid lip-service to the conclusion offered him by his Pope, but in a perfunctory and unsatisfactory manner, leaving little doubt as to the real conclusion to be drawn from the discussion. Presumably Galileo thought that the Maffeo Barberini he had known would recognise the validity of the dialogue while the Urban VIII Barberini had become would be mollified by its conclusion.

In fact the "Dialogue" was a splendidly unambiguous piece of Renaissance literature which sold out as fast as it was printed. Its true impact was perceived by all, including the theologians, and its perfunctory conclusion fooled nobody, including the Inquisition. Galileo was summoned to come to Rome, where in April 1633 he was tried by the Inquisition for disobeying the injunction of 1616, and thus for heresy. On the morning of June 22, 1633, after some behind-the-scenes negotiation over his sentence, he was pronounced guilty but absolved, on the condition that he publicly recant his statements supporting the heliocentric world-view and the motions of the Earth. This the seventy-year-old Galileo did, on his knees and clad in the white shirt of penitence, reciting a text agreed to in advance and signing a written copy of the same words.

To have murmured the legendary "eppur si muove"--"but it does move"--at the end of the public ceremony would have been a reckless act from which Galileo would have gained little, and the remark is likely apocryphal. But while spending the rest of his life under house arrest as part of his sentence he made the far more effective "murmurs" of his "Discourses", in which he outlined his theories of motion in full detail. By the time of his death in 1642 the hollowness of his recantation was clear to the intellectual world. The main effect of the prohibition of his and Koppernigk's books was to move the principal theatre of progress in understanding the Solar System from Catholic Europe to Protestant England, where the final pieces of the puzzle were assembled by the members of the Royal Society for the Promotion of Natural Knowledge.

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