THE ASTROPHYSICAL JOURNAL, 257:63–74, 1982 June 1

SYNCHROTRON BRIGHTNESS DISTRIBUTION OF TURBULENT RADIO JETS

R. N. HENRIKSEN[1]
Institute for Plasma Research, Stanford University

A. H. BRIDLE[1]
National Radio Astronomy Observatory[2] VLA Program; and Department of Physics and Astronomy, University of New Mexico

AND

K. L. CHAN
Applied Research and Systems Landover, Maryland
Received 1981 June 30; accepted 1981 December 14

ABSTRACT

In this paper we introduce the notion of radio jets as turbulent mixing regions. We further propose that the essential small-scale viscous dissipation in these jets is by Lighthill emission of MHD waves and by their subsequent strong damping due, at least partly, to gyroresonant acceleration of suprathermal particles. The equilibrium eddy, wave, and particle spectra are not found exactly in this paper, but the problem is defined and rough estimates of the spectra are given to aid in the observational interpretation.

A formula relating the synchrotron surface brightness of a radio jet to the turbulent power input is deduced on very general and simple grounds from our physical postulates, and it is tested against the data for NGC 315 and 3C 31 (NGC 383). The remarkable predictive power of this formula is the major justification for presenting our model at this time. The predicted brightness depends essentially on the collimation behavior of the jet, and, to a lesser extent, on the Chan and Henriksen picture of a "high" nozzle with accelerating flow. The conditions for forming a large-scale jet at a high nozzle from a much smaller scale jet are discussed. In particular, the optimum condition for retaining the memory of the initial jet direction is given as that of a turbulent jet.

The effect of entrainment on our prediction is discussed with the use of similarity solutions. Although entrainment is inevitably associated with the turbulent jet, it may or may not be a dominant factor depending on the ambient density profile.

Subject headings: galaxies: structure — hydromagnetics — radiation mechanisms — radio sources: galaxies — turbulence

I. INTRODUCTION

It is becoming clear that relativistic particle reacceleration occurs at distances from 1 to 40 kpc from the active nuclei of some radio galaxies, in the rapidly expanding bases of radio jets linking these nuclei to their distant radio lobes. The evidence for this particle reacceleration is of three kinds. First, the observed surface brightnesses of radio jets decrease with jet width much less rapidly than expected if they were adiabatically expanding fluxes of particles and fields without particle replenishment or field amplification. This effect has been found in most of the well-studied radio jets (3C 31: Burch 1979; Fomalont *et al.* 1980; NGC 315: Willis *et al.* 1981; HB 13: Masson 1979; 3C 296 and 3C 449: Birkinshaw, Laing, and Peacock 1981). Second, a polarized optical continuum coincides with the bright radio knots in the jets of M87 (Owen, Hardee, and Bignell 1980) and 3C 277.3 (Bridle *et al.* 1981). If this optical continuum is indeed synchrotron radiation, the lifetimes of the radiating electrons in the equipartition fields in these knots are much less than the light-travel times to the knots from the galactic nuclei. This is clear evidence for relativistic particle replenishment within the knots. Third, the longer radio jets show little or no gradient in radio spectral index along their lengths (e.g., Burch 1979; Willis *et al.* 1981, hereafter WSBF; Willis, Perley, and Bridle 1982). The evidence for large-scale particle reacceleration from such radio spectral data is more dependent on assumptions about particle transport velocities in the jets than the first two lines of evidence but is nevertheless suggestive.

In this paper we report encouraging fits to the observed synchrotron brightness distributions of the well-observed radio jets in 3C 31 and NGC 315, based on a simple theory derived from the scaling laws applicable to "fully developed" volume turbulence. This turbulence is anisotropic because the mean velocity of a jet imposes a preferred direction on the flow, but we suppose that it may be treated as homogeneous and isotropic in the comoving frame of the jet.

[1] On leave from Queen's University at Kingston, Ontario, Canada.
[2] Operated by Associated Universities, Inc., under contract with the National Science Foundation.

The volume turbulence invoked in our model is generated in the free mixing layer that develops following the emergence of a jet with a large Reynolds number from a high "nozzle" (e.g., Chan and Henriksen 1980, hereafter CH; Bridle, Chan, and Henriksen 1981, hereafter BCH) or when the jet encounters a density "plateau" in its path (see § V). The essential features of this turbulence for our purposes are that it exhibits large-scale structure (for a review, see, e.g., Brown and Roshko 1974; Brown and Thomas 1977; Cantwell 1981) and soon fills the entire volume of the jet, eliminating the laminar (potential) core altogether. The growth of this large-scale structure may be understood at each stage in terms of ring vortex pairing, mutual orbiting, and merging, followed by these processes repeating with the just-merged eddies on a larger scale. (This is really a description of the nonlinear growth of a Kelvin-Helmholtz instability.) During this development, ambient material is entrained and intense smaller scale turbulence is generated in the regions between the vortices (Cantwell), presumably establishing the turbulent cascade to higher wavenumbers, which is eventually dominated by viscosity on the Kolmogorov microscale, l_K. The small-scale of the stretched turbulence-generating regions between the vortices can be associated with the Taylor microscale, l_T (Cantwell).

