THE ASTRONOMICAL JOURNAL VOLUME 82, NUMBER 8 AUGUST 1977

Variability of extragalactic sources at 2.7 GHz. III. The nature of the variations in different source classes

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(Received 17 March 1977)

The incidence of variability at 2.7 GHz of 365 sources over periods of 2–10 yr is found to depend mainly on the spectral class of the source rather than on the optical identification. Virtually all sources with Q-type spectra in the radio two-color diagram are variable. Furthermore, about one-fourth of the sources with "normal" class G spectra exhibit low-amplitude variations. Most of these class G variables are known to contain some small-scale structure, but their overall spectra are not dominated by their opaque components. Selection effects are such that the true incidence of variability in class G sources probably exceeds our present estimates. Variability amplitudes at 2.7 GHz are compared with those at higher frequencies. There are significant and systematic discrepancies with the predictions of the standard expanding cloud model. An interpretation is suggested wherein no variable sources *intrinsically* conform to the standard model, but some sources *appear* to do so because of opacity in previously ejected material. The variable components which coexist with transparent nonvariable emission do not appear to be systematically different from those which exist in isolation. The variability of class G sources may present problems for accurate calibration of flux-density measurements at centimeter, and shorter, wavelengths.

INTRODUCTION

FLUX-density monitoring of 365 extragalactic sources at 2.7 GHz for two years using the NRAO 300-ft telescope was described by Kesteven *et al.* (1976, Paper I). In that paper we showed (Figs. 2 and 3) that the internal consistency of the new flux-density measurements was in good agreement with their small estimated errors, which are typically of order ±0.03 Jy or ±1%.

The sources studied in the two-year monitoring program were (1) complete samples drawn from the 178-MHz revised 3C Catalogue (Bennett 1962, the "C" sample), the 1.4-GHz BDFL Catalogue (Bridle *et al.* 1972; Bridle and Fomalont 1974, the "B" sample), and the 8-GHz Michigan Catalogue (Brandie and Bridle 1974, the "M" sample); (2) known variable sources; and (3) other sources with flux densities known (in 1972) to exceed 1 Jy at 2.7 GHz. Weaker sources were observed when no source satisfying these criteria could be scheduled. The complete samples and the known variable sources were given the highest priority throughout the program.

Other published 2.7-GHz flux densities were normalized by Bridle *et al.* (1977, Paper II) to the scale adopted in Paper I. The small scatter in this normalization confirms the consistency of the new 300-ft measurements over the declination range −19° to +80° and implies that our adopted flux-density scale is within a few percent of an absolute scale at 2.7 GHz. The good agreement between data from different telescopes, obtained using different calibration and reduction procedures, justifies combining results from different 2.7-GHz observing programs to examine the variability of the 365 sources over periods of up to 10 yr, as was done in Paper II.

In this paper we examine the incidence of variability at 2.7 GHz and other frequencies as a function of source spectrum, optical identification, and survey frequency. Section I of this paper combines the results of Papers I and II with data on the radio spectra of the sources and discusses possible selection effects in the variability assessment. Section II examines the incidence of variability at 2.7 GHz in various classes of source. Section III relates the amplitudes of the variations observed at 2.7 GHz to those found at higher frequencies. Section IV discusses the variations found in different spectral classes, with particular emphasis on variable sources which do not have dominant flat-spectrum components at high frequencies. The implications of the results are considered in Sec. V.

I. CLASSIFICATION OF VARIABLE SOURCES

A. Spectral Classification

We classify the 1- to 10-GHz spectra of the program sources according to the location of each source in the

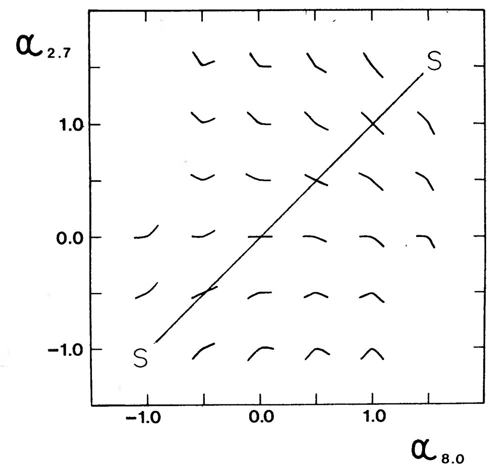

FIG. 1. Schematic radio two-color diagram showing spectral shapes characterized by different regions of the diagram. The line labeled S is the locus of power-law (zero curvature) spectra. For our definition of the spectral classes see also Fig. 2.

radio "two-color" (two spectral index) diagram with the spectral windows used by Brandie and Bridle (1974, hereafter referred to as B[2]). This diagram, originated by Bolton (1969), displays both spectral curvature and spectral index simultaneously (Fig. 1). Spectral classification based on this diagram is more useful than the traditional (S, C^+, C^-) scheme for our present purpose, because two-color classification can separate spectral types according to the relative importance of "flat-spectrum" and "steep-spectrum" ("transparent") emission in the adopted spectral windows. The traditional scheme, which used the spectral curvature but not the absolute value of the spectral index, cannot achieve this to the extent needed to separate sources whose spectra are dominated by different physical mechanisms. For example, the traditional C^- class (all of Fig. 1 *below* the line of unit slope, marked S) mixes the characteristic spectral shapes produced by synchrotron losses and by self-absorption acting on similar electron energy distributions. The ability of the two-color diagram to organize the basic spectral shapes in a way which can reflect underlying source physics may ultimately account for the different clustering in the diagram of galaxies, Lacertids, and quasars of low and high redshift (Bolton 1969; B[2]; Kraus and Gearhart 1975; Wall 1975; Pacht 1976).

Figure 2 shows the radio two-color diagram of the 316 program sources whose spectra (or mean spectra, see below) can be defined in both of the B[2] spectral windows using data available to us. The nomenclature given to the spectral classes in Fig. 2 follows the identification-related scheme provisionally introduced by B[2], with two exceptions: (1) The elliptical boundary of class G has been determined by the statistical procedure of B[2], but using the larger sample of BDFL sources observed in this program (the new class G ellipse extends farther into the "GQ region" than did the B[2] ellipse); and (2) class GQ of B[2] is subdivided into class GS (both spectral indices > 0.5) and class GQ (one or both spectral indices < 0.5). In this paper we define the spectral index α through $S \propto \nu^{-\alpha}$.

Table I lists the major references used in determining the radio spectra of the sources. Unpublished observations at 3.2, 6.6, and 10.6 GHz obtained by A.H.B.,

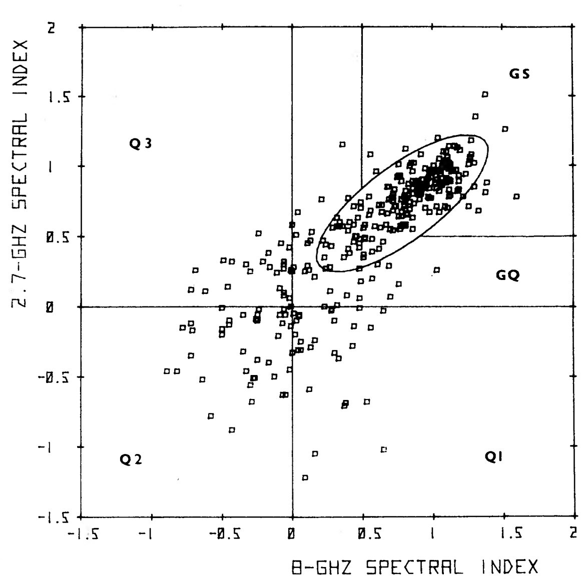

FIG. 2. Radio two-color diagram of the 316 program sources whose spectra in the 2.7- and 8.0-GHz spectral windows are known. The ellipse defining spectral class G has center (0.79, 0.73); semimajor axis 0.74; semiminor axis 0.26; inclination to the $\alpha_{8.0}$ axis, 36°.5.

TABLE I. Sources of radio spectral data.

Reference	Frequencies (GHz)
Andrew *et al.* (1973)	0.6, 1.4, 2.65, 3.2, 6.6, 10.7
Bell *et al.* (1971)	6.6, 10.7
Brandie and Bridle (1974)	1.4, 2.7, 6.6, 8.0, 10.6, 13.5
Bridle *et al.* (1972)	1.4
Bridle and Fomalont (1974)	1.4
Condon and Jauncey (1974)	0.3, 0.6
Conklin *et al.* (1972)	0.6, 1.4, 2.65, 3.2, 6.6, 10.6, 13.5, 31.4, 85.3
Davis (1967)	1.4
Doherty *et al.* (1969)	10.6
Guindon (1971)	3.2, 6.6, 10.6
Jauncey *et al.* (1970)	0.1, 0.3, 0.6
Kellermann and Pauliny-Toth (1973)	10.7
Kraus and Andrew (1970)	0.6, 1.4, 2.65, 3.2, 6.6, 10.6
Kraus *et al.* (1968)	0.6, 1.4, 2.65, 3.2, 6.5, 10.7
Medd *et al.* (1972)	6.6, 10.6
Pauliny-Toth and Kellermann (1968b)	5.0
Pauliny-Toth and Kellermann (1972a)	2.7, 5.0
Pauliny-Toth and Kellermann (1972b)	5.0
Pauliny-Toth *et al.* (1972)	5.0, 10.7
Shimmins and Bolton (1972)	5.0
Shimmins and Wall (1973)	8.9
Wall (1972)	0.18, 0.4, 0.47, 0.6, 1.4, 2.7, 5.0
Wills (1975)	0.47, 0.6, 0.96, 1.4, 2.65, 2.7, 5.0
Wills *et al.* (1971)	0.6, 1.4, 2.65, 3.2, 6.6, 10.6, 13.5

M.J.L.K., and B. Guindon with the Algonquin Radio Observatory 46-m telescope and at 8.1 GHz obtained by G.W.B., A.H.B., E.B. Fomalont, and B. Guindon with the NRAO interferometer have also been used for spectral determination. For variable sources, we have based our spectral classification on the smoothest spectrum passing through the mean of the flux-density ranges at each frequency.

The complete samples contain extragalactic sources from the surveys lying between declinations −10° and +60° and having angular diameters less than 1 arcmin. The radio two-color diagram of the BDFL sources *excluded* by these limits is shown in Fig. 3. Most of the excluded sources are in class G, but since the exclusion limits were set mainly by telescope parameters (see Paper I) and not by intrinsic source properties, the exclusion of these sources should not seriously bias the discussion below.

