THE ASTROPHYSICAL JOURNAL

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May 4, 1981

Dr. Alan H. Bridle
National Radio Astronomy Observatory
VLA Program
P. O. Box O
Socorro, NM 87801

Dear Dr. Bridle:

ORBITAL MOTION OF THE HEAD-TAIL RADIO GALAXY IC708
by J. P. Vallée, A. H. Bridle, and A. S. Wilson

We are pleased to report that the above paper has been accepted for publication in The Astrophysical Journal. It is scheduled for the November 1, 1981 issue. You will receive the edited manuscript by June 25. Please return it to our production office in Chicago within forty-eight hours. Galley proofs will be sent to you by August 6. These also must be returned to our production office within forty-eight hours. If you will not be at the present address during these periods, please provide Mr. Bilsens with a forwarding address or arrange for a review of the manuscript and galleys during your absence. Your cooperation will expedite the publication of your paper in the scheduled issue of the Journal.

Enclosed is the Publication Agreement pertaining to copyright assignment, which should be signed and returned as addressed.

Sincerely,

Helmut A. Abt

Helmut A. Abt /Kar
Managing Editor

HAA:kar

Enclosure

cc: Mr. Elmars Bilsens
Production Manager

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NATIONAL RADIO ASTRONOMY OBSERVATORY

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TELEPHONE 505 835 2924 TWX 910 988 1710 VLA SITE 505 772 4011

27 April 1981

Dr. Helmut A. Abt,
Managing Editor,
The Astrophysical Journal,
Kitt Peak National Observatory,
P.O. Box 26732,
Tucson, AZ 85726.

Dear Dr. Abt,

We enclose a revised version of the article ORBITAL MOTION OF THE HEAD-TAIL RADIO GALAXY IC708 by Vallee, Bridle and Wilson which takes account of the referee's comments.

We have shortened Section IV by about 30% in response to the referee's criticism. We have also strengthened the statement on p.14 which explains why we use an ad hoc model for the variation of emissivity along the radio trails. The suggestion of the referee that "standard synchrotron radiation theory" be used to predict the emissivity variation has been known since the work of Jaffe and Perola (1973) to fail to match observed trail properties unless an ad hoc particle replenishment scheme is also introduced. There is presently no consensus about the mechanisms for, much less the parameterisation of, particle replenishment in the trails. As this topic is clearly stated to be outside the scope of the present article, we feel justified in adopting a purely empirical model for the emissivity variations.

We do not wish to shorten Sections III or V similarly. Section III contains a careful discussion of the possible centers of attraction which can be responsible for the distortions of this radio source; all that follows rests on the nature and location of these centers and we do not wish the reduce the weight of this discussion. It is precisely the "elementary" nature of the considerations given in Equations (1) to (7) that is likely to make the conclusions drawn from them in Sections III and V fairly independent of modelling details. In this case we feel that it is worth making the elementary nature of the argument very clear by writing the equations.

The referee suggests that we give even less attention to models which fail to account for the structure (most of the discussion is now only a few sentences and some entries in Table 3). Our conclusion that continuous-jet dynamics are much better able to describe the trail structure is not yet generally-accepted wisdom, so we feel that the precise reasons for the rejection of other dynamical models are still of interest. The referee's criticism itself attests to this, as it urges us to emphasise the JP model, which is not the preferred dynamics over most of the trail length.

We have reformatted the title page, and have altered the nomenclature of the Tables, as requested in your letter. We request that Figure 6 be printed on text stock within the paper, and that the Tables be typeset.

We trust that the article as revised is now acceptable for publication in the Astrophysical Journal.

Please continue to send all correspondence in connection with this article to Dr A. H. Bridle, NRAO VLA Program, P.O. Box O, Socorro, NM 87801.

Yours sincerely,

Alan H Bridle

Dr A. H. Bridle

A.S. Wilson

Dr A. S. Wilson

N.B.

VLA Site
8 December 1980
Original

Dear Jacques, Dear Andrew,

4th Feb [this one]

Here is what I hope will be the final draft of IC708. This takes account of the most recent iterations between JPV and AHB regarding interpretation. In particular see new discussion of the mass discrepancy on pages 19 and 21.

we are using APPENDIX (A') following earlier APPENDIX (A).

Appendix A was scratched after JPV noted that we are not now computing with those equations anyway and after various people commented to me that it was difficult to follow without further diagrams.

Jacques is having various small scruffinesses in the diagrams tidied up.

Jacques - could you provide your favorite brief definition of the angles quoted in Table III ? → Enclosed here (I thought I had done so)... Sorry!

Please notate these copies with final proofreading errors, and return to me. The original is in a horrendous state (as the VLA Text Editor has a 6-week backlog I have cut/paste this version together and retyped parts of it myself - main reason for the delay), so I will munge together a set of tidy Xeroxes for actual submission.

Jacques is having photography of the final diagrams done in Canada.

Andrew - we thought A.J. would be most appropriate purely because of the long delays now encountered in Ap.J., and because A. and A. can only give rapid publication if you have a camera-ready manuscript. This manuscript is now a horrible montage that any camera would barf at.

My immediate schedule is such that you should probably send your comments to me at U.N.M. (Dept. of P. and A., 800 Yale Blvd. N.E., ABQ, NM 87131).

Merry Christmas,

Jacques. Could you try to provide this before the photographs arrive back here from the far East?

Alan

P.S.: any photo made for ASTRONOMERS is not done in HIA nor in NRC nor in MINISTRY in which NRC resides; it is done in the MINISTRY of "SUPPLY AND SERVICES"!

ASTRONOMY PROGRAM

University of Maryland, College Park, Md. 20742 tel: 301-454-3001

15 November 1980

Dr. A.H. Bridle
Department of Physics and Astronomy
University of New Mexico
800 Yale Blvd. N.E.
Albuquerque, NM 87131

Dr. J.P. Vallée
Herzberg Institute of Astrophysics
100 Sussex Drive
Ottawa, K1A OR6
Canada

Dear Alan and Jacques,

I enclose a copy of the draft of the IC 708 paper with the proof reading corrections in. My remaining comments are:

→ me too; without APPENDIX $\boxed{A'}$ added earlier $\boxed{\text{by hand}}$ > NOBODY can DUPLICATE OUR RESULTS. NOBODY.

1) I am sorry to see the end of Appendix A. It does (did) describe the correct solution to the problem and, although may require a little mental effort to understand, is worth retaining in my opinion. Someone may need the answer to this one day and it would be helpful to find it in a published paper. Only small modification to the text would be required to put it back in.

Of course, Appendix $\boxed{A'}$ reduces to Appendix A when the velocity around the CLUSTER orbit is $\boxed{\text{zero.}}$

→ NO.

2) P.21, paragraph beginning "The Double-Orbit...". Surely the mass here should be the mass within the "cluster (large) orbit" so the agreement of 3.3 x 10^{11} $M_\odot$ with the initial expectations (Section IIIb) has no significance. I would expect M $\sim$ 10^{14} $M_\odot$ within the cluster orbit. The low masses remain a disquieting thing.

$\boxed{3.3 \times 10^{11}\ M_\odot \text{ is as stated in text}}$ (binary elliptical orbit).

3) My preference is still for Ap.J. even if it may take a little longer to publish. The problem with A.J. is that it is considered a lower quality journal and is read by few people, and even fewer in Europe. Theoreticians ignore it (and they should see ~~it~~ the paper) whereas everybody scans Ap.J.

I have no preference in this matter.

MAYBE STATE THAT "all intermediate equations are available from JPV" ???

I don't need to see the corrected version before submission but please send me a final copy and tell me where you sent it.

Merry Christmas to you too!

Andrew

A. S. Wilson

copy to: Dr. J.P. Vallée

ASTRONOMY PROGRAM

University of Maryland, College Park, Md. 20742 tel: 301-454-3001

Dr. A.H. Bridle
Department of Physics and Astronomy
University of New Mexico
800 Yale Blvd. N.E.
Albuquerque, NM 87131

15 November 1980

Dr. J.P. Vallée
Herzberg Institute of Astrophysics
100 Sussex Drive
Ottawa, K1A OR6
Canada

Dear Alan and Jacques,

I enclose a copy of the draft of the IC 708 paper with the proof reading corrections in. My remaining comments are:

1) I am sorry to see the end of Appendix A. It does (did) describe the correct solution to the problem and, although may require a little mental effort to understand, is worth retaining in my opinion. Someone may need the answer to this one day and it would be helpful to find it in a published paper. Only small modification to the text would be required to put it back in.

2) P.21, paragraph beginning "The Double-Orbit...". Surely the mass here should be the mass within the "cluster (large) orbit" so the agreement of $3.3 \times 10^{11} M_\odot$ with the initial expectations (Section IIIb) has no significance. I would expect $M \sim 10^{14} M_\odot$ within the cluster orbit. The low masses remain a disquieting thing.

3) My preference is still for Ap.J. even if it may take a little longer to publish. The problem with A.J. is that it is considered a lower quality journal and is read by few people, and even fewer in Europe. Theoreticians ignore it (and they should see ~~it~~ our paper) whereas everybody scans Ap.J.

I don't need to see the corrected version before submission but please send me a final copy and tell me where you sent it.

Merry Christmas to you too!

A. S. Wilson
Andrew

copy to: Dr. J.P. Vallée

Department of Physics and Astronomy
Division of Mathematical and Physical Sciences and Engineering

TO: DR. A. H. BRIDLE
FROM: J.P.V.
DATE: 1980 OCT. 31
RE: YOUR PHONE CALLS ON MONDAY 27 OCT. '80, RE: IC708 PAPER

1) HOW FAR UP CAN THE ECCENTRICITY GO? CAN IT BE A HYPERBOLIC ORBIT (i.e. ENCOUNTER)? ANSWER: NO, IT NEEDS AN APOGALACTICON TO TURN AROUND AND GIVE TWO HOOKS IN THE RADIO RIDGE STRUCTURE. THERE ARE NO KNOWN HYPERBOLIC ORBITS WITH AN APOGALACTICON... ERGO, THE ECCENTRICITY CANNOT REACH $\boxed{\text{UNITY}}$, TO SEE THE HOOKS.

(a) BRB (double orbit) HAS AN ECCENTRICITY OF $\boxed{0.873}$, SEMIMAJOR AXIS OF 41 kpc, SEMIMINOR AXIS OF 20 kpc. IF YOU WANT, THE CLUSTER DIAMETER = 6 Mpc (ABELL DEFINITION) CAN BE USED AS AN UPPER LIMIT ON SEMIMAJOR AXIS, USING SAME SEMIMINOR AXIS OF 20 kpc. THIS WILL GIVE AN ECCENTRICITY OF $\boxed{0.999}$.