We envisage that a primarily hydrodynamic turbulent cascade exists in a radio jet because (*a*) the Debye length is small ($\gtrsim 2 \times 10^6$ cm) and yet there are many particles in a Debye sphere ($\gtrsim 4 \times 10^{16}$), (*b*) we expect the magnetic field to be dynamically weak (CH; BCH) except in the smaller eddies (CH; De Young 1980), and (*c*) the small gyroradii of the particles (~ 0.1 AU for fields $\sim 10^{-5}$ gauss) should impart a fluid character to otherwise collisionless systems. The central idea which we use throughout for coupling the eddy cascade to the MHD wave spectrum is the Lighthill MHD radiation from the turbulent eddies (for a summary, see, e.g., Kulsrud 1955; Kato 1968; Stein 1981). We further suppose that the viscous subrange of the eddy cascade is established by this wave emission and subsequent damping, principally by particle acceleration. We defer to a later paper the detailed calculation of self-consistent eddy, wave, and particle spectra that is required to justify our view and proceed here instead by plausibility arguments and by direct appeal to the observations of radio jets (§ III).

The radiated MHD waves may accelerate electrons directly by a resonant interaction (e.g., Lacombe 1977) or by developing into collisionless shock waves with scattering wakes which can mediate the first-order Fermi acceleration process (e.g., Bell 1978*a*, *b*; Blandford and Ostriker 1978; Bicknell and Melrose 1982). Our estimates suggest that nonlinear waves and shock waves can occur at $l \gtrsim l_T$, while true dissipation of the turbulent energy occurs in a range of smaller scales which are in resonance with typical high-energy electrons present in the jets.

This nonlinear scheme for particle acceleration avoids the difficulty of propagating surface-generated resonant waves away from the limb of a jet, which was encountered in the pioneering work of Pacholczyk and Scott (1976; see Eilek 1981). In our view, the strong damping of the resonant MHD waves is not a problem but rather a statement of the strong coupling between particle energy and turbulent energy spectra to be expected locally. Section III gives observational evidence for this view. Our model should also be distinguished from that of a turbulent wave cascade driven at the top end by Kelvin-Helmholtz instabilities (see Benford, Ferrari, and Trussoni 1980 for a recent view).

Section II of this paper derives (from simple scaling arguments) our basic expression for the variation of the brightness B of a jet with jet radius R. Section III tests this relationship against observations of NGC 315 and 3C 31. Sections IV and V discuss entrainment and the relation between parsec-scale and kiloparsec-scale radio jets in the light of our general principles. Section VI contains our conclusions and indicates what remains to be done. Much of § II*a* is used to define carefully our proposed Lighthill mechanism for maintaining a turbulent spectrum of MHD waves. We retain this section as an announcement of what we feel is an important idea to be developed subsequently, but the reader interested primarily in our predicted fits may bypass much of § II*a*.

II. RADIO BRIGHTNESS PROFILE OF TURBULENT JETS

a) *Principles*

We assume for definiteness that the spectrum of the turbulent inertial range is Kolmogorov (1968) in the frame comoving with the jet. It should be clear how to proceed with a slight variation on the theme, however, such as a Kraichnan (MHD) spectrum (e.g., De Young 1980 and references therein). There is thus local homogeneity in the comoving frame as well as a kind of transverse incompressibility (CH). The range is taken to extend from $l \sim R(Z)$ (Brown and Roshko 1974), the "radius" of the jet at a distance Z from a "nozzle" (or from a point where strong turbulent interaction with the surroundings begins), down to the effective damping or Kolmogorov scale l_K. We suppose further that the "internal" or true dissipative viscosity in the jet is provided largely by the resonant interaction between MHD waves and the radiating electrons, for a sufficiently low density of thermal particles. This should be true even for the shock-wave Fermi mechanism because the energy gained by the particles must derive ultimately from the turbulence through the mechanism of the pre-shock and postshock scatterers, of which the resonant waves are the most efficient.