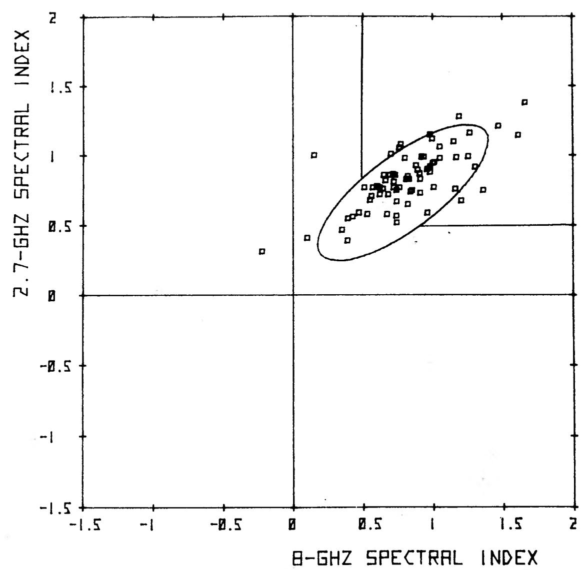

FIG. 3. Radio two-color diagram of sources in the BDFL complete sample (Bridle and Fomalont 1974) which were excluded from this program. Eleven excluded sources have not been observed over a sufficiently wide frequency range to allow their spectra in the two "windows" to be determined.

B. Variability Classification and Variability Indices

In Papers I and II we used the statistic

$$x^2 = \sum_{i=1}^{n} (S_i - \bar{S})^2/\sigma_i^2, \quad \bar{S} = \sum_{i=1}^{n} (S_i/\sigma_i^2)/ \sum_{i=1}^{n} (1/\sigma_i^2)$$

to assess the variability of each source. In the absence of variability x^2 should be distributed as χ^2 with $(n-1)$ degrees of freedom. A source was classified "variable" if $p(x^2)$, the probability of exceeding the observed x^2 by chance, was <0.1%, and "possibly variable" if $0.1\% < p(x^2) \leq 1\%$.

The two quantities which would best characterize a source's variability amplitude are the *mean flux-density range* $\overline{\Delta S}$ of its outbursts and its *fractional variability* $\bar{V} = \overline{\Delta S}/S$(nonvarying part). Our data are generally too coarsely sampled to define the amplitudes of individual outbursts (even if bursts did not overlap), so we must use approximations to these variability measures.

In Papers I and II we tabulated values of the extreme flux densities S_{max} and S_{min} observed for each source to give a general indication of the range of variability observed. The range of these values ($S_{max} - S_{min}$) distinguishes highly variable from relatively flux-stable sources, but is not a good parameter for detailed statistical studies, as it is sensitive to the effects of noise and occasional nonrandom errors in the individual flux densities S_{max} and S_{min}. In what follows, we use the quantity

$$\Delta\hat{S} = [8(x^2 - \langle\chi^2\rangle)/\Sigma(1/\sigma_i^2)]^{1/2}$$

to estimate the flux-density range of each source; here x^2 is the *observed* value for the source and $\langle\chi^2\rangle$ is the *expectation* value of χ^2 appropriate to the number of observations made of the source. The factor of 8 is introduced to obtain approximate equivalence between $\Delta\hat{S}$ and the "peak-to-peak" variation in Gaussian noise sampled seven or eight times, as in our monitoring program. The $\Delta\hat{S}$ statistic thus has the physical significance of (and is numerically similar to) the quantity ($S_{max} - S_{min}$), but is derived from *all* the data weighted according to their errors, and so is less susceptible to distortion by "accidents of measurement." It has the further advantage that it does not include apparent variability due to system noise alone (because of the subtraction of $\langle\chi^2\rangle$); this feature is particularly important for sources whose variations are not much greater than system noise.

Our introduction of this more stable statistic $\Delta\hat{S}$ means that for some sources the flux-density extrema listed in Papers I and II are more disparate than the flux-density ranges used in this paper. For sources classified as nonvariable, we compute a "99% confidence" upper limit to $\Delta\hat{S}$ by setting x^2 (observed) $= \chi_1^2$, where χ_1^2 is the value of χ^2 with a 1% probability of being exceeded by chance.

In each case, we estimate the fractional variability at 2.7 GHz as $\hat{V}_{2.7} = \Delta\hat{S}/2\bar{S}$.

Table II combines the spectral and variability data for sources included in the two-year monitoring program (Paper I); the results from the longer-term study (Paper II) are given where these supersede those of Paper I. Column 1 gives the IAU designation of each source (alternate source names may be found in Paper I). Membership in one or more of the complete samples is indicated in column 2. The radio spectral class of each source based on its location in the two-color diagram (Fig. 2) is given in column 3; where this location is uncertain due to lack of data the spectral index at 2.7 GHz is given. Column 4 gives our final assessment (Paper II) of the 2.7-GHz variability of the source, and column 5

TABLE II. Spectral and variability data for the 365 sources.

(1) SOURCE	(2) SAMPLE	(3) SP. CLASS	(4) 2.7-GHz VARIABLE?	(5) $\hat{V}_{2.7}$	(6) $\Delta\hat{S}_{2.7}$
0003-003	C B	G	No	<0.02	<0.09
0010+005		G	No	<0.04	<0.07
0012+319		G	No	<0.02	<0.05
0019-000	B	G	No	<0.02	<0.07
0026+346		GQ	No	<0.03	<0.09
0026-014		-0.06	No	<0.14	<0.08
0028-012		0.72	No	<0.10	<0.11
0029+013		1.03	No	<0.16	<0.10
0030+196		G	No	<0.03	<0.08
0033+183	C	G	No	<0.04	<0.08
0034-014	C B M	G	Yes	0.02	0.12
0035-024	C B M	G	No	<0.02	<0.16
0038+328	C B	G	No	<0.02	<0.07
0038+097	C B	G	No	<0.01	<0.05
0048+509	C	G	No	<0.03	<0.07
0048-097		Q2	Yes	0.18	0.63
0051-038	B	G	No	<0.04	<0.09
0055+300		Q3	No	<0.02	<0.05
0056-001	B M	G	No	<0.03	<0.12
0059+144		G	No	<0.04	<0.07
0100+146		0.89	Yes	0.07	0.08
0106+013	M	Q2	Yes	0.32	2.40
0111+021	M	Q3	No	<0.05	<0.06
0112-017	M	Q1	No	<0.05	<0.11
0114+074		G	No	<0.04	<0.08
0116+082	B	G	No	<0.02	<0.07
0116+319	B	G	No	<0.02	<0.08
0119+115		Q3	No	<0.05	<0.08
0119+041	M	Q1	Yes	0.10	0.32
0122-003	M	Q3	Yes	0.13	0.30
0123+329	C B	G	No	<0.02	<0.09
0125+287	C B	G	No	<0.02	<0.06
0127+233	C B	G	No	<0.02	<0.07
0128+250		G	No	<0.03	<0.05
0132+079	B	G	No	<0.03	<0.08
0133+476		Q2	Yes	0.32	1.22
0134+329	C B	G	Possibly	0.02	0.25
0138+136	C B	G	Yes	0.05	0.16
0145+532	C	G	No	<0.02	<0.09
0146+056		Q1	No	<0.07	<0.10
0154+286	C B	GS	No	<0.02	<0.05
0202+319		Q2	Yes	0.10	0.19
0202+149	B	GQ	Yes	0.09	0.62
0204+067		0.61	Yes	0.05	0.10
0206+355	B	GS	No	<0.03	<0.08
0208+210		0.77	No	<0.03	<0.04
0216+011		-0.43	No	<0.09	<0.09
0218-021	C B	GS	No	<0.02	<0.06
0220+397	C	G	No	<0.03	<0.10
0221+276	C B	G	No	<0.02	<0.07
0223+341	B	G	No	<0.02	<0.07
0229+341	C B	G	No	<0.02	<0.06
0229+131		Q1	Yes	0.04	0.10
0234+285		Q2	Yes	0.05	0.12
0235+164		Q3	Yes	0.54	1.29
0237-027	M	Q2	Yes	0.27	0.16
0240-002	C B M	G	Yes	0.05	0.32
0256+075		Q2	Yes	0.12	0.20
0258+350		G	No	<0.03	<0.04
0307+169	C B	G	No	<0.02	<0.05
0312+100		0.66	Yes	0.08	0.15
0316+413	C	Q2	Yes	0.32	11.11
0316+162	B	G	No	<0.01	<0.03
0317+188		Q1	No	<0.05	<0.07
0319+176		0.88	No	<0.07	<0.09
0319+121		G	No	<0.03	<0.08
0320+053	B	G	No	<0.03	<0.09
0326+277		G	No	<0.03	<0.05
0331-013	C B	G	No	<0.03	<0.08
0333+321		GQ	Yes	0.05	0.28
0336-019	B M	Q2	Yes	0.23	1.06
0340+048	C B	G	No	<0.02	<0.06
0345+337	C	G	No	<0.03	<0.08
0347+057	B	G	No	<0.02	<0.08
0355+508		Q2	Yes	0.13	1.37
0408+070		0.90	No	<0.05	<0.09
0411+141	B	G	No	<0.03	<0.08
0411+054		G	No	<0.04	<0.09
0420+417		GQ	Yes	0.08	0.23
0420-014	M	Q2	Yes	0.16	0.47
0422+004	M	Q3	Yes	0.29	0.41
0428+205		G	No	<0.01	<0.06
0430+052	B	Q2	Yes	0.18	3.03
0433+295	C	G	Yes	0.02	0.93
0438+252		0.67	No	<0.08	<0.06
0440-003	B M	GQ	Yes	0.17	0.98
0446+112		Q3	Yes	0.12	0.20
0450+314	C	G	No	<0.03	<0.09
0453+227	C	G	No	<0.02	<0.08
0454+066		Q3	Yes	0.18	0.19
0458-020		Q3	Yes	0.05	0.19
0459+252	C	G	No	<0.01	<0.07
0500+019	B M	GQ	Possibly	0.02	0.10
0507+179		Q2	Yes	0.12	0.14
0515+508	C	G	No	<0.03	<0.06
0518+165	C	G	Yes	0.03	0.38
0528+064	C	G	No	<0.03	<0.10
0529+075		Q3	Yes	0.07	0.19
0531+194		Q3	No	<0.01	<0.08
0538+498	C	G	Yes	0.02	0.60
0540+187		GS	No	<0.02	<0.05
0548+165		G	No	<0.02	<0.06
0552+398		Q1	Yes	0.05	0.39
0554-026		0.77	No	<0.06	<0.06
0559+024		0.99	No	<0.12	<0.07
0605+480	C	G	No	<0.02	<0.09
0605-085		Q1	Yes	0.11	0.75
0624-058		G	No	<0.02	<0.45
0640+233	C	G	No	<0.03	<0.08
0642+449		GQ	Yes	0.10	0.21
0642+214	C	G	Yes	0.07	0.21
0651+542	C B	G	No	<0.02	<0.08
0659+445	B	1.27	No	<0.02	<0.05
0711+146	C	G	No	<0.04	<0.09
0723-008	M	GQ	Yes	0.15	0.66
0725+147	C	G	No	<0.03	<0.07
0732+332	B	G	No	<0.03	<0.08
0735+178		Q2	Yes	0.07	0.29
0736+017	M	GQ	Yes	0.10	0.39
0738+313	B	GQ	Yes	0.06	0.24
0741-063		G	No	<0.02	<0.20
0742+103		Q1	Possibly	0.02	0.14
0743-006	M	Q1	No	<0.03	<0.08
0748+333		Q1	Yes	0.06	0.10
0758+143	C B	G	No	<0.02	<0.05
0801+303		0.82	No	<0.05	<0.07
0802+212		GQ	No	<0.04	<0.08
0802+103	C	G	No	<0.04	<0.07
0805-077		Q3	Yes	0.20	0.46
0806+426	C B	G	No	<0.03	<0.07
0808+019		Q2	Yes	0.33	0.49
0809+483	C B	G	No	<0.02	<0.31
0811+131		G	No	<0.04	<0.08
0814+425	B	Q1	Yes	0.13	0.50
0820+225	B	GQ	Possibly	0.03	0.10
0824+294	C	G	No	<0.04	<0.08
0827+378	B	G	No	<0.03	<0.08
0828+493		Q1	Yes	0.14	0.26
0829+046	M	Q2	No	<0.07	<0.09
0831+557	B	GQ	No	<0.01	<0.30
0831+171		GS	No	<0.04	<0.08
0835+580	C B	G	No	<0.04	<0.09
0838+133	C B	G	Possibly	0.03	0.10
0839+187		Q3	No	<0.03	<0.07
0850+140	C B	G	No	<0.03	<0.07
0851+202		Q2	Yes	0.25	1.51
0855+143	C B	G	No	<0.03	<0.09
0905+380	C B	GS	No	<0.02	<0.04
0906+430	C B	G	No	<0.02	<0.09
0906+015	M	Q3	Yes	0.14	0.28