(b) dP (SINGLE ORBIT) HAS AN ECCENTRICITY OF $\boxed{0.70}$, SEMIMAJOR AXIS OF 125 kpc, SEMIMINOR AXIS OF 90 kpc. IF YOU WANT, THE CLUSTER DIAM. = 6 Mpc CAN BE USED FOR AN UPPER LIMIT ON SEMIMAJOR AXIS, USING SAME SEMIMINOR AXIS OF 90 kpc. THIS WILL GIVE AN ECCENTRICITY OF $\boxed{0.999}$.

AS A RULE, YOU DESTROY THE BEST FIT BY DOUBLING THE SEMIMAJOR AXIS (FROM 41 kpc → 82 kpc IN (a) GIVING ECCENTRICITY OF 0.969; FROM 125 kpc → 250 kpc in (b) GIVING ECCENTRICITY OF 0.933). SO, CONCLUDING, A TYPICAL $\boxed{\text{ERROR}}$ ESTIMATE OF THESE ECCENTRICITIES IS $\boxed{\pm 0.15}$.

---

2) WIDTH OF NARROW JET WITHIN 3 ARCSEC OF OPTICAL CENTROID OF GALAXY? ANSWER: EQUAL TO HPBW OF 20-km TAPER (0.63 ARCSEC), ON THE (CONTOUR) FIGURE 1. MY PRINTOUT DISPLAY AT 20-km TAPER HAS A VERY CRUDE INTENSITY SCALE (1-2-3) ONLY, SO I CAN SAY LITTLE...

(a) THE INTENSITY PER BEAM AREA (20-km TAPER) VARIES FROM THE CENTRE ALONG THE JET, BEING 10 mJy/beam area at 1" distance, BEING $\boxed{4}$ mJy/beam area at 2" distance, and BEING 1.5 mJy/beam area at 3" dist.

(b) THE INTENSITY PER BEAM AREA (5-km TAPER) VARIES SIMILARLY AS FOLLOWS, BEING $\boxed{18}$ mJy/beam area at 2" distance, BEING 9 mJy/beam area at 3" dist. (resolved with HPBW = 1.35 ARCSEC). N.B.: (a) ≠ (b), SO SENSITIVITY IS A PROBLEM WITH 20-km TAPER...

ASTRONOMY PROGRAM
University of Maryland, College Park, Md. 20742 tel: 301-454-3001

Sep 29 1980

Dear Alan and Jacques,

Yes, its almost there. I am unhappy about having "M in the range 1.5×10^{11} to $1.5 \times 10^{12} M_\odot$" (top. p. 10) but only 1×10^{10} in the model (Table III) with a single orbit. Ideal would be a computer run with $M = 5 \times 10^{11} M_\odot$. Also, I would have thought that $M \sim 10^{14} M_\odot$ or more would be appropriate for a double orbit interpretation in which the whole cluster bends the IC708 orbit.

If we wanted to sound like true orbital theory buffs we could use barycenter instead of center of mass when we are discussing 2 body dynamics. Yes, I'll pay 25% of the page charges (how pleasant not to have those European "free loaders"!). I think this paper is much more suited to Ap. J. than A. J. - why do you prefer the latter?

Cheers
Andrew

Department of Physics and Astronomy
Division of Mathematical and Physical Sciences and Engineering

ASTRONOMY PROGRAM

University of Maryland, College Park, Md. 20742 tel: 301-454-3001

MEMORANDUM

TO: Alan and Jacques

FROM: Andrew ASW

SUBJECT: IC708 Paper

DATE: September 17, 1980

I've now been through your drafts of IC708 and enjoyed reading them both. Although the content is similar I found Alans to be easier going and have therefore made my suggested corrections on his version (enclosed). I'll send the top copy (in which the blue corrections are easier to see) to you Alan since you volunteered to do the remaining rewriting.

My comments are numerous and mainly small. The major point concerns the redshift difference of IC708 and IC709 (see page 9). Since the errors on the radial velocities are $\pm$ 150 km s^{-1} (1 x r.m.s. error from one of the authors) the velocity difference is 64 $\pm$ 212 km s^{-1}. With such a large error, some remarks about the angle of view (page 10) and possibly the doubt about the single orbit model (page 18) may be questioned. I leave it to you Alan to rework the text somewhat to allow for this unfortunate uncertainty (I plan to get better redshifts for these 2 galaxies but the vagaries of telescope time, weather etc. have precluded this so far).

After regarding the whole thing I'm left vaguely dissatisfied. The motion of IC708 seems to be nicely towards IC709 but in the 2 orbit interpretation it is motion around the cluster center (to which the motion of IC708 does not point) which dominates. Perhaps this is just "state of the art". If we stretch the redshift difference to 276 km s^{-1} can we get over the timescale problem?

Revised by AHB
23 Sept 1980.

OBSERVED RADIAL VELOCITIES IN A1314

Cluster Centre: $\langle V_r \rangle_c$ = 10150 km/s

Cluster Dispersion: σ_r = 708 km/s

IC708 - $\langle V_r \rangle_c$ = ~~547 km/s~~ 536 km/s

IC709 - $\langle V_r \rangle_c$ = ~~611 km/s~~ 600 km/s

PROBABILITY FOR ONE GALAXY TO FALL within 64 km/s of 600 km/s, i.e.

Between -536 km/s and -664 km/s is given by:

$$P_g(x,\mu,\sigma)dx = \frac{1}{\sigma\sqrt{2\pi}} \exp[-\frac{1}{2}(\frac{x-\mu}{\sigma})^2]dx$$

← BEVINGTON, P.R. (PAGE 44) "DATA REDUCTION AND ERROR ANALYSIS FOR THE PHYSICAL SCIENCES", (c) 1969, McGRAW-HILL

where: dx = 128 km/s

x = ~~(-547 km/s, -611 km/s)~~

= -600 km/s

μ = 0 km/s

σ = 708 km/s

∴ PROBABILITY = 0.035, but

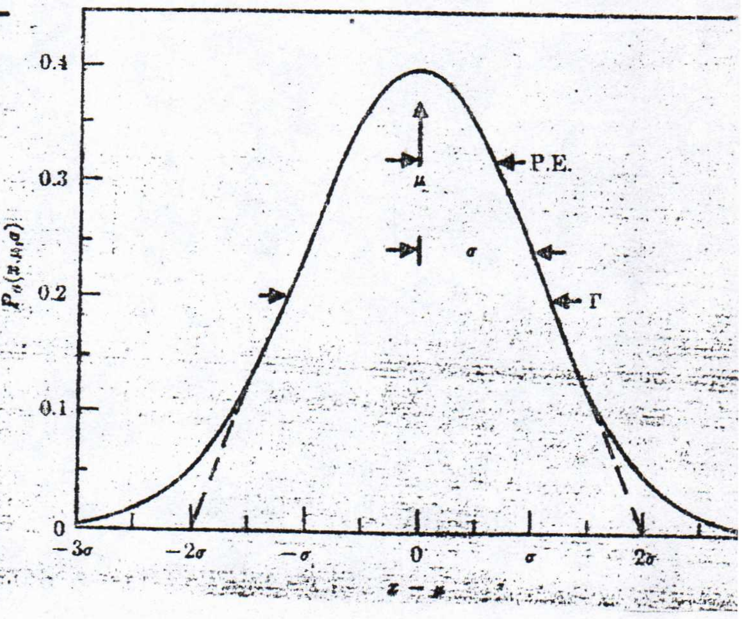
Characteristics The Gaussian distribution function is defined as

$$P_G(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \qquad (3\text{-}9)$$

It is a continuous function describing the probability that from a parent distribution with a mean μ and a standard deviation σ, the value of a random observation would be x. Since the distribution is continuous, we must define an interval in which the value of the observation x will fall. The probability function is properly defined such that the probability $dP_G(x,\mu,\sigma)$ that the value of a random observation will fall within an interval dx around x is given by

$$dP_G(x,\mu,\sigma) = P_G(x,\mu,\sigma)\,dx$$

considering dx to be an infinitesimal differential.



this isn't true.

$P(>536) = 0.2245$
$P(>664) = 0.1742$] Hence $P(536 < V < 664) = .0503$ or $\boxed{5.0\%}$

using normal integral distribution

Interoffice

National Radio Astronomy Observatory

Very Large Array

To: Jacques, Andrew

From: Alan

Subject: IC708 redraft (sorry, Jacques)

I really think Jacques' last draft was becoming very hard to read because of its intermingling of the computational details with the astrophysical flow of ideas, and suggest that we separate them by some re-ordering and by putting certain details into Appendices as in this (rough) redraft. I also feel we need to emphasise more our firm (model-independent) conclusions and explain the merits of the double-orbit picture (not mere complexity but probably essential astrophysics). At the same time, I felt the paper was getting too long for its content and have attempted to shorten it. The result is rough and bumpy, but I'd like to hear your comments on this direction for revision before I work on it some more. I visualise another complete circulated draft before it goes for "final" typing. Despite Jacques' long "history" on the cover of the last draft, I do feel more work is needed to make a readable and concise paper. I'll do the remaining rewriting if you could both send me your opinions.

I've given greater emphasis to the "twin-jet" analogy in the concluding remarks because I believe the direct observations of the intensity and polarization distribution were leading us to the same conclusions that we reached by detailed modelling of the orbital and ejection parameters - namely that analogy with the straight jets may be the right astrophysics for the object. Possibly I'm prejudiced and doing conclusion-jumping that is unwarranted. I will rely on you two to tell me so.

I'd like to see the observed and predicted ridge-line intensities for the double-orbit Begelman model in a Figure such as we had at one stage for the single-orbit JP model. It would help pick out the intensity maxima that we're talking about.

I also thought that at several points in the text it would have been useful to have labelled features along the trails in Figure 3(a). This would make it easier to refer to important features later in the discussion.

At the present rate of progress we ought to think of second-epoch observations to check the model directly. (My fault, Jacques !)

IC 708

JPV? 1 Reprint of Jacques' 1977 Review paper

JPV? From print plot? 2. High Resolution Map

a) Is jet unresolved? It appears to be

b) Why don't you see the counter-jet? Is it

? really very diffuse? Similar to 3C31

⟹ ~~One~~ Northern Tail appears to have a strong radio core + diffuse outer halo.

— Could the jet have "turned" off on one side & electrons are now diffusing away producing a halo of emission?

? 3. Explain why $\vec{v}_e$ w/r to $\vec{v}_g$ changes with time

✓ 4. Equation (1) should be $\ddot{\vec{r}}_1 = \dfrac{-G(m_1+m_2)}{[1+m_1/m_2]^3} \cdot \dfrac{\vec{r}_1}{|\vec{r}_1|^3}$

Co-ord diagram Figure?