TABLE II (*continued*)

(1) SOURCE	(2) SAMPLE	(3) SP. CLASS	(4) 2.7-GHz VARIABLE?	(5) $\hat{V}_{2.7}$	(6) $\Delta\hat{S}_{2.7}$
0911+174		G	No	<0.04	<0.06
0912+029		0.13	Yes	0.29	0.19
0922+005	M	GQ	Yes	0.06	0.11
0923+392	B	Q2	No	<0.01	<0.10
0932+022		0.78	No	<0.06	<0.06
0936+022		0.65	No	<0.12	<0.06
0937+033		0.78	No	<0.21	<0.11
0941+100	C B	G	No	<0.03	<0.06
0947+145	C B	G	No	<0.02	<0.08
0953+254		Q2	Yes	0.07	0.15
0954+556	B	G	Possibly	0.02	0.10
1003+351	C B	G	No	<0.02	<0.08
1005+077	C B	G	Possibly	0.02	0.11
1008+066	C B	G	Yes	0.05	0.14
1012+022		0.59	No	<0.08	<0.06
1014+018		0.94	No	<0.13	<0.09
1021+028		0.83	No	<0.14	<0.06
1021-006		0.28	No	<0.05	<0.10
1027+008		G	No	<0.05	<0.06
1039+029	B	G	No	<0.02	<0.06
1040+123	C B	G	No	<0.02	<0.08
1049+215		GQ	No	<0.03	<0.06
1055+201	B	GQ	Possibly	0.03	0.10
1055+018	B M	Q1	Yes	0.04	0.25
1056+432	C B	G	No	<0.03	<0.10
1059-010	C B	G	No	<0.03	<0.08
1111+408	C B	G	No	<0.03	<0.09
1116+128	B	G	No	<0.02	<0.07
1117+146	B	G	Possibly	0.03	0.08
1119+183		Q1	No	<0.04	<0.06
1123+264		Q2	No	<0.03	<0.05
1128+455	B	G	No	<0.03	<0.07
1138+015	B	G	No	<0.03	<0.09
1140+233	C B	G	No	<0.03	<0.09
1147+130	C B	G	No	<0.04	<0.10
1148-001	B M	GQ	Possibly	0.02	0.11
1150+498		GQ	Yes	0.05	0.17
1153+317	B	G	No	<0.02	<0.07
1156+295		Q3	Yes	0.09	0.20
1201-041	B	G	No	<0.04	<0.12
1206+439	C B	G	No	<0.03	<0.07
1213+350		GQ	No	<0.03	<0.07
1218+339	C B	G	No	<0.03	<0.09
1219+045	M	Q1	Yes	0.14	0.19
1225+368	B	G	No	<0.02	<0.06
1226+023	C B M	Q2	Yes	0.08	7.34
1229-021		G	Possibly	0.04	0.10
1237-101		G	No	<0.05	<0.15
1239-044	C B	G	Yes	0.07	0.25
1241+166	C B	G	Yes	0.04	0.15
1242+410		G	No	<0.03	<0.06
1250+568	C B	GS	No	<0.04	<0.11
1251+278	C B	G	No	<0.02	<0.04
1253+185		Q1	No	<0.06	<0.06
1253-055		Q1	Yes	0.07	1.81
1254+476	C B	G	No	<0.02	<0.11
1306-095		G	No	<0.03	<0.18
1318+113	B	G	No	<0.02	<0.05
1323+321	B	G	No	<0.01	<0.07
1328+307	C B	G	No	<0.01	<0.20
1328+254	C B	G	No	<0.02	<0.18
1335-061		G	No	<0.04	<0.15
1336+391	C B	G	No	<0.02	<0.07
1340+053		G	No	<0.05	<0.11
1343+500	C B	G	No	<0.03	<0.08
1345+125	B	G	No	<0.01	<0.08
1354+013	B	GS	No	<0.04	<0.10
1354+195	B	G	Yes	0.11	0.38
1403-085		Q3	No	<0.08	<0.11
1404+286		Q1	No	<0.02	<0.08
1409+524	C B	G	No	<0.01	<0.24
1413+349	B	G	No	<0.03	<0.10
1416+067	C B	GQ	No	<0.02	<0.11
1419+419	C B	G	Possibly	0.03	0.09
1420+198	C B	G	No	<0.02	<0.08
1420-005		0.87	No	<0.14	<0.07
1425-011	C B	G	No	<0.03	<0.10
1434+036	B M	G	No	<0.02	<0.07
1441+522	C B	G	No	<0.03	<0.09
1442+101	B	G	Yes	0.05	0.15
1453-109		G	No	<0.06	<0.30
1456+044	M	Q3	No	<0.06	<0.09
1502+106		Q2	Yes	0.07	0.25
1502+036		Q2	Yes	0.38	0.38
1509+015	B	0.97	No	<0.03	<0.07
1510-089		Q3	Yes	0.32	1.58
1511+238		G	No	<0.03	<0.07
1514+072	C B	GS	No	<0.02	<0.08
1517+204	C B	G	No	<0.02	<0.05
1518+046	B	GS	No	<0.02	<0.09
1522+546	C B	G	No	<0.03	<0.08
1523+033		0.79	No	<0.02	<0.04
1532+016	M	Q3	Yes	0.04	0.09
1535+139		0.55	No	<0.04	<0.08
1535+004	M	Q1	Yes	0.09	0.17
1538+149		Q2	Yes	0.14	0.43
1543+005		G	No	<0.04	<0.09
1545+210	C B	G	No	<0.02	<0.05
1546+027	M	Q2	Yes	0.08	0.18
1547+215	C B	G	No	<0.03	<0.08
1548+056	B	Q3	Yes	0.08	0.31
1553+202	C B	G	No	<0.02	<0.05
1555+001	M	Q1	Yes	0.15	0.42
1600+335	B	GQ	Yes	0.14	0.60
1602+014	C B	G	Yes	0.04	0.18
1603+001	B	GQ	No	<0.03	<0.09
1607+268	B	GS	Yes	0.07	0.42
1611+343	B	GQ	No	<0.02	<0.10
1614+269		G	No	<0.03	<0.05
1618+177	C B	G	No	<0.02	<0.05
1622+238	C B	G	No	<0.02	<0.06
1624+416	B	G	No	<0.02	<0.07
1626+396	C B	GS	No	<0.03	<0.08
1626+278	C B	G	No	<0.04	<0.08
1627+444	C B	G	No	<0.03	<0.10
1627+234	C B	G	No	<0.03	<0.08
1635-035	M	Q1	No	<0.09	<0.09
1638+398		Q1	Yes	0.12	0.26
1638+124	B	G	No	<0.02	<0.06
1641+399	C B	Q1	Yes	0.22	4.24
1641+173	C B	G	No	<0.02	<0.09
1643+022		G	No	<0.04	<0.09
1645+174	B	G	No	<0.02	<0.05
1648+015	M	Q3	Yes	0.09	0.15
1649-062		G	No	<0.05	<0.12
1656+053		Q2	No	<0.02	<0.06
1702+298		G	No	<0.04	<0.07
1708+006		G	No	<0.06	<0.10
1711+006		0.82	No	<0.08	<0.10
1712-032		1.13	No	<0.10	<0.12
1716+006	B	G	No	<0.02	<0.05
1735+240		0.79	No	<0.03	<0.06
1741-038	M	Q1	Yes	0.23	1.02
1749+096		Q2	Yes	0.19	0.32
1756+134		G	No	<0.02	<0.05
1759+138		1.46	No	<0.06	<0.09
1801+010		Q3	No	<0.03	<0.05
1807+279		GQ	No	<0.03	<0.05
1810+046		0.96	No	<0.02	<0.05
1819+396	B	1.04	No	<0.02	<0.07
1821+017		G	No	<0.03	<0.09
1828+487	C B	G	Possibly	0.02	0.31
1829+290		G	No	<0.02	<0.07
1832+474	C B	G	No	<0.02	<0.09
1832+315		G	No	<0.02	<0.06
1835+134		GQ	No	<0.04	<0.09
1842+455	C B	G	No	<0.02	<0.13
1843+098	C	GQ	Yes	0.03	0.16
1901+319		GQ	Yes	0.07	0.39
1914+302	C	G	Yes	0.03	0.09

TABLE II (*continued*)

(1) SOURCE	(2) SAMPLE	(3) SP. CLASS	(4) 2.7-GHz VARIABLE?	(5) $\hat{V}_{2.7}$	(6) $\Delta\hat{S}_{2.7}$
1939+605	C	G	No	<0.02	<0.11
1953+035		0.95	No	<0.17	<0.09
1957-013		0.33	No	<0.05	<0.04
2003-025		G	No	<0.03	<0.09
2012+234	C	G	No	<0.02	<0.26
2019+098	C	G	No	<0.01	<0.03
2030+257		0.91	No	<0.04	<0.08
2037+511	C	G	Yes	0.05	0.38
2037-029		0.75	No	<0.16	<0.13
2044-027	B M	G	No	<0.03	<0.09
2045+068	C B	G	No	<0.03	<0.07
2050+363		G	Yes	0.02	0.19
2052+005		0.51	No	<0.05	<0.05
2059+283		1.25	No	<0.04	<0.08
2059+034	M	Q2	Yes	0.07	0.09
2113+293		Q1	Yes	0.12	0.24
2121+248	C	G	No	<0.01	<0.13
2126+073	C B	G	No	<0.03	<0.06
2128+048	B M	G	No	<0.01	<0.06
2131-021	M	Q2	Yes	0.22	0.87
2134+004	B M	Q1	Yes	0.03	0.43
2141+279	C	G	No	<0.02	<0.07
2142+042		0.81	No	<0.05	<0.08
2144+092		Q2	Yes	0.17	0.29
2145+151	C B	G	No	<0.03	<0.09
2145+067	B	Q2	Yes	0.04	0.23
2147+145	B	G	No	<0.03	<0.08
2148+143	B	0.85	No	<0.03	<0.08
2153+376	C	G	No	<0.01	<0.07
2200+420		Q2	Yes	0.37	3.29
2201+315		Q2	Yes	0.14	0.51
2203+292	C B	G	Possibly	0.03	0.09
2210+016	B M	G	No	<0.02	<0.07
2216-038	M	Q2	Yes	0.13	0.32
2223+210	B	GQ	Yes	0.04	0.14
2223-052		Q3	Yes	0.07	0.64
2230+114	B	G	Yes	0.03	0.34
2234+282		Q1	Yes	0.18	0.36
2236+124		-0.22	Yes	0.17	0.12
2244+366		G	No	<0.03	<0.07
2247+140	B	G	No	<0.02	<0.06
2247+132		G	No	<0.04	<0.07
2249+185	C B	G	No	<0.03	<0.07
2251+244		G	No	<0.04	<0.10
2251+158	C B	Q1	Yes	0.08	1.60
2252+129	C B	G	No	<0.03	<0.08
2254+024	M	Q2	No	<0.09	<0.06
2300-013		-0.18	No	<0.16	<0.06
2302-025		0.91	No	<0.18	<0.10
2303-008		0.72	No	<0.13	<0.09
2304+006		0.63	No	<0.10	<0.06
2309+090	C B	GS	No	<0.02	<0.05
2314+038	C B	G	No	<0.02	<0.09
2318+049	M	Q2	Yes	0.10	0.24
2320+079		G	No	<0.04	<0.06
2324+405		G	No	<0.03	<0.09
2332-017	M	GQ	No	<0.07	<0.08
2335-027	M	Q1	No	<0.08	<0.10
2338+042		0.94	No	<0.05	<0.08
2344+092	B	Q3	Yes	0.03	0.09
2347-026		G	No	<0.06	<0.12
2349-014		0.72	No	<0.05	<0.10
2351+456		Q3	No	<0.02	<0.06
2351-006		0.04	No	<0.09	<0.09
2352+495		G	Yes	0.03	0.13

gives the estimate of the fractional variability $\hat{V}_{2.7}$. Column 6 gives the corresponding estimate of the flux-density range $\Delta\hat{S}_{2.7}$.

C. Selection Effects

The data from our two-year monitoring program do not form an entirely homogeneous set: Some sources were observed more often than others within a single observing period. This means that the flux-density errors vary from source to source, as well as from period to period and from declination to declination. Approximately, however, the typical error of the measurements quoted in Paper I is the larger of $\overline{\sigma} = 0.03$ Jy or 1%. It should be noted that if these *quoted* errors underestimated the *true* errors by a factor f, then the horizontal scale of the observed x^2 distributions shown in Fig. 2 of Paper I would be wider than that of the theoretical χ^2 distributions by a factor f^2. On the basis of the actual *agreement* between the observed x^2 and theoretical χ^2 distributions, we concluded in Paper I that our quoted errors reflect the true errors to within ~20%.

The typical quoted errors can be used to establish the levels at which variability can in principle be detected using our data, in terms of the fractional variability $\hat{V}_{2.7}$, the range $\Delta\hat{S}$, and the mean flux density $\overline{S}$. The least-detectable flux-density range is approximately

$$\Delta\hat{S}(\min) = [8(\chi_i^2 - \langle\chi^2\rangle)/\Sigma(1/\sigma_i^2)]^{1/2},$$

i.e.,

$$\Delta\hat{S}(\min) \sim \overline{\sigma}\{8[(\chi_i^2/n) - 1]\}^{1/2} \qquad \text{for } n \sim 7,$$

and

$$\hat{V}_{2.7}(\min) = \Delta\hat{S}(\min)/2\overline{S}.$$

Success in detecting a given level of $\Delta\hat{S}$ therefore depends on whether or not a *nonvarying* source component determines $\overline{\sigma}$ for the source concerned. For the stronger sources (>3 Jy), calibration uncertainties (proportional to flux density) dominate those due to noise and confusion; it is therefore harder to detect given levels of variability in sources with stronger nonvarying emission in addition to their variable component. On conventional source models, such objects are more likely to be found in spectral class G than in Q3, and in Q3 rather than Q2 or Q1 (see Fig. 1). Thus, *a given variability range may be more likely to escape detection in class G than in classes Q2 and Q1,* and we may *under*-estimate the incidence of variability in G-type sources.

Conversely we may overestimate the incidence of variability in Q-type sources. Suppose, for example, that a class of variable source exists which is active for some small fraction of its total lifetime, say f, being radio quiet for the remaining time. If such objects coexisted with nonvariable emission, then our observations would detect a fraction $\lesssim f$ as variable. Of those which existed in isolation, however, *only the recently active examples would be included in radio surveys*. Thus, we might erroneously estimate that sources of this "isolated" class are *always* variable and conclude that $f \sim 100\%$. In relation to conventional expanding cloud models of variability, such bias may lead to overestimation of the incidence of variability in sources with class Q spectra compared with those of class G spectra.

D. Are the Low-Level Variables Real?

Several phenomena discussed below could in principle result from gross underestimation of our errors, or serious deviation of the flux-density error statistics from Gaussian. If unaccounted-for errors were widespread in our data, our results on the incidence of low-level variability (Secs. II and IV) and on discrepancies from the "standard" expanding cloud model (Sec. III) would be especially suspect. We are confident that our errors, though small, are substantially as claimed, for the following reasons.

Firstly, *our flux-density calibration is unusually extensive and free from prior assumptions.* As described in Paper I, we did not assume that any group of sources would show long-term flux stability, but observed a large complete sample from which a *post hoc* calibration was derived by an iterative procedure. One result of this procedure is that over half of the observing time was, retrospectively, used to calibrate telescope performance; our calibration, as it affects temporal variations (as opposed to the absolute flux-density scale at different declinations), is based on the *mean* properties of 190 sources ultimately judged nonvariable. Another result is that possible low-level variations of "well-known" calibration sources have not been allowed to generate spurious "noise" in the calibration process and thus mask similar variations of program sources. We do, indeed, suggest that 3C 123 ($\hat{V} = 0.02$) and 3C 147 ($\hat{V} = 0.02$) are low-level variables, and that 3C 48 ($\hat{V} \lesssim 0.02$) is possibly variable. [Several of these sources have been assumed to be flux stable in other monitoring programs (e.g., Altschuler and Wardle 1976; Medd *et al.* 1972; Brandie 1972), so that very little corroboration of our results on these sources is *possible*.]

Secondly, we showed in Paper I that the scatter in our measurements for nonvariable sources substantially obeys the expected Gaussian statistics for errors within ~20% of our estimates. If our errors are significantly underestimated or largely non-Gaussian, the success of this procedure would be hard to understand.

Thirdly, *we have protected ourselves against misinterpreting occasional unaccounted-for errors by always discarding our most discrepant measurements* (i.e., largest contributions to x^2) if these were based on single transits.

We believe, therefore, that every reasonable precaution has been taken to ensure that our estimates of source variability (based on the x^2 statistic) are statistically valid appraisals of real fluctuations in the sources themselves.

II. THE INCIDENCE OF VARIABILITY

A. The Significance of "Possibly Variable" Sources

In Papers I and II and in Table II of this paper, we have distinguished "variable" sources (probability of the observed scatter in flux densities arising by chance <0.1%) from "possibly variable" sources (probability between 1% and 0.1%). The random flux-density errors should cause approximately four sources in our 365-source sample to be erroneously classified as "possibly variable" and less than one source to be erroneously classified as "variable"; in fact we find 14 "possibly variable" and 104 "variable" sources. We conclude that ~70% of the "possibly variable" sources in Table II are, indeed, variable sources whose observed range of flux densities is small. This conclusion is supported by the number of reclassifications of variability when the time base was extended from 2 to 10 yr in Paper II, Sec. III-C. The *under*estimate of the true incidence of variability, which would result from grouping "possibly variable" sources with the nonvariable, would therefore be greater than the *over*estimate made if these sources are grouped with the "variable" sources. We have, therefore, grouped the "possibly variable" and "variable" sources together as "variable" in what follows.

B. The Incidence of Variability in the Complete Samples

The incidence of variability at 2.7 GHz in each of the complete samples is shown in Table III. As at higher frequencies (e.g., Andrew *et al.* 1972), most of the sources of spectral classes Q1, Q2, and Q3 are variable. Variability is also commonplace in class GQ, whose spectra might be termed "flat," though not necessarily "centimeter excess" above 2.7 GHz.