(5) You might note (maybe its obvious) that this distance d is ~~in the rest frame of~~ w/r to the initial blob ejection point. Must do a coordinate tranform into rest frame of galaxy to get true tail shape.

(6) Explain I_T, J_T, K_T to me, & A^2.
Maybe a picture to explain coordinate systems.

? (7) A bit difficult to interpret Figures 7 & 8

(8) You might want to rewrite equation (B6) to show more precisely where the typo came from i.e.,

$$g = -\left[\frac{P_0}{\rho_{j_0} h_0}\right]\left[1 + y^{-2}\right]^{\left(\frac{1}{2\gamma}-1\right)}$$

(9) You might note existence of Jones & Owen Model. The only real difference with Begelman et al. is the value of the ~~sca~~ scale height.

Begelman et al. use $h_0 \sim r_j$

⇒ Jones & Owen use $h_0 \sim r_{galaxy}$ ⇒ i.e., cocooned their jet with an ISM, thereby decrease lost of stability.

✓

(10) Not clear ~~a~~ how emmissivity varies with tail radius. Did you assume constant emissivity-filled cylinders?

did not model the radius.

✓

(11) You might just mention that an equally successful model could be an unbound orbit of IC 708/709 (i.e. hyperbolic) since Faber & Gallagher claim there are no true E/E binary pairs.

especially as e=0.7

✓

(12) Figure 9. Error bars ~~on~~ on flux density points. What level of structure can I believe?

DEPARTMENT OF PHYSICS
STIRLING HALL
Physics
Engineering Physics
Astronomy

Queen's University
Kingston, Canada
K7L 3N6

July 29, 1980

Dr. Alan H. Bridle
Dept. of Physics & Astronomy
University of New Mexico
800 Yale Boulevard N.E.
Albuquerque, New Mexico 87131
U.S.A.

Dr. Andrew S. Wilson
Astronomy Program
University of Maryland
College Park, Maryland 20742
U.S.A.

Dear Alan and Andrew:

PDP11/34 computing time for BOWMAP and RAMMAP is directly proportional to the number N_c of cubic blocs used, times the number N_T of trace steps along the radio ridges of a source: $TIME = N_c \cdot N_T$

Trials made this weekend when nobody used the computer indicated the following computing times, using identical input parameters except as given below.

Trial #	N_c	Cube Side	N_T	Degree Per Step	Pixel HPBW	PDP11/34 Computing Time
1	64 millions	2 kpc	10	3.0°	4" = 4 kpc	1 hour
2	64	2	50	0.6°	4 4	5
3	64	2	100	0.3°	4 4	11
4	64	2	200	0.17°	4 4	20
5	125	1.5	170	0.20°	4 4	42

The degree of smoothness of the intensity distribution (RA, DEC) for trial #3 is about equal to that of the actual VLA distribution observed (see figure A attached). For comparison, trial #4 is displayed in figure B attached, and trial #5 is displayed in Figure C1. Figure C2 is a Versatec contour plot of Figure C1 (courtesy of M.J. Kesteven).

Please accept, dear Alan and Andrew, the expression of my best sentiments.

Amicalement,

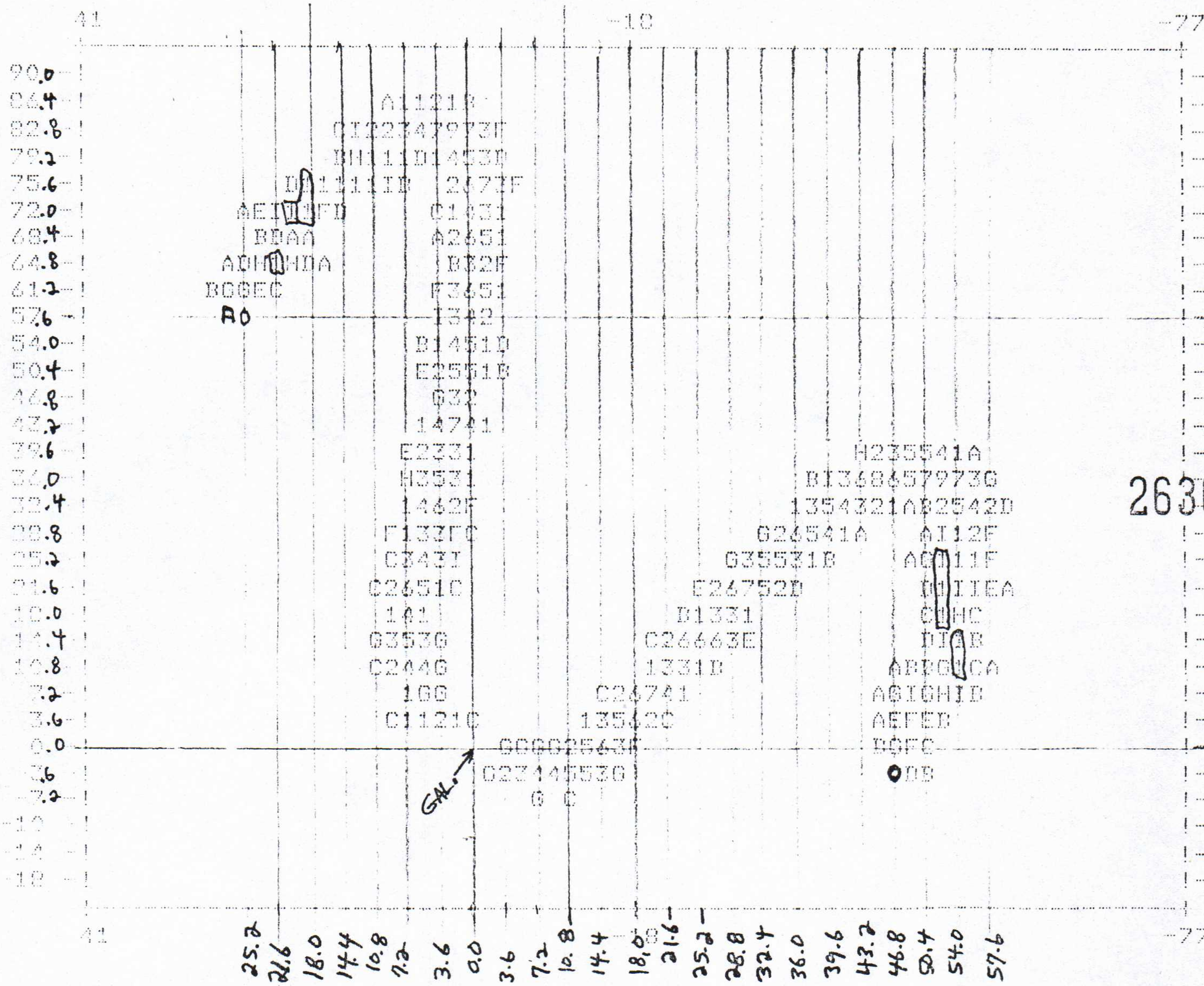
Jacques

JPV/ih

Jacques Vallée

```
                                          -0.72E+00-0.18E+04 0.51E+04 J=15
                                          -0.10E+01-0.30E+04 0.66E+04 J=20
                                          -0.18E+02-0.45E+05 0.16E+05 J=52
SOL= 0.63E-01-0.25E-05-0.25E-05-0.25E-05 0.62E-01 0.16E+05 0.45E+05 J=1
                                           0.62E-01 0.16E+05 0.45E+05 J=2

BEAM CONVOLUTION= 0.40E+01 ARCSEC
     SPACING/PT= 0.18E+01 ARCSEC
     MAX.INTEN.= 0.513E+01 AT ( -1 ARCS, 81 ARCS)
```



SYNTHETIC MAP - NUMERICAL SIMULATION OF THE ORBITAL MODEL OF I700-DOD

```
INPUT LISTING - NUMERICAL PARAMETERS: CELLSZ= 0.20E+04  NCEL1D=400
                                      DPPT= 0.3 IMAX= 100
                CLUSTER PARAMETERS:   XACC=-0.20E+05 YACC=-0.25E+06
                                      ZACC=-0.39E+06 ANGLE=   3.3 CIRV= 0.59E
                BINARY  PARAMETERS:   XO= 0.20E+05 ZO=-0.70E+04 ELLAX= 0.21E+0
                                      COMASS= 0.33E+12 NTURN= APOGALACTICON
                DYNAMICAL PARAMETERS: AZIMA=   90.  POLARA=  130.
                                      VN= 0.78E+04 VS= 0.77E+04 STOPD= 0.90E+0
                                      AU= 0.25E+04
                                      ABOW=  0.8E-20 ADR= 87.0 BOWMAP MODEL=JA
                EVOLUTION PARAMETERS: FLUX DENSITY DECREASES AS EXP (-2S/SMAX)
                OBSER...R PARAMETERS: XOB= 0.20E+09 YOB=-0.20E+08 ZOB= 0.40E+0
                PHOTOMAP PARAMETERS:  SELECF=0.C0  CONVF= 2.2  J0=-10
```

FIGURE A

```
UAL.   ABOVE STAG., K4=   1
```

OUT OF REGELMAN

```
BEAM CONVOLUTION= 0.40E+01 ARCSEC
      SPACING/PT= 0.18E+01 ARCSEC
      MAX.INTEN.= 0.786E+01 AT (   0 ARCS, 81 ARCS)


      41                               -18                               -77
      +---------------------------------+---------------------------------+
  90 -!                                                                   !-
  86 -!                  B1211                                            !-
  82 -!               DI22347963B                                         !-
  79 -!               E111IE2564H                                         !-
  75 -!            DG1111HC B3672E                                        !-
  72 -!         BEI1IGD     F2431                                         !-
  68 -!          ACCC       D2652A                                        !-
  64 -!         BFIGEA       F32E                                         !-
  61 -!          AB        AG3651                                         !-
  57 -!                     1342                                          !-
  54 -!                    A2452C                                         !-
  50 -!                    C2551C                                         !-
  46 -!                     132A                                          !-
  43 -!                    14731                                          !-
  39 -!                   C2341                       H135531             !-
  36 -!                   13531                     I368667973E           !-
  32 -!                  A1462I                  B2355431ED2441A          !-
  28 -!                  D133FB                 G36641C   DI21FA          !-
  25 -!                  14631                C24531B     H111E           !-
  21 -!                 F2661F              H257531       DHIHEA          !-
  18 -!                  121               C2341          D1IC            !-
  14 -!                 I453D            C25753C          CIIB            !-
  10 -!                 D2541           A2442C         ACCEIIC            !-
   7 -!                   1IG         F35742B          AFFFEEB            !-
   3 -!                  BI121G      13653B            AEFDA              !-
   0 -!                     GG111IGI35631                 A               !-
  -3 -!                       B12345653G                                  !-
  -7 -!                           DDB                                     !-
 -10 -!                                                                   !-
 -14 -!                                                                   !-
 -18 -!                                                                   !-
      +---------------------------------+---------------------------------+
      41                               -18                               -77
```