More surprising is the significant fraction of variable sources found in spectral class G: About 20% of the class G sources in each sample varied at 2.7 GHz. By the definition of class G the overall microwave spectra of these sources do not suggest the presence of *strong* (i.e., dominant) compact components. We believe these variations to be real and our monitoring to have detected weak variable emission whose presence has only a small effect on the overall microwave spectrum of the source concerned. The nature of this variable emission is discussed in Sec. IV.

Some sources not found to be variable at 2.7 GHz are known to vary at other frequencies (references to other monitoring of our program sources were given in Table

TABLE III. 2.7-GHz variability in complete samples.

Sp. class	No. observed	Not variable	Variable
	(a) 3CR ("C") sample		
GS	7	7 (100%)	0
G	101	82 (81%)	19 (19%)
GQ	2	1 (50%)	1 (50%)
Q1	2	0	2 (100%)
Q2	2	0	2 (100%)
Q3	0	...	...
Total	114	90 (79%)	24 (21%)
	(b) BDFL ("B") sample		
GS	11	10 (91%)	1 (9%)
G	112	94 (84%)	18 (16%)
GQ	13	4 (31%)	9 (69%)
Q1	5	0	5 (100%)
Q2	5	1 (20%)	4 (80%)
Q3	2	0	2 (100%)
Unknown	4	4 (100%)	0
Total	152	113 (74%)	39 (26%)
	(c) Michigan ("M") sample		
GS	0	...	...
G	8	6 (75%)	2 (25%)
GQ	7	1 (14%)	6 (86%)
Q1	11	4 (36%)	7 (64%)
Q2	12	2 (17%)	10 (83%)
Q3	7	2 (29%)	5 (57%)
Total	45	15 (33%)	30 (67%)

II of Paper I). The incidence of variability at *any* frequency in the complete samples is shown in Table IV, where the ubiquity of variations in spectral classes GQ, Q1, Q2, and Q3 is even more evident than in Table III. The percentage of variables in spectral class G also rises significantly in all three samples when the higher-frequency data are included.

TABLE IV. Variability at any frequency in complete samples.

Sp. Class	No. observed	Variable at 2.7 GHz	Variable elsewhere	Total variable
		(a) 3CR ("C") sample		
GS	7	0	0	0
G	101	19 (19%)	3 (3%)	22 (22%)
GQ	2	1 (50%)	0	1 (50%)
Q1	2	2 (100%)	...	2 (100%)
Q2	2	2 (100%)	...	2 (100%)
Q3	0	...	...	...
Total	114	24 (21%)	3 (3%)	27 (24%)
		(b) BDFL ("B") sample		
GS	11	1 (9%)	1 (9%)	2 (18%)
G	112	18 (16%)	10 (9%)	28 (25%)
GQ	13	9 (69%)	1 (8%)	10 (77%)
Q1	5	5 (100%)	...	5 (100%)
Q2	5	4 (80%)	1 (20%)	5 (100%)
Q3	2	2 (100%)	...	2 (100%)
Unknown	4	0	0	0
Total	152	39 (26%)	13 (9%)	52 (34%)
		(c) Michigan ("M") sample		
GS	0	...	...	...
G	8	2 (25%)	3 (38%)	5 (63%)
GQ	7	6 (86%)	1 (14%)	7 (100%)
Q1	11	7 (64%)	4 (36%)	11 (100%)
Q2	12	10 (83%)	2 (17%)	12 (100%)
Q3	7	5 (71%)	0	5 (71%)
Total	45	30 (67%)	10 (22%)	40 (89%)

TABLE V. Variability of all sources in this program.

Sp. Class	No. ob-served	Not variable (at 2.7 GHz)	Variable (at 2.7 GHz)	Variable at other frequencies	Total variable
GS	13	12	1 (8%)	1	2 (15%)
G	185	158	27 (15%)	22	49 (26%)
GQ	29	11	18 (62%)	2	20 (69%)
Q1	28	9	19 (68%)	8	27 (96%)
Q2	37	5	32 (86%)	4	36 (97%)
Q3	24	8	16 (67%)	5	21 (88%)
Unknown	49	44	5 (10%)	0	5 (10%)

The fact that the *overall* fraction of variable sources in the samples increases with survey frequency in both Tables III and IV is mainly a reflection of the increasing fraction of Q-type spectra going from the 3C to the BDFL to the Michigan samples. Indeed, the division of each sample according to spectral class shows directly that *the incidence of variability depends more on spectral class than on survey frequency*. This being so, it is valid to examine the statistics of variability using all 365 sources monitored, whether members of complete samples or not. The importance of the "complete" sampling in our two-year monitoring program is, then, not that completeness to a given flux density is worthwhile for its own sake, but that it has provided us with *representative* samples of *each* of the various spectral populations.

C. The Incidence of Variability in all Sources Studied

Table V shows the incidence of variability (at 2.7 GHz and at other frequencies) of all 365 sources monitored; columns 1 and 2 give the spectral class and number of sources observed; columns 3 and 4 give the variability statistics at 2.7 GHz as in Table III; column 5 gives the number of sources found to be variable or possibly variable only at other frequencies; column 6 gives the number of program sources variable or possibly variable at any frequency. The columns are not totaled because this overall group of sources is biased towards the inclusion of Q-type spectra, and it is not representative of any particular survey frequency.

Again, virtually all sources of spectral classes Q1, Q2, and Q3 are variable at some frequency, and about three-fourths of the sources of the intermediate class GQ are variable. *Table V provides the most significant evidence for our new finding: About one-fourth of the sources of class G are variable.*

Figure 4 shows the two-color diagram of sources found to be variable at 2.7 GHz; the class G variables are distributed *throughout* the class G ellipse, and are not concentrated among the flatter-spectrum objects of this class. The exclusion of G-type spectra from earlier studies of radio variability may therefore have systematically discriminated against variable source components associated with more intense, steep-spectrum emission.

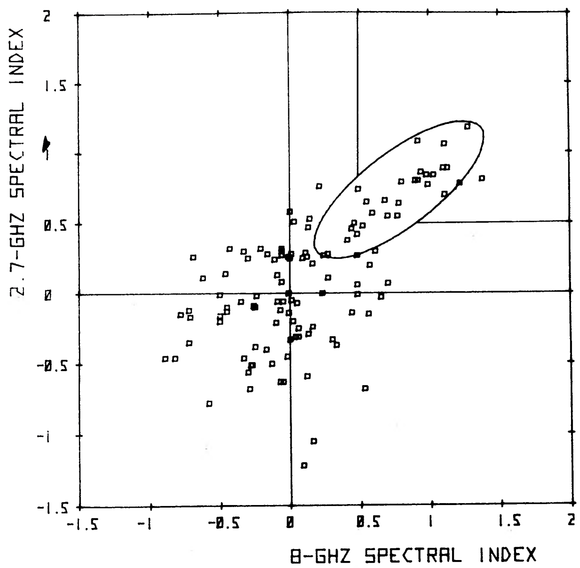

FIG. 4. Radio two-color diagram of the 114 sources with known spectra found to be possibly variable or variable at 2.7 GHz.

The optically identified sources of classes Q1, Q2, and Q3 are predominantly stellar objects (quasistellar objects with measured redshifts and stellar objects without published redshifts; B^2; Wall 1975) whereas class G contains both radio galaxies *and* stellar objects. Table VI shows how the incidence of variability at 2.7 GHz depends on the type of optical identification within each spectral class. [Identifications have been drawn mainly from the compilations of Véron and Véron (1974, 1976).] Variability is more common among the stellar objects of class G than among the radio galaxies of this class, but this difference is not statistically significant. Thus, *the nature of the optical identification appears secondary to the spectral classification in determining whether or not a source is found to vary.* Stellar objects are more likely than radio galaxies to be variable primarily because they are more likely to have Q-type spectra.

III. VARIABILITY AMPLITUDES AT DIFFERENT FREQUENCIES

In the "standard" instantaneous injection, adiabatic expansion model of radio source variation (e.g., van der Laan 1966), the amplitude of an outburst varies with observing frequency as

$$\Delta S_{\nu_1}/\Delta S_{\nu_2} = (\nu_1/\nu_2)^{\Gamma},$$
$$\Gamma = (7\gamma + 3)/(4\gamma + 6),$$

where γ is the exponent of the energy distribution of the injected electrons. Analyses of individual outbursts of some variable sources (Pauliny–Toth and Kellermann 1968a, 1968b; Dent 1968; Locke *et al.* 1969; Peterson and Dent 1973) and of spectra of compact source components (e.g., Kellermann and Pauliny-Toth 1971; Kellermann 1966) have suggested $\gamma \geq 1$, for which the standard model gives $\Gamma \geq 1$. The validity of the model's assumptions can be examined in a representative population of variable sources by comparing the flux-density ranges $\Delta\hat{S}$ computed from (1) our two-year monitoring, and (2) the monitoring of the same sources at higher frequencies at the Algonquin Radio Observatory by Andrew *et al.* (1976) and at NRAO by Altschuler and Wardle (1976). Comparison with the Algonquin data at 10.6 GHz tests the model over a nearly 4:1 frequency range, for which the standard model with $\gamma \gtrsim 1$ predicts the ratio $R_{10.6}$ $(= \Delta S_{10.6}/\Delta S_{2.7}) \gtrsim 3.9$; comparison with the NRAO data at 8.1 GHz tests a predicted ratio $R_{8.1}$ $(=\Delta S_{8.1}/\Delta S_{2.7}) \gtrsim 3$.

Inspection of the Algonquin data at 10.6 and 6.6 GHz suggests that the time delay between appearance of a given burst peak at these two frequencies rarely exceeds two months. Extrapolation to 2.7 GHz, using the standard model, implies that the delay between 10.6 or 8.1 GHz and 2.7 GHz would rarely exceed six months. We have therefore computed $\Delta\hat{S}_{10.6}$ and $\Delta\hat{S}_{8.1}$ from the Algonquin and NRAO data for a 30-month period roughly coincident with out two-year program at 2.7 GHz. Because few observations will coincide exactly with the times of flux-density extrema of the sources, the observed ranges $\Delta\hat{S}$ must underestimate the amplitudes of the average outburst by amounts depending both on the nature of the variations and on the timing of the observations. As the 2.7-GHz data are generally more widely spaced than the 10.6- and 8.1-GHz data, *such underestimates should be more pronounced at the lower frequency.*

In contrast to this expected bias, we find (Fig. 5) that for more than half the observed population the ratios $\hat{R}_{10.6}$ and $\hat{R}_{8.1}$ between our range *estimates* $\Delta\hat{S}_{10.6}$,

TABLE VI. Incidence of 2.7-GHz variability as a function of optical identification.

Sp. class	Galaxies No. observed	Galaxies No. variable	Lacertids No. observed	Lacertids No. variable	Stellar objects No. observed	Stellar objects No. variable	Unidentified No. observed	Unidentified No. variable
GS	7	0	0	...	1	0	5	1
G	80	10	3	1	50	12	52	4
GQ	2	0	0	...	20	14	7	4
Q1	2	0	0	...	22	17	4	2
Q2	2	2	8	6	25	22	2	2
Q3	5	2	2	2	14	11	1	1
Unknown	10	1	0	...	12	1	27	3

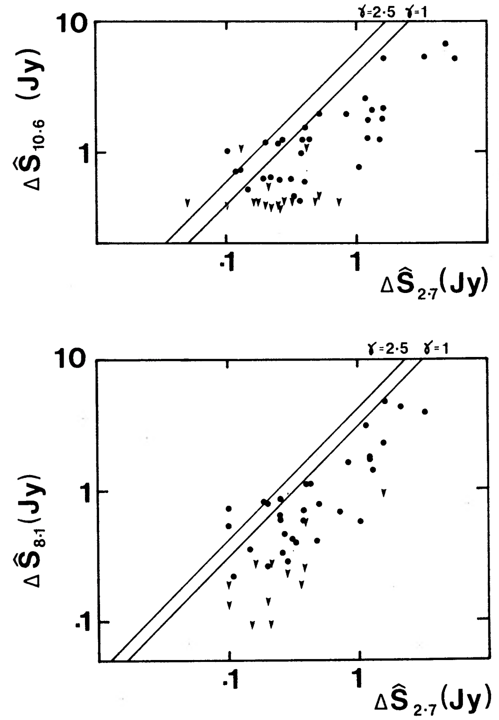

FIG. 5. Plots of (a) $\Delta\hat{S}_{10.6}$ against $\Delta\hat{S}_{2.7}$ and (b) $\Delta\hat{S}_{8.1}$ against $\Delta\hat{S}_{2.7}$ for variable sources observed in the monitoring programs of Andrew *et al.* (1976), Altschuler and Wardle (1976), and Paper I. Upper limits (computed as described in Sec. I-B) are denoted by arrows. The straight lines indicate the ratios predicted by the standard expanding cloud model for two characteristic values of γ.

$\Delta\hat{S}_{8.1}$, and $\Delta\hat{S}_{2.7}$ are less than the R values predicted by the standard model with $\gamma \gtrsim 1$. The implied discrepancy of much of the source population from the standard model is unlikely to be due to sampling bias, particularly as the major bias should be in the sense opposite to the effect found.

Others (e.g., Locke *et al.* 1969) have shown that multifrequency observations of some individual sources are hard to understand in terms of standard van der Laan models, and more complex, and possibly more realistic, models have since been proposed (e.g., Peterson and Dent 1973; de Bruyn 1976; Pacholczyk and Scott 1976). As our data do not define individual outbursts in detail, analysis of them using more complex models is not warranted. Figure 5 implies, however, that *deviations from the instantaneous injection, adiabatic expansion model with $\gamma \gtrsim 1$ are probably widespread* and that this model's apparent success in accounting for the variations in such sources as 3C 120 (Pauliny-Toth and Kellermann 1968a) may be atypical.

We formed a "conformity index" C from the data shown in Fig. 5 as $C = (\hat{R}_{10.6}/2 + \hat{R}_{8.1}/1.7)/2$. For $\hat{R}_{10.6} \geq 2$ and $\hat{R}_{8.1} \geq 1.7$, the implied value of γ in the standard model would be ≥ 0. We therefore regard $\hat{R}_{10.6} < 2$ and $\hat{R}_{8.1} < 1.7$ as *significant* discrepancies from the standard model. Thus, sources with $C < 1$ are, *on average*, significantly discrepant from the standard model in both $\hat{R}_{10.6}$ and $\hat{R}_{8.1}$. Figure 6 shows radio two-color diagrams for (a) sources with $C > 1$ and (b) sources with $C < 1$. It is clear that the two classes of source do not occupy the same regions of the two-color diagram. The sources whose amplitude ratios are consistent with the model preferentially occupy the Q2 region of the diagram. On conventional source models, these sources have optical depths $\tau \gtrsim 1$ throughout most of the frequency range 1–10 GHz. In contrast, the sources whose variability amplitudes do not conform to the standard model have spectral indices (G, some Q1) or spectral curvature (Q3) which indicate that a substantial part of their emission in this frequency range comes from regions with $\tau < 1$.

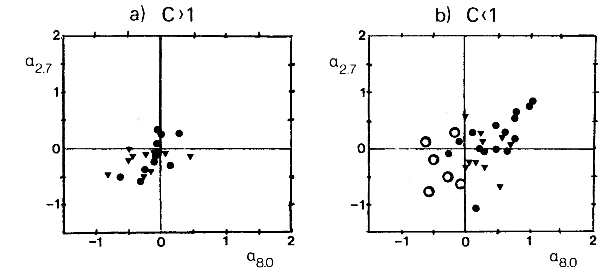

FIG. 6. Radio two-color diagrams for (a) sources with $C > 1$ (variability amplitudes not significantly deviant from the standard expanding cloud model) and (b) sources with $C < 1$ (significantly deviant). Symbols: ●: $\hat{V}_{2.7} < 0.1$; ▼: $0.1 \leq \hat{V}_{2.7} < 0.3$; O: $\hat{V}_{2.7} \geq 0.3$.

On models wherein components are "born" optically thick with flat injection spectra and subsequently evolve by expansion and E^2 losses, new components would be "born" in the Q2 region of the two-color diagram and would migrate first to the Q1, then to the G region; the exact two-color path would depend on the injection, expansion, and loss-rate parameters. Significant recurrent activity in such sources would produce G, GQ, Q3, or Q2 overall spectra (in order of increasing dominance of the later events). Table VII presents the results, diagrammed in Fig. 6, as a contingency table based on this evolutionary sequence in the two-color diagram. The decrease in the fraction of "conforming" sources going from Q2 to the remaining spectral classes is significant at the $\lesssim$1% level when the usual χ^2 tests are applied. The number of Q1 sources is too small for it to be demonstrable that class Q1 *is* intermediate between Q2 and the remainder, but the data are evidently consistent with this view. It can also be seen from Fig. 6 that the sources with the highest fractional variabilities $\hat{V}_{2.7}$ do *not* conform

TABLE VII. Conformity with the standard model in different spectral classes.

	Q2	Q1	G/GQ	Q3
$C > 1$	14	3	2	3
$C < 1$	5	9	14	3

to the standard model—i.e., if the variable emission is a large fraction of the total, its low-frequency amplitude is higher than would be predicted.

The circumstances under which the variability amplitudes at different frequencies conform to the standard model thus appear to be (1) spectral dominance of "young" optically thick source regions and (2) modest fractional variability of these regions. These conclusions are discussed further in Sec. V.

IV. VARIABILITY IN CLASS G AND GS SPECTRA

A. The Fractional Variability $\hat{V}_{2.7}$

Figure 7 shows histograms of $\hat{V}_{2.7}$ (defined in Sec. I-B) for variable sources in the various spectral classes, ordered according to increasing dominance of optically thick emission in the 1–10-GHz window. The median value of $\hat{V}_{2.7}$ increases systematically through the spectral classification, i.e., the proportion of variable flux density increases with the proportion of Q-type flux density. Even for the Q2 sources, however, $\hat{V}_{2.7}$ does not exceed 0.4. It is unlikely that the fractional values of $\hat{V}_{2.7}$ among the Q2 sources arise soley from our discrete sampling of these sources' light curves; it is more likely either that only a fraction of the Q-type emission varies significantly on time scales $\lesssim 2$ yr, or that superposition of bursts simulates a quasisteady background emission.

The observed variation of $\hat{V}_{2.7}$ with spectral class is broadly consistent with the simple hypothesis that class G variables contain weak components similar to the regions seen in isolation as Q2 sources.

Suppose that *on average* the observed variable luminosity ΔL bears a proportionality $\Delta L = \lambda \bar{L}_Q$ to the average luminosity $\bar{L}_Q$ of Q-type emission at 2.7 GHz. (λ may be a function of L_Q or of time in a given source—this argument depends only on *average* properties.) The average total luminosity at 2.7 GHz will be composed of Q-type emission plus some steep-spectrum (G type) luminosity L_G; if we assume that G-type source regions do not vary significantly, then $\bar{L} = \bar{L}_Q + L_G$. For Q2 *sources*, $\bar{L}_Q \gg L_G$; for G *sources*, $\bar{L}_Q < L_G$. Then

$$\hat{V} \sim \Delta S/2\bar{S} = \Delta L/2\bar{L} = \lambda \bar{L}_Q/2(\bar{L}_Q + L_G),$$

i.e.,

$$\hat{V} = \frac{1}{2}\lambda\,(1 + \ell)^{-1},$$

where $\ell = L_G/\bar{L}_Q$. We find $0.04 < \hat{V}_{2.7} < 0.4$ with median 0.13 for the Q2 *sources* ($\ell \sim 0$), so we conclude that $0.08 < \lambda < 0.8$ with median 0.26 for these objects. If the variable components of the class G sources were physically similar to the Q2 sources and had the same range of λ, we should expect to find (for $\ell \sim 3$) the $\hat{V}_{2.7}$ values in the G sources to be <0.1 with median 0.03—as is in fact observed.

An alternative hypothesis might be that G-type variables contain strong *slowly varying* G-type components

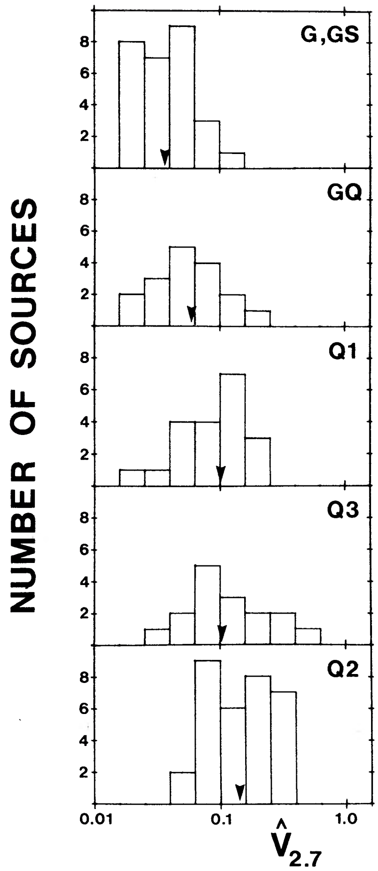

FIG. 7. Distributions of variability indices $\hat{V}_{2.7}$ of variable and possibly variable sources, medians indicated by arrows.