SYNTHETIC MAP - NUMERICAL SIMULATION OF THE ORBITAL MODEL OF I708-DOB

INPUT LISTING - NUMERICAL PARAMETERS: CELLSZ= 0.20E+04 NCEL1D=400
DPPT= ~~0.2~~ 0.17 IMAX= 200
CLUSTER PARAMETERS: XACC=-0.20E+05 YACC=-0.25E+06
ZACC=-0.39E+06 ANGLE= 3.3 CIRV= 0.59E+(
BINARY PARAMETERS: X0= 0.20E+05 Z0=-0.70E+04 ELLAX= 0.21E+0
COMASS= 0.33E+12 NTURN= APOGALACTICON
DYNAMICAL PARAMETERS: AZIMA= 90. POLARA= 130.
VN= 0.78E+04 VS= 0.77E+04 STOPD= 0.90E+0
AW= 0.25E+04
ABOW= 0.8E-20 ABR= 87.0 BOWMAP MODEL=JA
EVOLUTION PARAMETERS: FLUX DENSITY DECREASES AS EXP (-2S/SMAX)
OBSERVER PARAMETERS: XOB= 0.20E+09 YOB=-0.20E+08 ZOB= 0.40E+0
PHOTOMAP PARAMETERS: SELECF=0.08 CONVF= 2.2 J0=-10

FIGURE B

148968

OUT OF BEGELMAN

```
ITH= 16 - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -


BEAM CONVOLUTION= 0.40E+01 ARCSEC
      SPACING/PT= 0.18E+01 ARCSEC
      MAX.INTEN.= 0.172E+02 AT ( -1 ARCS, 81 ARCS)


      41                                -18                                -77
      +----------------------------------+----------------------------------+
  90 -!                                                                    !-
  86 -!                  AH111C                                            !-
  83 -!               AD122369963F                                         !-
  79 -!               AE1111H36851A                                        !-
  75 -!            BE11121IC  2552D                                        !-
  72 -!          AF11HEB     H3662B                                        !-
  68 -!           BDDC       D2551                                         !-
  64 -!            ACB       B2551A                                        !-
  61 -!                      I3641A                                        !-
  57 -!                     B1452G                                         !-
  54 -!                     B1441B                                         !-
  50 -!                     B2441                                          !-
  46 -!                     C2541A                                         !-
  43 -!                     14741                          B               !-
  39 -!                    E3651                       I234542B            !-
  36 -!                    H453I                    D1358878973G           !-
  32 -!                   A1452E                 A13576542G13541D          !-
  28 -!                   C2551                  135531B   C1221A          !-
  25 -!                   I463H                135642G    AG121IA          !-
  21 -!                  D2763E              G35742C      AG11HD           !-
  18 -!                  H253G              13653C         E11C            !-
  14 -!                  H343F            B24652D          C11C            !-
  10 -!                  E232G           136531          CDEHIC            !-
   7 -!                  C1331D        B24642            CDEGGB            !-
   3 -!                   A1221C      136531                               !-
   0 -!                     BDBAE1FBD35762D                                !-
  -3 -!                        B13456642D                                  !-
  -7 -!                          EBCEB                                     !-
 -10 -!                                                                    !-
 -14 -!                                                                    !-
 -18 -!                                                                    !-
      +----------------------------------+----------------------------------+
      41                                -18                                -77


      SYNTHETIC MAP  - NUMERICAL SIMULATION OF THE ORBITAL MODEL OF I708-DOB


 INPUT LISTING - NUMERICAL PARAMETERS: CELLSZ= 0.15E+04  NCEL1D=500
                                       DPPT= 0.2 IMAX= 170
               CLUSTER PARAMETERS:     XACC=-0.20E+05 YACC=-0.25E+06
                                       ZACC=-0.39E+06 ANGLE=   3.3 CIRV= 0.59E+(
               BINARY  PARAMETERS:     X0= 0.20E+05 Z0=-0.70E+04 ELLAX= 0.21E+0
                                       COMASS= 0.33E+12 NTURN= APOGALACTICON
               DYNAMICAL PARAMETERS:   AZIMA=   90.  POLARA=  130.
                                       VN= 0.78E+04 VS= 0.77E+04 STOPD= 0.90E+0
                                       AW= 0.26E+04
                                       ABOW=  0.8E-20 ABR= 87.0 BOWMAP MODEL=JA
               EVOLUTION PARAMETERS:   FLUX DENSITY DECREASES AS EXP (-2S/SMAX)
               OBSERVER PARAMETERS:    XOB= 0.20E+09 YOB=-0.20E+08 ZOB= 0.40E+0
               PHOTOMAP PARAMETERS:    SELECF=0.08  CONVF= 2.2  J0=-10
```

149621

FIGURE C1

DESC. : IC 708 - 42 HOURS ON PDP11/

CONTOUR LEVELS AT: 1.0%, 10.0%, 50.0%, 75.0%

OF PEAK VALUE OF: 17.186

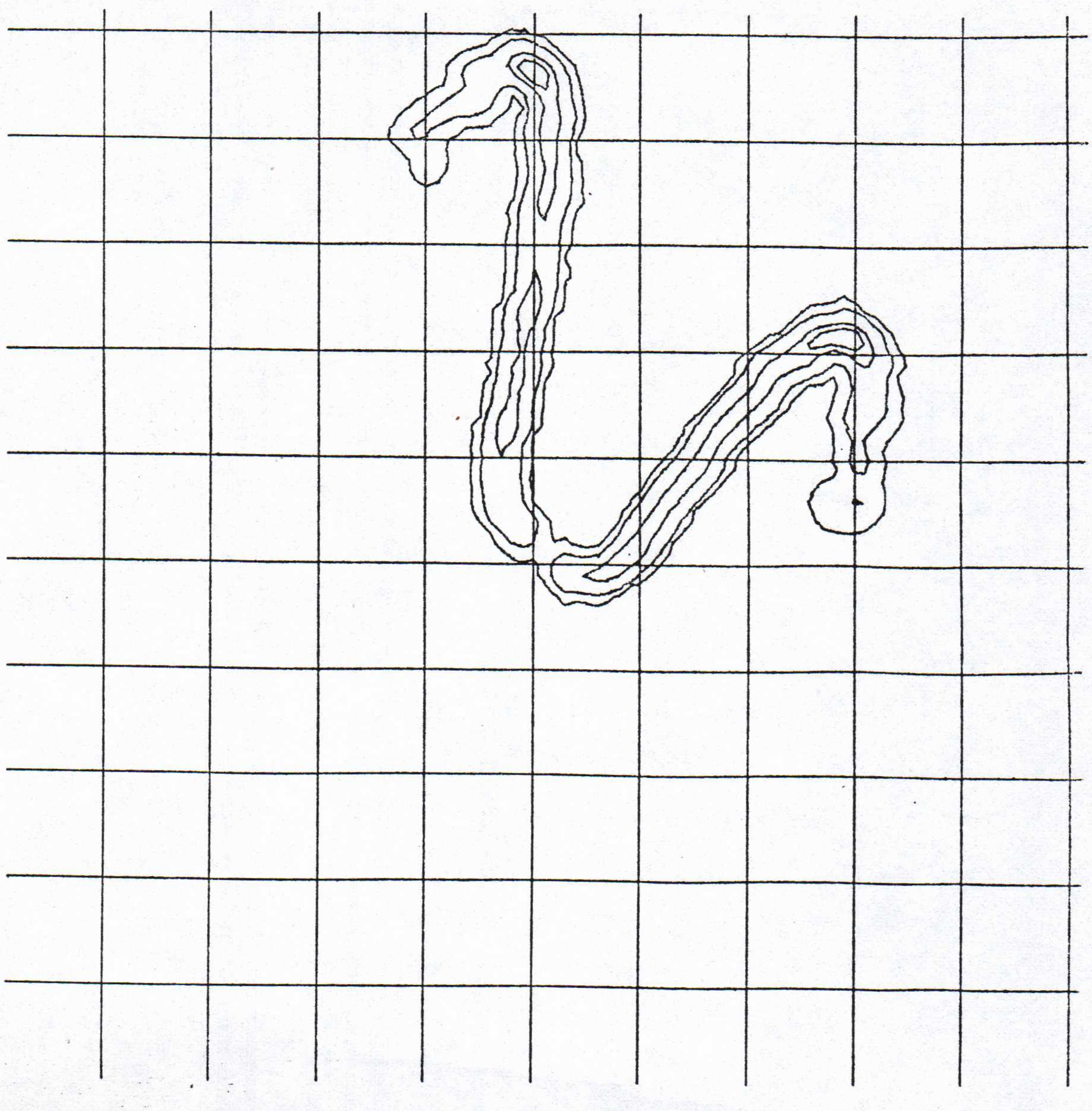


FIGURE C2

Dr. Andrew S. Wilson
Astronomy Program,
University of Maryland
USA

Kingston 20 May 1980

DEPARTMENT OF PHYSICS
STIRLING HALL
Physics
Engineering Physics
Astronomy

Queen's University
Kingston, Canada
K7L 3N6

Dear Andrew,

Enclosed you will find more equations pertaining to our paper on IC708-VLA at λ6cm.

SUMMARY: On modelling Begelman et al (Nature 279 770-773), three inconspicuous errors krept up in. The correct equation is:

$$g = V_j^2\, y''\,(1+y'^2)^{-1.5} = \frac{-P_0}{n_{j0} m_p H_0}\,(1+y'^{-2})^{\left(\frac{1}{2\gamma}-1\right)}$$

The proof (due in part to R.N.Henriksen) is attached (Pages A,B,C,D).

SUMMARY: On modelling Dual orbital motion (IC708 around IC709 in an ellipse, and the system IC708/709 around cluster centre in a circle), may I join the Whole orbiting scheme (Pages E,F,G,H,I) as well as the Full equations for the radio trails (Pages J,K).

The beauty of this Dual orbital motion is to give shorter overall time scales (10**8 years, as opposed to 10**10 years for only one orbital motion).