(so that our two-year monitoring detected only a small fraction of their long-term variation). This hypothesis is ruled out by the fact that most of the class G variables varied sporadically rather than showing long-term trends during our monitoring. The further alternative that the G-type variables contain strong rapidly variable G-type components would require unusual assumptions about the source physics.

B. The Variable Luminosity

With one exception, there is no significant separation between the luminosity distributions of the variable emission in the different spectral classes (Fig. 8). The exceptional variable component is a low-luminosity variable in the class G source 0240 − 002 (= 3C 71 = NGC 1068) which may be associated with two compact emission regions near the nucleus of this Seyfert galaxy (see notes to Table VIII in Sec. IV-C). Apart from this source, the range of variable luminosities in each spectral class is derived more from the common range of redshifts than from the observed range in $\Delta\hat{S}$—i.e., there are low-redshift galaxies (e.g., 3C 84, 3C 120) among the high-amplitude Q2 variables and high-redshift quasars (e.g., CTA 102) among the G variables. The similarity

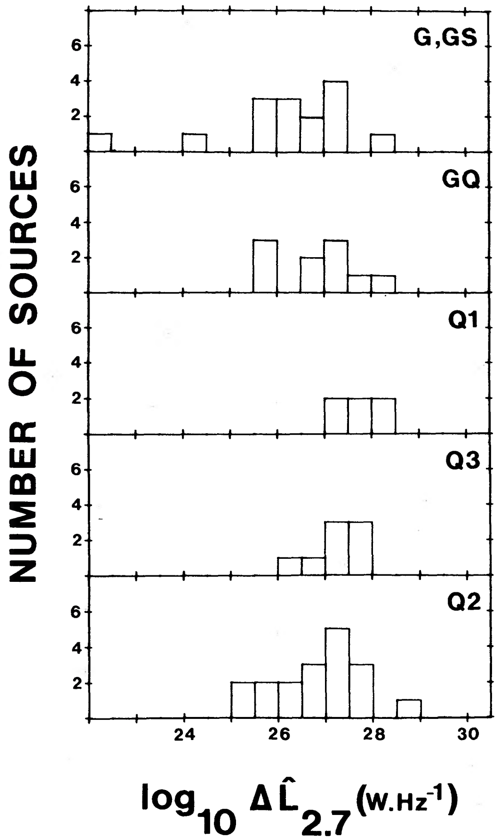

FIG. 8. Distributions of variable luminosity $\Delta\hat{L}$ at 2.7 GHz (based on variable flux density $\Delta\hat{S}$) of variable and possibly variable sources whose redshifts are known. An Einstein-de Sitter cosmology with Hubble constant 50 km sec^{-1} Mpc^{-1} has been assumed.

of the broad luminosity distributions of the G and Q2 variables is again consistent with the hypothesis that the variable regions in both spectral classes are of the same nature.

C. Independent Evidence for Weak Compact Components

Table VIII and Fig. 9 summarize the available high-resolution structural and spectral observations of the class GS and G sources which definitely vary at 2.7 GHz. (The "possibly variable" sources are excluded here.) Most of the sources contain structure with angular scale less than 1 arcsec (in many cases *much* less than 1 arcsec) as well as more extended structure. Where the positions of the fine structure are known, the compact components are roughly centrally located in the source structures and coincide with the optical objects identified with the radio sources. The compact components rarely contain the majority of the total flux density; this reinforces our interpretation of the low *fractional* variability found in these class G sources, namely that their variations are large proportional changes in weak components rather than weak variations of strong components.

Our spectral classification is based on the spectral indices in the two windows centered on 2.7 and 8.0 GHz; it does not take account either of spectral detail within these windows or of the spectral shape outside these windows. Figure 9 shows directly that in most cases the G classification is nevertheless a fair representation of the spectral shape. About half of the class G variables have spectral cutoffs below 2 GHz or spectra which flatten significantly at the highest frequencies, but the others show no such direct spectral evidence for opaque, presumably compact, components. It is noticeable, however, that the spectra of the class G variables with the *largest* fractional variabilities $\hat{V}_{2.7}$ *do* show some evidence for weak opaque components.

In summary, the available details of the source structures and spectra support the interpretation that the class G variables contain weak compact (and probably optically thick) source components, in at least some cases

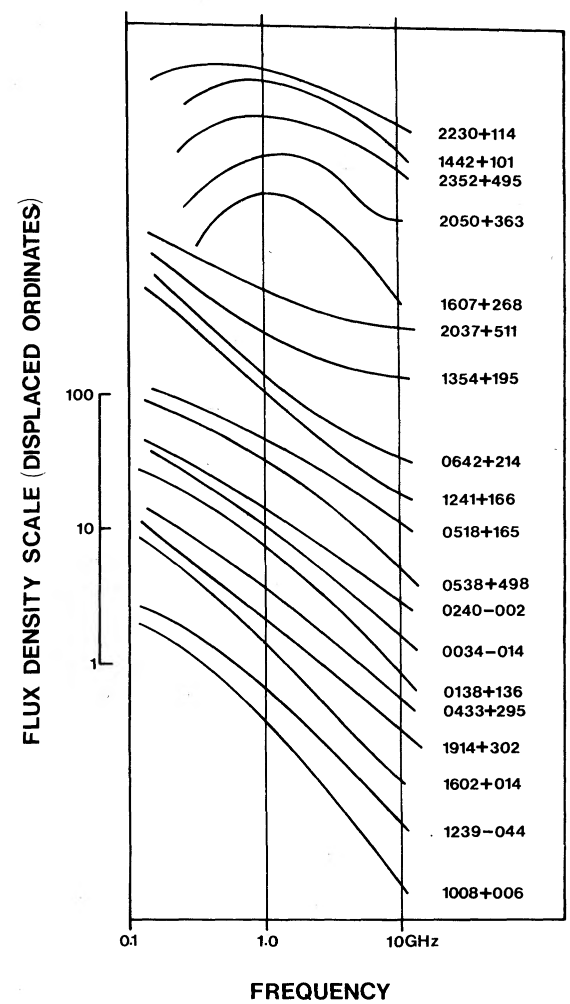

FIG. 9. Representations of the spectra from 0.1 to 10 GHz of the class G sources which definitely varied at 2.7 GHz. It should be noted that the systematic spectral classification is based on the spectrum in the window from 1 to 10 GHz.

TABLE VIII. Small-scale structure in variable sources of classes GS and G.

Source	Sp. class	Frequency	Information on small-scale structure	Ref.	Note
1607 + 268	GS	2.3 GHz	Overall diameter 0.″0019 ± 0.″0002	VLB 1	*
0034 − 014	G	8.1 GHz	8% of flux in unresolved (≲ 0.″4) component coincident with galaxy	I 1	
0138 + 136	G	430 MHz	Diameter ∼0.″03	VLB 2	*
0240 − 002	G	430 MHz	<1% unresolved at 3.5 × 10⁶ λ	VLB 2	*
		2.3 GHz	<9% unresolved at 81 × 10⁶ λ	VLB 1	
		8.1 GHz	19% in two unresolved (<2″) components	I 1	
0433 + 295	G	81.5 MHz	15% in component 0.″8	S 1	*
		195, 430, 611 MHz	5% in component ≤0.″2	S 2	
		430 MHz	<0.3% unresolved at 3.5 × 10⁶ λ	VLB 2	
		5 GHz	39% in barely resolved (1.″2 × <1.″8) component	I 2	
		15 GHz	∼0.1 Jy in compact component coincident with identification	I 11	
0518 + 165	G	2.7 GHz	>50% in two components, each <0.″04	I 3	*
0538 + 498	G	448 MHz	50% in two components ∼ 0.″007	VLB 3	*
		2.7 GHz	80% in two components <0.″07 × 0.″04	I 4	
0642 + 214	G	2.7 GHz	30% in component <0.″1	I 1	*
		8.1 GHz	48% in component <0.″1	I 1	
1008 + 006	G	81.5 MHz	65% in component 0.″35	S 1	
		195, 430, 611 MHz	30% in region ≲0.″2	S 2	
1239 − 044	G	81.5 MHz	Entire source scintillates; diameter 0.″7	S 1	
		408 MHz	40% unresolved (<1.″2)	I 7	
1241 + 166	G	430 MHz	20% in component <0.″2	S 4	
		610 MHz	∼20% unresolved on 24-km baseline	I 8	
1354 + 195	G	430 MHz	40% in component <0.″25	S 4	*
1442 + 101	G		No high-resolution data available		*
1602 + 014	G	81.5 → 611 MHz	No scintillating component seen	S1, S2, S4	
		2.7 GHz	Equal double, 12″ separation	I 9	
1914 + 302	G	81.5 → 611 MHz	No scintillating component seen	S1, S4	
		430 MHz	<2% unresolved at 3.5 × 10⁶ λ	VLB 2	
2037 + 511	G	1.7 GHz	≥ 70% in component ≲ 0.″015	VLB 5	*
2050 + 363	G	430 MHz	<6% unresolved at 3.5 × 10⁶ λ	VLB 2	*
2230 + 114	G	2.3 GHz	≳20% in component ≲0.″001	VLB 5	*
2352 + 495	G		No high-resolution data available		*

References:

VLB 1 Kellermann *et al.* (1970).
VLB 2 Broderick and Condon (1975).
VLB 3 Clarke *et al.* (1969).
VLB 4 Kellermann *et al.* (1975).
VLB 5 Kellermann *et al.* (1971).
I 1 E. B. Fomalont, G. W. Brandie and A. H. Bridle (unpublished observations with the NRAO interferometer).
I 2 Pooley and Henbest (1974).
I 3 Donaldson *et al.* (1971).
I 4 Donaldson and Smith (1971).
I 5 Wilkinson (1972).
I 6 Macdonald and Miley (1971).
I 7 Wraith (1972).
I 8 Wilkinson *et al.* (1974).
I 9 Adgie *et al.* (1972).
I 10 Longair (1975).
I 11 Pooley (1976).
S 1 Readhead and Hewish (1974).
S 2 Cohen *et al.* (1967).
S 3 Harris (1973).
S 4 Harris and Hardebeck (1969).