Cheers,

Jacques

J.P.Vallée

2629 A

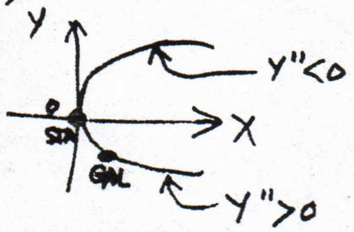
RE: 3 CORRECTIONS TO BEGELMAN EQU. (1) (NATURE, 279, 770-773)

a) SIGN ERROR

b) PROJECTION FACTOR ERROR

c) SCALE HEIGHT ERROR

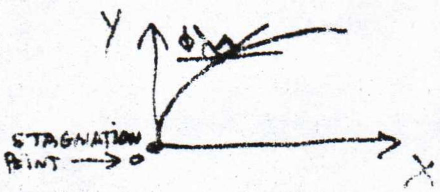
a) TO BEND THE JET TOWARDS X-AXIS (POSITIVE), ONE NEEDS (-1):



$$g = V_0^2 \cdot y'' \cdot (1+y'^2)^{-3/2} = (-1) * \overbrace{\left[\frac{\rho_0 \cos^2\phi \, H \cdot L}{n_0 m_p H_0 \cdot L}\right]}^{\text{ALWAYS POSITIVE}}$$

b) PROJECTION FACTOR $= (\cos\phi)^2$, WHERE ϕ IS THE ANGLE BETWEEN SURFACE NORMAL AT (X,Y) AND SURFACE NORMAL AT $(0,0)$

SO: $\cos^2\phi$ is to be written in terms of the slope y'



$$y' = \tan(90° - \phi) = \cot\phi$$

$$\cos^2\phi = (y')^2 \, (1+y'^2)^{-1}$$

ONE NEEDS THIS TERM (y'^2).

c) RNH HAS SHOWN THAT: $H = (1+y'^2)^{\frac{1}{2\gamma}} (y'^2)^{\frac{-1}{2\gamma}} H_0$

ONE NEEDS THIS TERM: $(y')^{-\frac{1}{\gamma}}$.

FINAL DERIVATION OF BEGELMAN EQUATION (WITH 3 ERRORS CORRECTED)

(B)

$$g = \frac{\text{FORCE}}{\text{MASS}} = (-1)\frac{\text{PRESSURE} \times \left(\begin{smallmatrix}\text{PROJECTED}\\ \text{AREA}\end{smallmatrix}\right)}{\text{MASS INVOLVED}} = (-1)\frac{P_o \cdot \left(\text{PROJECTION FACTOR}\right)\left(\begin{smallmatrix}\text{WHOLE}\\ \text{AREA}\end{smallmatrix}\right)}{\text{DENSITY} \times \text{VOLUME INVOLVED}}$$

(a) (b) (c) SCALE HEIGHT IS: H

$$= (-1)\frac{P_o \times (\text{Cos}\,\phi)^2 \times (H \times \text{LENGTH ALONG JET})}{(n_{JET})_0 \cdot m_{PROTON} \cdot (H_0^2 \cdot \text{LENGTH ALONG JET})}$$

[illegible]HERE: $V_{JET} \equiv$ CONSTANT,
LENGTH ALONG JET $\equiv$ CONSTANT
MASS INVOLVED $\equiv$ CONSTANT

⟵ TWO FACES OF UNIT VOLUME STAY AT SAME DISTANCE FROM EACH OTHER; SO:
WIDTH = SCALE HEIGHT = H,
AREA = H × H,
VOLUME = H × H × LENGTH ALONG JET.

NOW:

(a) (b) (c)

$$g = \frac{(-1)\cdot P_o \cdot \left[(y'^2)(1+y'^2)^{-1}\right] \cdot \left[(1+y'^2)^{\frac{5}{8}} (y')^{\frac{1}{8}} H_o\right]}{(n_{JET})_0 \cdot m_{PROTON} \cdot H_0^2}$$

SINCE WE HAVE:

$$g \equiv \frac{(\text{JET VELOCITY})^2}{\text{RADIUS OF CURVATURE}} = (V_j)^2 \left(\frac{d^2y}{dx^2}\right)\left(1 + \left[\frac{dy}{dx}\right]^2\right)^{-3/2}$$

THEN:

$$\boxed{g = V_j^2\, y''(1+y'^2)^{-3/2} = \frac{-P_o}{n_{j0} m_p H_o}\left(1 + y'^{-2}\right)^{\left(\frac{1}{2\gamma}-1\right)}}$$

OR:

$$y'' = \frac{-P_o}{V_j^2 \cdot n_{j0} m_p H_o}\left(1+y'^2\right)^{0.5+\frac{0.5}{\gamma}} \left(y'^2\right)^{1-\frac{0.5}{\gamma}}$$

C

HENCE:

$$y''' = \frac{P_0}{V_j^2 n_0 m_p H_0}\cdot\frac{d}{dx}\left[(y')^{2-\frac{1}{\gamma}}\,(1+y'^2)^{\frac{1}{2}+\frac{1}{2\gamma}}\right] = \frac{P_0}{V_j^2 n_0 m_p H_0}\,\frac{d}{dx}\left[(y')^{1.4}\,(1+y'^2)^{0.8}\right]$$

5/3

$$= \frac{P_0}{V_j^2 n_0 m_p H_0}\cdot\left[(y')^{1.4}\,0.8\,(1+y'^2)^{-0.2}\,2y'y'' + (1+y'^2)^{0.8}\,1.4\,(y')^{0.4}\,y''\right]$$

$$\boxed{\frac{P_0}{n_0 m_p H_0} = \frac{A\cdot V_G^2}{V_j^2}} \Rightarrow \boxed{y'' = \overset{\text{always}\downarrow}{-}\frac{A\cdot V_G^2}{V_j^2}\,(1+y'^2)^{0.8}\,(y')^{1.4}}$$

$$\therefore y''' = -\left(\frac{A\cdot V_G^2}{V_j^2}\right)^2\left[1.6\,(y')^{\overset{+1.4}{2.4}}\,(1+y'^2)^{-0.2+0.8} + 1.4\,(1+y'^2)^{\overset{+0.8}{0.8}}\,(y')^{\overset{+1.4}{0.4}}\right]$$

$$= \overset{\text{ALWAYS}\downarrow}{-}\left(\frac{A\cdot V_G^2}{V_j^2}\right)^2\left[1.6\,(1+y'^2)^{0.6}\,(y')^{3.8} + 1.4\,(1+y'^2)^{1.6}\,(y')^{1.8}\right]$$

In Rotated Coord. System, $y_R = -x$, $x_R = +y$, $\frac{dy}{dx} = -1/y_R'$, $y'' = \frac{-y_R''}{(y_R')^3}$

So.:

$$\frac{-y_R''}{(y_R')^3} = -\left(\frac{A\cdot V_G^2}{V_j^2}\right)\left(1+\left(\frac{-1}{y_R'}\right)^2\right)^{0.8}\left(\frac{-1}{y_R'}\right)\left(\frac{+1}{y_R'}\right)^{0.4}$$

$$\frac{-y_R''}{(y_R')^3} = \frac{+\left(\frac{A\cdot V_G^2}{V_j^2}\right)\left(y_R'^2+1\right)^{0.8}}{(y_R')^{1.6}\,(y_R')^{1.4}}$$

$$\boxed{y_R'' = \overset{\text{ALWAYS}\downarrow}{-}\left(\frac{A\cdot V_G}{V_j^2}\right)\left(1+y_R'^2\right)^{0.8}}$$

← "ROTATED EQUATION" FOR NUMERICAL INTEGRATION ACROSS STAGNATION POINT

$$y_R''' = -\left(\frac{A\cdot V_G^2}{V_j^2}\right)0.8\,\left(1+y_R'^2\right)^{-0.2}\,2\,y_R'\cdot y_R''$$

$$= \underset{\uparrow\text{ALWAYS}}{+}\left(\frac{A\cdot V_G^2}{V_j^2}\right)^2 1.6\,\left(1+y_R'^2\right)^{0.6}\,y_R'$$

DATE: 17 APRIL 1980

CIRCULAR MOTION OF COMB AROUND COMC

(E)

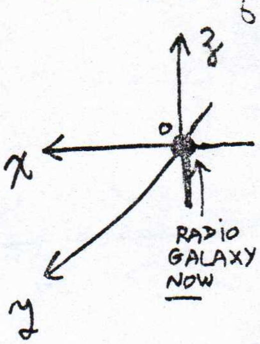
DEFINITION:

COMB $\equiv$ Center-Of-Mass of Binary Galaxies.

COMC $\equiv$ Center-of-Mass of Cluster Centre.

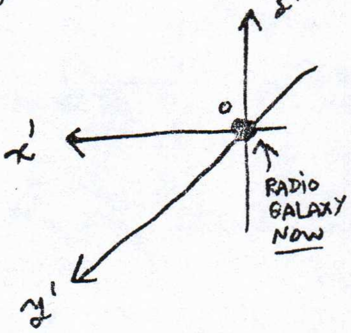
RECALL:

The two binary galaxies are in ELLIPTICAL motions around COMB.
The ORIGIN of coordinates is at the present location of the RADIO ga



} OBSERVER'S SYSTEM OF COORDINATES (SAME AS CLUSTER CENTRE'S SYSTEM OF COORDINATES)

COMB MOTION IS A CIRCLE AROUND A FIXED POINT



} BINARY GALAXIES' SYSTEM OF COORDINATES (SAME AS COMB'S SYSTEM OF COORDINATES)

ELLIPTICAL MOTIONS ARE ALL IN (x', z') PLANE.

PROBLEM:

Find past locations (x_G, y_G, z_G) of radio galaxy.

$$x_G = x_{COMB} + x'_G - (x_0)$$

$$y_G = y_{COMB} + y'_G - (0)$$

$$z_G = z_{COMB} + z'_G - (z_0 - \mathrm{ELLBZ.ECC})$$

} KNOWN ELLIPTICAL ORBITS $\Rightarrow$ KNOWN $x'_G, y'_G, ?$

$$x_{COMB} = x_0 + f(t)$$

$$y_{COMB} = 0 + g(t)$$

$$z_{COMB} = z_0 - \mathrm{ELLBZ.ECC} + h(t)$$

} FIND: $f(t), g(t), h$
WITH: $f(0) = g(0) = h($

SOLUTION (BROKEN INTO SEVERAL PARTS):

F

PART 1) FIND DIR. COS. OF PLANE (IN WHICH COMB MOVES):

$\boxed{\text{INPUT}}$ ⟶ (x_A, y_A, z_A) ≡ LOCATION OF CLUSTER CENTRE

$\boxed{\text{OUTPUT}}$ ⟶ $(\cos\alpha, \cos\beta, \cos\gamma)$ = Direction Cosines of Plane in Which COMB moves

We have EQUATION of PLANE (in Which COMB moves) given by:

$$x\cos\alpha + y\cos\beta + z\cos\gamma = p$$

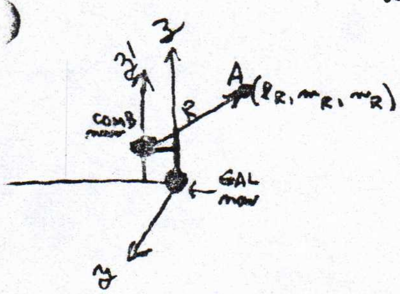
with: $$p = x_A\cos\alpha + y_A\cos\beta + z_A\cos\gamma$$

The radius R of the circular motion of COMB is:

$$R = \sqrt{(x_0 - x_A)^2 + (0 - y_A)^2 + (z_0 - \text{ELLBZ.ECC} - z_A)^2}$$

Now, $\vec{R}_{NOW}$ is:

$$l_R = \frac{x_A - x_0}{R}; \quad m_R = \frac{y_A}{R}; \quad n_R = \frac{z_A - (z_0 - \text{ELLBZ.ECC})}{R}$$



The "plane ⊥ to $\vec{R}_{NOW}$" has direction Cosines: l_R, m_R, n_R

Also, We know that the "plane of $(\hat{z}', \vec{R}_{NOW})$" is given by three points in it:

$$(x_0, 0, z_0 - \text{ELLBZ.ECC})_1, \ (x_A, y_A, z_A)_2, \ (x_0, 0, z_0 - \text{ELLBZ.ECC} + 1)_3$$

$$\left[(x - x_0)\begin{vmatrix} y_A & z_A - z_0 + \text{ELLBZ.ECC} \\ 0 & 1 \end{vmatrix} + (y - 0)\begin{vmatrix} z_A - z_0 + \text{ELLBZ.ECC} & x_A - x_0 \\ 1 & 0 \end{vmatrix} + (z - z_0 + \text{ELLBZ.ECC})\begin{vmatrix} x_A - x_0 & y_A \\ 0 & 0 \end{vmatrix} = 0 \right]$$

$$y_A x - y_A x_0 - x_A y + x_0 y = 0$$

whose direction Cosines are for the "plane of $(\hat{z}, \vec{R}_{NOW})$":
$$l_z = y_A / \sqrt{y_A^2 + (x_0 - x_A)^2}$$
$$m_z = (x_0 - x_A) / \sqrt{y_A^2 + (x_0 - x_A)^2}$$
$$n_z = 0$$

These two planes meet into a junction, ie: a line given by the direction cosines (l_P, m_P, n_P):

$$l_P \cdot l_R + m_P \cdot m_R + n_P \cdot n_R = 0$$
$$l_P \cdot l_Z + m_P \cdot m_Z + n_P \cdot n_Z = 0$$
$$l_P^2 + m_P^2 + n_P^2 = 1$$

BY CONSTRUCTION, WE TAKE $n_P > 0 \Rightarrow$ angle between this line and $\hat{z}$ is: $< 90°$.

Thus:

$$l_P = \frac{n_P (n_Z m_R - n_R m_Z)}{l_R m_Z - l_Z m_R} = \frac{n_P(-n_R m_Z)}{l_R m_Z - l_Z m_R}$$

$$m_P = \frac{n_P (n_Z l_R - n_R l_Z)}{m_R l_Z - m_Z l_R} = \frac{n_P(-n_R l_Z)}{m_R l_Z - m_Z l_R}$$

$$n_P = + \frac{|m_R l_Z - m_Z l_R|}{DNP}$$

where: $DNP = + SQRT\left[(l_R m_Z - l_Z m_R)^2 + (\underbrace{n_Z}_{\to 0} m_R - n_R m_Z)^2 + (\underbrace{n_Z}_{\to 0} l_R - n_R l_Z)^2 \right]$

$\boxed{\text{INPUT}} \longrightarrow$ Angle ε (between 0 and 180 degrees)

ANY PLANE WITH $\vec{R}$ in it can now be constructed, whose normal is located in the "plane $\perp$ to $\vec{R}_{NOW}$" and is at an angle ε to (l_P, m_P, n_P); let this plane be the one COMB moves in, with dir. Cos.: $\cos\alpha, \cos\beta, \cos\gamma$.

So one has: $\boxed{n_P \cdot \cos\varepsilon = \cos\gamma}$

Furthermore, $l_R \cos\alpha + m_R \cos\beta + n_R \cos\gamma = 0$

and: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

So: $\boxed{\cos\alpha = \frac{-1}{l_R}\left[n_R \cos\gamma + m_R \cos\beta \right]}$

Hence:

$$\boxed{\cos\beta = \frac{- n_R m_R \cos\gamma \pm l_R \sqrt{-\cos^2\gamma + m_R^2 + l_R^2}}{(m_R^2 + l_R^2)}}$$

SIGN: Watch for $(-\varepsilon)$, giving same $\cos\gamma$ but different $(\cos\alpha, \cos\beta)$.
CONSTRUCT ε rotation (0° to 180°) on same side of plane $(l_Z, m_Z, 0)$, $\Rightarrow l_Z \cos\alpha + m_Z \cos\beta > 0$

PART 2) FIND NEW COORD. SYSTEM, WITH 2 AXES IN COMB – MOTION PLANE.

One of these 2 axes shall be: l_R, m_R, n_R

The other axis should be $\perp$ to the first one, and $\perp$ to: $\cos\alpha, \cos\beta, \cos\gamma$.

So, calling it: l_d, m_d, n_d, one has:

$$
\begin{aligned}
&l_d \cdot l_R + m_d \cdot m_R + n_d \cdot n_R = 0 \\
&l_d \cdot \cos\alpha + m_d \cdot \cos\beta + n_d \cdot \cos\gamma = 0 \\
&n_d^2 = 1 - l_d^2 - m_d^2
\end{aligned}
$$

Solving,

$$l_d = \frac{n_d \left(n_R \cos\beta - m_R \cos\gamma \right)}{m_R \cos\alpha - l_R \cos\beta}$$

$$m_d = \frac{n_d \left(n_R \cos\alpha - l_R \cos\gamma \right)}{l_R \cos\beta - m_R \cos\alpha}$$

$$n_d = \frac{\pm \left[m_R \cos\alpha - l_R \cos\beta \right]}{DEM}$$

where: $DEM = SQRT\left[(m_R \cos\alpha - l_R \cos\beta)^2 + (m_R \cos\gamma - n_R \cos\beta)^2 + (l_R \cos\gamma - n_R \cos\alpha)^2 \right]$

The SIGN of n_d is chosen to give a RIGHT-ANGLE system (x', y', z'), where
$\hat{x}' = -l_R, -m_R, -n_R$; $\hat{y}' = -l_d, -m_d, -n_d$; $\hat{z}' = \cos\alpha, \cos\beta, \cos\gamma$.

Since $\hat{x}' \times \hat{y}' \equiv \hat{z}'$, one has: $\pm(l_R m_d - m_R l_d) = \cos\gamma$. Just Compute: $SIV \equiv \left(\frac{\cos\gamma}{l_R m_d - m_R l_d} \right)$
an Set: $n_d = (SIV) \cdot n_d$.

So one has:

$$
\begin{aligned}
x_{FUT} &= x_{COMB} \pm l_d \\
y_{FUT} &= y_{COMB} \pm m_d \\
z_{FUT} &= z_{COMB} \pm n_d
\end{aligned}
$$

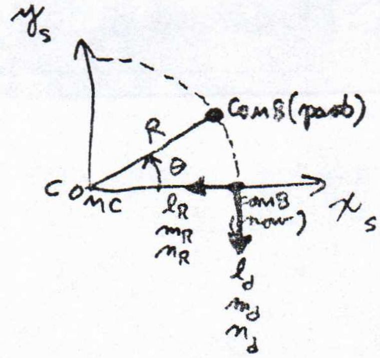
so one can use the sign of the circular speed around COMC:

if: $v_c < 0$, take $-|\theta|$

if: $v_c > 0$, take $+|\theta|$

$\boxed{\text{INPUT}} \longrightarrow$ Velocity (+ or −) of COMB around COMC in plane: V_{CIR}

PART 3) FIND CIRCULAR MOTION IN COMB PLANE, AND IN 3D SPACE



$$y_s = +R\cdot\sin\theta$$

$$x_s = +R\cdot\cos\theta$$

$$x_{COMB} = -l_R\cdot R\cos\theta - l_d\cdot R\sin\theta + x_A \equiv x_0 + f(t)$$
$$y_{COMB} = -m_R\cdot R\cos\theta - m_d\cdot R\sin\theta + y_A \equiv g(t)$$
$$z_{COMB} = -n_R\cdot R\cos\theta - n_d\cdot R\sin\theta + z_A \equiv z_0 - \text{ELLBZ.ECC} + h(t)$$

i) START AT $\theta \equiv 0^\circ$

Using Δt from program, Compute: $\Delta\theta = \overset{+\text{ or }-}{(V_{CIR})}\cdot\Delta t/R$;⇒ NEW θ.

ii) NOTE: $R\cdot V_{CIR}^2 = G$ (Mass inside Orbit of COMB)

$\boxed{\text{SUMMARY}}$ OF PARAMETERS TO COMPUTE: R, (l_R, m_R, n_R), (l_z, m_z, n_z), (l_p, m_p, n_p), $(\cos\alpha, \cos\beta, \cos\gamma)$, (l_d, m_d, n_d), θ, $(x_{COMB}, y_{COMB}, z_{COMB})$

While the INPUTS WERE: (x_A, y_A, z_A), angle ε, V_{CIR}.

The Relations for computing purposes are:

$$R = \sqrt{(x_0 - x_A)^2 + y_A^2 + (z_0 - \text{ELLBZ.ECC} - z_A)^2}$$

$$\begin{cases} l_R = (x_A - x_0)/R \\ m_R = y_A/R \\ n_R = [z_A - (z_0 - \text{ELLBZ.ECC})]/R \end{cases}$$

$$\begin{cases} l_z = y_A/\sqrt{y_A^2 + (x_0 - x_A)^2} \\ m_z = (x_0 - x_A)/\sqrt{y_A^2 + (x_0 - x_A)^2} \\ n_z = 0 \end{cases}$$

$$\begin{cases} n_p = +|m_R\cdot l_z - m_z\cdot l_R| \Big/ \sqrt{(l_R m_z - l_z m_R)^2 + (-n_R m_z)^2 + (-n_R l_z)^2} \\ m_p = n_p\,(-n_R\cdot l_z)/(m_R\cdot l_z - m_z\cdot l_R) \\ l_p = n_p\,(+n_R\cdot m_z)/(m_R\cdot l_z - m_z\cdot l_R) \end{cases}$$

$$\begin{cases} \cos\gamma = n_p\cdot\cos\varepsilon \\ \cos\beta = \left[-n_R m_R\cos\gamma \pm l_R\sqrt{l_R^2 + m_R^2 - \cos^2\gamma}\right] / \left[l_R^2 + m_R^2\right] \\ \cos\alpha = [-n_R\cos\gamma - m_R\cos\beta]/l_R \end{cases}$$