Notes to Table VIII

1607 + 268 The spectrum shows self-absorption below ∼1 GHz and progressive steepening above this frequency.

0138 + 136 The spectrum flattens below ∼400 MHz.

0240 − 002 The unresolved components have 1950 positions and 8.1-GHz flux densities:
$02^h40^m7^s.26$ $-0°\ 13'\ 28''.2$ (0.20 ± 0.03 Jy)
$02^h40^m6^s.57$ $-0°\ 13'\ 32''.4$ (0.06 ± 0.01 Jy)
The existence of two compact components is confirmed by more extensive synthesis observations with the NRAO interferometer (P. Crane, private communication).

0433 + 295 The source has been identified with a galaxy with redshift 0.637 (Spinrad 1975); the main compact component is displaced from the galaxy (Longair and Gunn 1975), but the weak, high-frequency, compact component (Pooley 1976) coincides with it.

0518 + 165 Variability and possible variability have been noted by Shimmins and Wall (1973) and Wills (1975). The spectrum may show a cutoff below 100 MHz.

0538 + 498 The spectrum shows a cutoff between 38 and 178 MHz.

0642 + 214 The spectrum flattens slightly above ∼2 GHz.

1354 + 195 The spectrum is flatter at frequencies above 1.4 GHz than at lower frequencies. Variability has been found by Medd *et al.* (1972) and by Shimmins and Wall (1973).

1442 + 101 There is a spectral component with a cutoff below ∼1 GHz and a strong millimeter-wave component dominating the spectrum above ∼20 GHz (Gearhart *et al.* 1974). Variability has been found by Ross (1972).

2037 + 511 Variability has been found by Dent and Kojoian (1972), Medd *et al.* (1972), Dent and Hobbs (1973), and Kellermann and Pauliny-Toth (1973).

2050 + 363 The spectrum has a cutoff below ∼1.4 GHz.

2230 + 114 The spectrum becomes flat at frequencies below ∼800 MHz. Variability has been found by Medd *et al.* (1972), Nicolson (1973), Shimmins and Wall (1973), and Wills (1975).

2352 + 495 The spectrum becomes flat or self-absorbed below ∼1.4 GHz. Variability has been found by D. E. Hogg (reported by Bridle *et al.* 1972).

as the central "core" components of extended emitting regions.

D. Discrepancies from the Standard Model

Figure 5 (Sec. III) shows that the variability amplitudes of the class G variables at different frequencies do not usually conform to those predicted by the standard expanding cloud model. If the variable components in class G sources are indeed weak components with high fractional variabilities, this nonconformity is concordant with our observation that the Q2 sources with high $\hat{V}_{2.7}$ do not obey the model. Thus, our results on both G and Q2 sources suggest that *highly variable* Q-*type source components,* whether coexisting with steep-spectrum emission (class G) or in isolation (class Q2), *deviate systematically from the standard expanding cloud model.*

E. Conclusion

Our observations support the conclusion that at least 25% of class G sources contain weak variable components which coexist with more intense, transparent, nonvariable emission. The properties of these variable *components* appear identical to those of isolated, compact, variable *sources* in all presently measurable respects.

V. DISCUSSION

A. What Fraction of Class G Sources Varies?

We have found $\sim 25\%$ of the observed class G sources to vary at 2.7 GHz. As discussed in Sec. I-C, *this probably underestimates the true incidence of variability in these systems.* The data in Fig. 7 illustrate the way in which selection effects will bring about this underestimate; the lowest value of $\hat{V}_{2.7}$ in a Q2 variable is 0.04 (2145 + 067); in a class G variable with $\ell \sim 3$ (Sec. IV-A), this would correspond to $\hat{V}_{2.7} \lesssim 0.01$, which could not normally be detected using our data. Indeed, to be reasonably sure of detection, we need $\hat{V}_{2.7} \gtrsim 0.04$; if this were assumed to correspond to the variations of a Q2 *component* in a source with $\ell \sim 3$, that component must have $\hat{V}_{2.7} \gtrsim 0.16$. Only about 50% of Q2 *sources* were found to have $\hat{V}_{2.7} > 0.16$; on such a model, the true incidence of variability in class G could therefore be $\gtrsim$ 50%. We note that this is comparable to the known incidence of compact "core" components in extended, extragalactic sources (e.g., Riley and Pooley 1976).

B. Do Quasars Vary More than Galaxies?

We noted in Sec. II-C that the spectral classification of a source is more important than its identification in determining whether or not it is found to vary. Within a given spectral class, quasars do not appear much more likely to vary than do galaxies; the distribution of quasars and galaxies among the spectral classes is, of course, very different (Brandie and Bridle 1974; Wall 1975). The only hint of a relationship between variability and identification in the parameters which we have examined is a trend for the variable luminosity in quasars to exceed that in galaxies (Fig. 10).

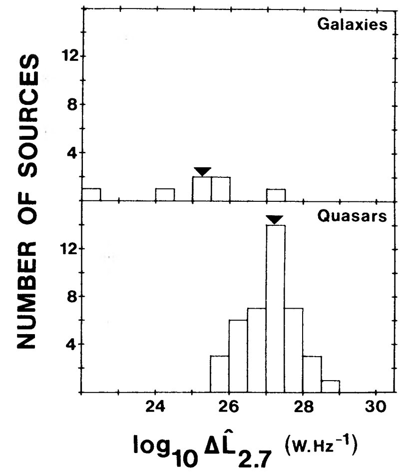

FIG. 10. Distributions of the variable luminosity $\Delta\hat{L}$ at 2.7 GHz for radio galaxies and quasistellar objects whose redshifts are known. An Einstein–de Sitter cosmology with Hubble constant 50 km sec^{-1} Mpc^{-1} has been assumed. Arrows indicate medians.

C. Does $\hat{V}$ Correlate with Luminosity?

On the most simple "shot-noise" models for multiple-burst sources, the total variable luminosity L_V would be proportional to the number N of bursts present and the fluctuations to $\overline{N}^{1/2}$. This might lead to a correlation between $\hat{V}$ and $L_Q^{-1/2}$. We do not find such a correlation among the class Q2 variables, in which L_Q dominates the total luminosity. Thus, on a "shot-noise" model either the bursts must be extremely dissimilar or N must be quite small.

D. Do *Any* Variables Obey the Standard Model?

We found (Sec. III) that the sources which appear to conform to the standard instantaneous injection, adiabatic expansion model have dominant Q-type, presumably optically thick regions but modest fractional variabilities. Conversely, the "deviant" systems are class Q2 sources with high $\hat{V}_{2.7}$, and the majority of the non-Q2 are variables.

We offer a simple model for these effects. We suggest:

(1) There is only one type of variable-source mechanism; it produces "deviant" amplitudes, i.e., the variability amplitudes at low frequencies are enhanced relative to those expected on the standard model.

(2) The bursts are observed *through* some fraction

of the nonvarying, optically thick emission; the variation of the interposed optical depth with frequency therefore modifies the observed variability amplitudes. On this scenario *all sources with little nonvarying, opaque emission would be observed, correctly, as "deviant,"* accounting, in particular, for the deviance of the class G variables and the class Q2 variables with high $\hat{V}_{2.7}$.

In Q2 sources of modest $\hat{V}_{2.7}$, we would observe the intrinsic burst amplitudes modified by the transmission properties of the intervening opaque material; *this would attenuate the observed amplitudes more at the lower frequencies, producing the appearance of conformity with the standard model.* In the typical Q2 source, the optical depth of the total spectrum would be $\tau \sim 1$ at 8 GHz and $\tau \sim 10$ at 2.7 GHz. If the apparent conformity with the standard model is indeed due to relative attenuation of the low-frequency variations by factors $\sim e^{-1}$, the difference between the optical depths in front of the variable emission at 2.7 and 8 GHz should be $\Delta\tau \sim 1$. As the difference between the optical depths in the total opaque spectrum at these frequencies is $\Delta\tau \sim 9$, only ~10% of the opaque material should lie in the line of sight to the variable emission. A number of plausible geometries for the optically thick emission—e.g., thin shells, dumbbells, or filaments—could produce such parameters. A natural source of this absorbing material would be ejecta from previous activity in the variable source itself.

It therefore appears possible that a single variation mechanism *could* explain all the observations, but that such a mechanism would not correspond to the standard model. If the variation of burst amplitude with frequency is indeed dominated by opacity external to the variable source itself, the form of the observed light curve would be governed more by the particle acceleration mechanism than by the transition from high optical depth to transparency, as in the standard model. We will elaborate on such models elsewhere.

E. Do Extended Emission Regions Affect Variability?

After considering the inevitable selection effects, we have found no convincing evidence of a significant difference between the mechanisms of variable emission in the different spectral classes. Our results are consistent with the occurrence of a single class of compact variable object, either essentially in isolation, or coexisting with large-scale, transparent nonvariable emission. Such properties as variable luminosity, time scale of variations, and fractional variability appear essentially unaffected by coexistence with transparent emission, suggesting that the variable objects are dynamically independent of the processes producing the latter (more extensive) emission. In view of the small linear sizes inferred for variable-source components by very-long-baseline interferometry, such independence is not altogether surprising.

Our observations have shown, however, that *the development of prominent transparent emission does not extinguish a compact source's capacity to vary.* The astrophysical implications of this result are that either the energy supply for the transparent emission is effectively independent of that of the variable sources, or that the regeneration of the energy reservoir in the compact regions is almost independent of the past history of the system. The practical implications are that the accurate calibration of flux-density measurements at centimeter, and shorter, wavelengths is even more difficult than previously thought, for even a "normal" class G spectrum does not guarantee an extragalactic source's flux-density stability.

We thank B. H. Andrew for communicating results of the 10.6-GHz monitoring program at the Herzberg Institute of Astrophysics in advance of publication; E. B. Fomalont for collaborating with us in studies of the fine structure of interesting sources; and M. P. Véron and P. Véron for providing an updated listing of their identification catalog. This work was supported by grants to A. H. B. and M. J. L. K. from the National Research Council of Canada.

The Algonquin Radio Observatory is operated as a national radio astronomy facility by the National Research Council of Canada; the NRAO is operated by Associated Universities, Inc., under contract with the National Science Foundation.

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