← SIGN FROM $l_z\cos\alpha + m_z\cos\beta > 0$

$$\begin{cases} l_d = n_d\cdot[n_R\cos\beta - m_R\cos\gamma]/[m_R\cos\alpha - l_R\cos\beta] \\ m_d = n_d\cdot[-n_R\cos\alpha + l_R\cos\gamma]/[m_R\cos\alpha - l_R\cos\beta] \\ n_d = (\text{SiV})\cdot(m_R\cos\alpha - l_R\cos\beta) \Big/ \sqrt{(m_R\cos\alpha - l_R\cos\beta)^2 + (n_R\cos\gamma - m_R\cos\beta)^2 + (l_R\cos\gamma - n_R\cos\alpha)^2} \end{cases}$$

MODIFICATIONS FOR COMB MOTION AROUND COMC TO RAM PRESSURE MODELS OF DOUBLE RADIO SOURCES

PART A) MODELS OF $\boxed{\text{JAFFE-PEROLA}}$, $\boxed{\text{COWIE-McKEE}}$, $\boxed{\text{ICKE-BLANDFORD}}$, WITH d EQUATION:

ICKE-BLANDFORD: $d = V_0 \cdot t$

COWIE-McKEE: $d = D \cdot \ln\left[1 + \frac{V_0 \cdot t}{D}\right]$

JAFFE-PEROLA: $d = D\left[1 - \mathrm{EXP}(-V_0 t/D)\right]$

Galaxy dir. Cos.:

$$l_G = \left(x_G(t-\Delta t) - x_G(t)\right)/DEP$$
$$m_G = \left(y_G(t-\Delta t) - y_G(t)\right)/DEP$$
$$n_G = \left(z_G(t-\Delta t) - z_G(t)\right)/DEP$$

where:

$$DEP = \mathrm{SQRT}\left[\left(x_G(t-\Delta t) - x_G(t)\right)^2 + \left(y_G(t-\Delta t) - y_G(t)\right)^2 + \left(z_G(t-\Delta t) - z_G(t)\right)^2\right]$$

$\vec{V}_0$ is obtained here via a vectorial SUM:

Strength of V_G is: $|V_G| = DEP/\Delta t$

$$\vec{V}_G = |V_G|\, l_g\, \hat{i} + |V_G|\, m_G\, \hat{j} + |V_G|\, n_G\, \hat{k}$$
$$\vec{V}_E = |V_E|\, l_E\, \hat{i} + |V_E|\, m_E\, \hat{j} + |V_E|\, n_E\, \hat{k}$$
$$\vec{V}_0 = \left(|V_G|\, l_g + |V_E|\, l_E\right)\hat{i} + \left(|V_G|\, m_G + |V_E|\, m_E\right)\hat{j} + \left(|V_G|\, n_G + |V_E|\, n_E\right)\hat{k}$$

Strength of V_0 is: $|V_0| = \mathrm{SQRT}\left[\left(|V_G| l_G + |V_E| l_E\right)^2 + \left(|V_G| m_G + |V_E| m_E\right)^2 + \left(|V_G| n_G + |V_E| n_E\right)^2\right]$

LOCATION OF BLOB AFTER TIME t HAS ELAPSED:

In these models, the blob went a distance d (above) in time t, ALONG vector $\vec{V}_0$, starting from (x_G, y_G, z_G).

So:
$$\vec{d} = |d| \cdot \frac{\left(|V_G|\, l_G + |V_E|\, l_E\right)}{|V_0|}\,\hat{i} + |d| \cdot \frac{\left(|V_G|\, m_G + |V_E|\, m_E\right)}{|V_0|}\,\hat{j} + |d| \cdot \frac{\left(|V_G|\, n_G + |V_E|\, n_E\right)}{|V_0|}\,\hat{k}$$

The END POINT of this vector is at:

$$x_{BLOB} = x_G + \frac{|d|}{|V_0|} \cdot \left(|V_G|\, l_G + |V_E|\, l_E\right)$$

$$y_{BLOB} = y_G + \frac{|d|}{|V_0|} \cdot \left(|V_G|\, m_G + |V_E|\, m_E\right)$$

$$z_{BLOB} = z_G + \frac{|d|}{|V_0|} \cdot \left(|V_G|\, n_G + |V_E|\, n_E\right)$$

(K)

PART $\mathbb{I}$

MODELS OF $\boxed{\text{BEGELMAN et al}}$, with/without mixing with JAFFE-PEROLA or COWIE-MCKEE

→ WITH DIFFERENTIAL EQUATION TO OBTAIN DISTANCE OF BLOB: d.

Gal. dir. Cos.: $l_G = \dfrac{x_G(t-\Delta t) - x_G(t)}{DEP}$

$m_G = \dfrac{y_G(t-\Delta t) - y_G(t)}{DEP}$

$n_G = \dfrac{z_G(t-\Delta t) - z_G(t)}{DEP}$

} CONTAINS TWO ORBITAL MOTIONS (ELLIP. + CIRC.).

where:

$$DEP = SQRT\left[\left(x_G(t-\Delta t) - x_G(t)\right)^2 + \left(y_G(t-\Delta t) - y_G(t)\right)^2 + \left(z_G(t-\Delta t) - z_G(t)\right)^2 \right]$$

The ram pressure equation is:

$$\frac{d^2 Y_Q}{dx_Q^2} = -(A\sigma) \cdot \left(1 + \left(\frac{dY_Q}{dx_Q}\right)^2\right)^{0.5 + \frac{0.5}{\gamma}} \left[\left(\frac{dY_Q}{dx_Q}\right)^2\right]^{1 - \frac{0.5}{\gamma}}$$

where: $\gamma = 5/3$

$A\sigma = \text{CONSTANT} * V_{GAL}^2 / V_{JET}^2$

$|V_{GAL}| = DEP/\Delta t$

Location of BLOB after time t has elapsed:

→ WITHOUT mixing (ie: Begelman model ONLY):

a) just find (Y_Q, X_Q) from integrating twice the equation above;

b) transfer to 3D coord. System using dir. Cos.

of Y_Q: (VNN.FLJ − A·GL/VGAL)/AA, (VNN.FMJ − A·GM/VGAL)/AA, (VNN.FNJ − A·GN/VGAL)

of X_Q: −GL, −GM, −GN

where $AA = SQRT[(\ )^2 + (\ )^2 + (\ )^2]$

i.e.: $x_{BLOB} = x_Q \cdot (-GL) + y_Q \cdot (\ )/AA + x_{GAL}$

$y_{BLOB} = x_Q \cdot (-GM) + y_Q \cdot (\ )/AA + y_{GAL}$

$z_{BLOB} = x_Q \cdot (-GN) + y_Q \cdot (\ )/AA + z_{GAL}$

N.B.: $d = SQRT[(x_{BLOB} - x_{GAL})^2 + (y_{BLOB} - y_{GAL})^2 + (z_{BLOB} - z_{GAL})^2]$

→ WITH mixing (ie: Begelman model ends before end of Trail, new model starts with d(t))

a) find (Y_Q, X_Q) from integrating twice the equation above (to model ends);

b) from last 2 points $(X_Q, Y_Q; X_{Q-1}, Y_{Q-1})$, find dir. Cos. of bent jet of Velocity $|V_E|$ = CONSTANT, ie: DCJX, DCJY, DCJZ

c) find $\vec{V_0} = \vec{V_E}(\text{bent}) + \vec{V_{GAL}}$, ie:

$[|V_E| \cdot DCJX + |V_G| \cdot GL]$, $[|V_E| \cdot DCJY + |V_G| \cdot GM]$, $[|V_E| \cdot DCJZ + |V_G| \cdot GN]$

d) Add $\left[\dfrac{\vec{V_0}}{|V_0|} \cdot d(t)\right]$ to end point: $x_{BLOB} = x_{ENDPOINT} + [|V_E| \cdot DCJX + |V_G| \cdot GL] \cdot \dfrac{d(t)}{|\vec{V_0}|}$, etc.

ASTRONOMY PROGRAM

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4 March 1980

Dear Jacques and Alan,

Thanks for the latest version of IC 708. I am basically pretty happy with Sections 1 and 2 (p.3 -- middle of 6) and am enclosing my textual modifications of these pages. I'm afraid I still regard the model of Section 4 as illustrative rather than definitive because:

a) At the jet velocity found (only 12 km s^{-1}), the gravitational force of IC 708 (and perhaps IC 709) on the jet itself must be very important.
b) With such a low speed, the jet must be subsonic and the Jaffe and Perola form $d = D(1 - \exp(- v_o t/D))$ cannot be used. For subsonic velocities the Cowie and McKee form $d = D \ln (1 + v_o t/D)$ is appropriate. There are, however, plenty of problems with this since the size of the blobs is about constant (they are contained by thermal pressure), so they would have to begin in the nucleus being a few kpc across!

Although point (b) is easy to get around, I fear point (a) is more difficult. With a relative velocity of the galaxy w.r.t. the surrounding gas of only a few tens of kilometers per second, the jet velocity must be comparably low for enough bending. All this means, presumably, is that gravitational forces may dominate the motion. Such low speeds make me think of Gull and Northovers bubble model of extragalactic sources. I see no way out but to include gravity in a numerical calculation.

In general, I feel the discussion is too much oriented pro Jaffe and Perola and too anti the more recent jet models. I think Section 5b is not really relevant to the main thread of the paper and raises other questions. For example, in part (iii) one has to arrange for the electrons to spend long enough in the enhanced field regions to give rise to the observed radio emission but not too long or they will suffer energy losses. There is a literature on this topic -- see Hughes Mon Nots 186, 853 (1979) and Burn (referenced by Hughes).

The most important worry, however, is the dynamical model itself, perhaps we could talk on the phone about the best way forward.

With best wishes,

A. S. Wilson

ASW/dmz

Comments on Nov '79 edition of "Orbital Motion of "Papillon" radio galaxy IC708"

Title. Drop "Papillon"? - it doesn't look much like one now.

P. 9. Delete top 2 sentences

P. 12. I do not believe the remark claiming Jaffe and Perola model is better than Begelman et al. Treat them equally!

P. 13. Let us not refer to $d \sin \delta$ as y_T. It is not in the y direction.

P. 15. Concerning the brightness distribution along the trails. Wouldn't the best thing be to take the intensity of a blob as $I(t)$ where t is the time since ejection? The use of S seems artificial.

P. 16. Section (v). I don't see why the period should, a priori, be below the age of the universe.

d. Section vii). In what sense is IC709 "ahead" of IC708.

Table 3 and p. 18.

a. I think a diagram of the 3d geometry of the "best" model would make it clearer.

b. We should discuss which of the parameters in ~~Section~~ Table 3 are well determined and which are uncertain. I am sceptical of the claim of uniqueness.

P.19 I don't understand why "if the system IC708/IC709 were to be at its projected distance to the cluster center, the environment of intracluster gas would likely be at rest w.r.t. the cluster center".

I have doubts about whether p 19 & 20 should be included at all.

P.21 It would be interesting to state what is the total tail length (along the orbit) in connection with the discussion of the "in

~300 kpc?

situ" acceleration.

P.23 I don't think a "2 phase" medium helps much in the energy loss problem, because the rate of energy loss in the 2 phases cannot be very different because of the inverse Compton losses in the μwave background. i.e. one can't "store" relativistic electrons in the second phase.

P21-23 seem to reach no conclusion.

P.25 The sentence at the bottom seems different to the earlier claim that Jaffe and Perola is better than Begelman et al.

Basically I think Begelmans jets are much more plausible than J & P plasmoids and think the model should ~~investigate their shapes too~~ be applied with their jet shape too.

P.26 It is unlikely that higher sensitivity observations will extend the length of the trail since our WSRT λ6 & λ21cm observations show a similar length to those

/

Astronomy Program,
2/18/80

Dear Jacques and Alan,

O.K.! Now agree with solution for x, y, z coordinates of blob:

$$x_b - x_T = d \sin\gamma \cdot (n_g^2 l_e - l_g n_e n_g)/A^{1/2}$$
$$y_b = d \sin\gamma \cdot m_e / A^{1/2}$$
$$z_b - z_T = d \sin\gamma \cdot (l_g^2 n_e - n_g l_e l_g)/A^{1/2}$$

with
$$x_T = x_g + l_g d \cos\gamma$$
$$z_T = z_g + n_g d \cos\gamma$$
and
$$A = (n_g l_e - l_g n_e)^2 + m_e^2$$

I must say I prefer the above way of presenting them than in your draft Jacques because
a) its shorter
and b) its meaning is more transparent. i.e. each blob coordinate is written as the sum of the coordinate for the "tangent point" plus some fraction of the length of the line ($d \sin\gamma$) from the "tangent point" to the blob. This fraction is just the appropriate direction cosine of the line from the "tangent point" to the blob.

I promise a quick turnaround on the next draft, Cheers Andrew OVER

P.S.

Christine Jones has informed me that the IPC data on Abell 1314 should be processed in a week or two. Our HRI observation of IC708 is, of course, contingent upon adequate detection in the IPC map.

To: Dr. Andrew S. Wilson

From: Jacques

Queen's University Memorandum

Date: 13 Nov. '79

Subject: Your phone call yesterday at 5p.m, about the IC708/IC709 paper (VLA, 6cm data).

Enclosed you will find the proofs concerning the system of equations (1) and (2), written as "Appendix A" here.

Also, you will find enclosed the proofs concerning the system of equations (6) and (7), written as "Appendix B" here.

Please accept, dear Andrew, the expression of my best sentiments.

APPENDIX A

IN WHICH WE DERIVE EQUATIONS (1) AND (2) IN THE MAIN PAPER.

<u>Center-of-Mass System</u>

$$\vec{R}_A m_A + \vec{R}_B m_B = 0 \qquad \text{(by definition)}$$

a) <u>Orbital Positions</u>:

$$\vec{F}_a = m_a \ddot{\vec{R}}_a = -\frac{G m_a m_b}{R^3}\vec{R} \qquad \text{(Newton's)}$$

with: $\vec{R} \equiv \vec{R}_a - \vec{R}_b$

Now: $\vec{R} = \left(1 + \frac{m_a}{m_b}\right)\vec{R}_a$

So: $$\ddot{\vec{R}}_a = -G m_b \frac{\vec{R}}{R^3} = -\frac{G(m_a + m_b)\vec{R}_a}{\left(1+\frac{m_a}{m_b}\right)^3 R_a^3}$$

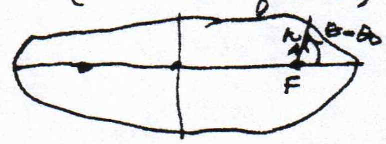
$$\boxed{\text{Define: } M_{eq} \equiv \frac{m_a + m_b}{\left(1 + \frac{m_a}{m_b}\right)^3} \qquad \text{Then: } \ddot{\vec{R}}_a = -G M_{eq} \cdot \frac{\vec{R}_a}{R_a^3}}$$

whose solution is:

HARWIT, Ap. CONCEPTS (1973), PAGES 75-76

$$\begin{cases} R_a = \dfrac{1}{B\cos(\theta-\theta_0) + \dfrac{GM_{eq}}{h^2}} \\[2ex] R_a^2 \dfrac{d\theta}{dt} = h \end{cases}$$

Rewrite: $$R_a = \frac{A(1-e^2)}{1+e\cos(\theta-\theta_0)} \qquad \text{(Conical Equation)}$$



with: $B = \frac{GM_{eq}\, e}{h^2} = \frac{e}{A(1-e^2)}$

and: $h^2 = GM_{eq} A(1-e^2)$

b) Orbital Velocities:

Similarly, we have: $\dot{\vec{R}} = \left(1 + \frac{m_a}{m_b}\right) \dot{\vec{R}}_a$

and, when $R = A$ (semimajor axis):

$$A = \left(1 + \frac{m_a}{m_b}\right) a_a$$

Now, when dealing with the potential energy, the two-body problem is often solved in a one-body system of reduced mass around a mass $(M_a + M_b)$ fixed in space, leading to the vis-viva equation:

$$V^2 = G\,(m_a + m_b) \left(\frac{2}{R} - \frac{1}{A} \right)$$ ← HARWIT, Ap. CONCEPTS (1973), P.78

and to the orbital period:

$$P = \frac{2\pi\, A^{3/2}}{\sqrt{G\,(m_a + m_b)}}$$ ← HARWIT, Ap. CONCEPTS (1973), P.78

But, those last two equations can be rewritten, since:

$$V^2 = \left(\dot{\vec{R}}\right)^2 = \left(1 + \frac{m_a}{m_b}\right)^2 \left(\dot{\vec{R}}_a\right)^2 = \left(1 + \frac{m_a}{m_b}\right)^2 v_a^2$$

or:

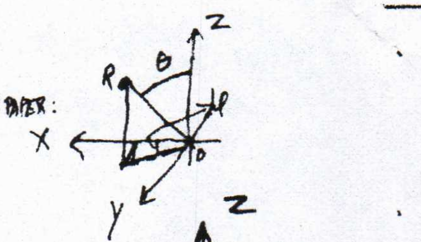
$$\boxed{v_a^2 = \frac{G\,(m_a + m_b)}{\left(1 + \frac{m_a}{m_b}\right)^3} \left[\frac{2}{R_a} - \frac{1}{A_a} \right] = G M_{eq} \left[\frac{2}{R_a} - \frac{1}{A_a} \right]}$$

and:

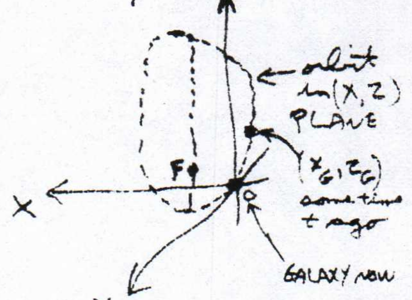
$$\boxed{P = \frac{2\pi \left(1 + \frac{m_a}{m_b}\right)^{3/2} A_a^{3/2}}{\sqrt{G\,(m_a + m_b)}} = \frac{2\pi\, A_a^{3/2}}{\sqrt{G M_{eq}}}}$$

APPENDIX B

IN WHICH WE DERIVE EQUATIONS (6) AND (7) IN MAIN PAPER:



Galaxy location sometime t ago: $(x_G(t), 0, z_G(t))$



Jet direction Cosines: $l_E = \sin\theta\cos\varphi$

$m_E = \sin\theta\sin\varphi$

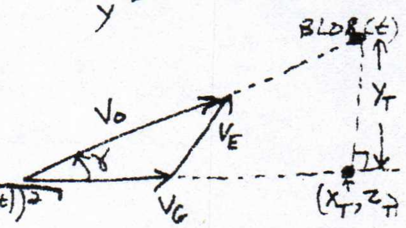
$n_E = \cos\theta$

Galaxy direction Cosines: $l_G = \left[x_G(t-\Delta t) - x_G(t)\right]/\Delta$

$m_G = 0$

$n_G = \left[z_G(t-\Delta t) - z_G(t)\right]/\Delta$

$\Delta \equiv \sqrt{(x_G(t-\Delta t) - x_G(t))^2 + (z_G(t-\Delta t) - z_G(t))^2}$



From last diagram, the BLOB's separation from tangent along $\vec{r}_G(t)$ is:

$Y_T \equiv d\sin\gamma$

where d is given in equation (5) of main paper, and the

BLOB's projected distance along the tangent is:

$d\cos\gamma$

which can be decomposed along the x-axis as:

$x_T - x_G = d\cos\gamma \cdot l_G$

and along the z-axis as:

$z_T - z_G = d\cos\gamma \cdot n_G$

To solve for the 3D location of a blob, we can set up 3 equations with $x_{BLOB}, y_{BLOB}, z_{BLOB}$ as the 3 unknown, as follows:

i) Equation for plane containing the vectors: $\vec{V_G}, \vec{V_E}, \vec{V_o}$

ii) Equation for plane perpendicular to: $\vec{V_G}$

iii) Equation for Y_T in plane (ii).

Solution for ~~x~~-location: (i)+(ii) $\Rightarrow m_E x = H6 + (H2)\, m_G \left[(Y1)\, m_E / \sqrt{(H2)^2 + m_E^2} \right]$

$$\therefore m_E x = m_G \left[x_G m_G - z_G l_G \right] m_E - l_G m_E [H5] + m_E [H2]\, m_G (Y1) / \sqrt{(H2)^2 + m_E^2}$$

$$\therefore \boxed{x_{Loc} = m_G^2 x_G - m_G z_G l_G - l_G (H5) + (H2)\, m_G (Y1) / \sqrt{(H2)^2 + m_E^2}}$$

← ok

Solution for z-location of radio blob:

from(ii) $\Rightarrow l_G x + m_G z + (H5) = 0$

$$\therefore m_G z = -H5 - l_G \left[m_G^2 x_G - m_G z_G l_G - l_G (H5) + (H2)\, m_G (Y1) / \sqrt{(H2)^2 + m_E^2} \right]$$

$$= (H5)\left[-m_G^2 \right] - l_G m_G \left[m_G x_G - z_G l_G + (H2)(Y1) / \sqrt{(H2)^2 + m_E^2} \right]$$

$$\therefore \boxed{z_{Loc} = -m_G (H5) - l_G \left[m_G x_G - z_G l_G + (H2)(Y1) / \sqrt{(H2)^2 + (m_E)^2} \right]}$$

← ok