We note that we expect the turbulent "viscosity" (i.e., the Reynolds stress) to provide the effective "external" or entrainment viscosity, but this is not, of course, to be used to calculate the Reynolds number.

The Alfvén wavelength in resonance with electrons may be written simply as $(2\pi\mu)r_{\mathrm{eg}}$ when the Alfvén speed is much less than that of light and the wave direction is close (i.e., $k_\perp \ll k_\parallel/\mu$) to that of the mean magnetic field (e.g., Eilek 1979). Here r_{eg} is the mean electron gyroradius and μ is the cosine of the mean electron pitch angle. Hence, the eddy scale l_K which will couple to the electrons by the emission of these waves is

$$l_K = f(2\pi\mu)r_{eg} = f\mu 2\pi\gamma_e m_e c^2/(e\bar{B}) , \quad (1)$$

where the factor $f(<1)$ allows for the difference between the scale of an eddy and its characteristic Lighthill wavelength (see below), $\bar{B}$ is the spatially averaged magnetic field strength, and the other symbols have their usual meanings. (Note that the effective damping scale is not uniquely defined in the absence of a self-consistent description of eddy, wave, and particle spectra. See also below.) Numerically, equation (1) gives $l_K \sim 10^{14}[\gamma_e(5)/\bar{B}(-5)]f\mu$ cm, where the numbers in brackets give the logarithmic unit of the standard measure in cgs units. The necessity for this latter scale to be considerably greater than the ion gyroradius ($\sim 10^9 T_8^{1/2}$ cm) but substantially less than the Taylor microscale suggests that the mechanism discussed here can accelerate electrons in the range $10^3 < \gamma_e < 10^7$. We therefore use $\gamma_e = 10^5$ for our estimates. Eilek (1981) has shown that Alfvén waves are absorbed on scales $(10^3\text{–}10^4)\lambda$ (the coefficient increases with λ) for conditions appropriate to the radio jets we seek to model, so that $\gamma_e \lesssim 10^7$ is also set by requiring that the waves are absorbed in less than a jet width ($\sim 10^{21}$ cm).

The Reynolds number, Re, of the jet flow is fixed by assumption (1) to be (e.g., Landau and Lifshitz 1959):

$$\text{Re} = (R/l_K)^{4/3} \sim 2 \times 10^9[\bar{B}(-5)R(21)/\gamma_e(5)]^{4/3}(f\mu)^{-4/3} . \quad (2)$$

Such a Reynolds number should be compared with those of $\sim 10^8$ obtaining in various ship-wake problems (Cantwell 1981); our postulated range in scales (the jet radius R to l_K) is evidently not far removed from experience.

One of the *a priori* arguments in favor of the observed radio jets being turbulent is the similarity of the observed "cone angles" near the bases of various jets. For example, in each of NGC 315 (WSBF), 3C 31 (Bridle *et al.* 1980), and NGC 6251 (Willis *et al.* 1982), dR/dZ is initially ~ 0.1 to 0.2. This behavior suggests qualitatively the "Reynolds number invariance" observed for laboratory turbulent shear layers (Cantwell 1981) and calculated in the turbulent jet models by Baan (1980). These slopes of 0.1 to 0.2 are even quantitatively similar to those observed in laboratory cases (e.g., Townsend 1976, p. 181 and Table 6.5). Moreover, the development of the large-scale nonlinear structure by vortex pairing leads to orbit velocities comparable to the eddy speeds, Δv. Because this process is ultimately associated with the jet entrainment or spreading rate, v_ϖ [we use cylindrical coordinates (ϖ, Z) as in CH] it is reasonable to set

$$\Delta v = v_\varpi \equiv v_z \, dR/dZ , \quad (3)$$

where v_z is the mean axial velocity and v_ϖ is defined to be the mean spreading rate at the boundary. In fact $R/\Delta v = (R/v_z)dZ/dR$ gives a good estimate of the period of the bursting phenomenon observed in boundary layer flow with dZ/dR between 6 and 10, and such bursting phenomena are associated with the growth of the large-scale structure in the layer (Cantwell, p. 483). Moreover, apparent *oscillations of the radio jet axis might be expected on this basis with wavelength* $v_z R/\Delta v = RdZ/dR$. There are preliminary indications (WSBF; Willis *et al.* 1982) that such oscillations exist in NGC 315 and NGC 6251 on about the scales expected, if the jets in these sources do not lie too close to the line of sight.

In a compressible jet (with the magnetic field unimportant in the large eddies), one expects the additional restriction $\Delta v \lesssim c_s$, the internal sound speed, because otherwise dissipative shocks can arise. Indeed if a jet had constant dR/dZ over a large range of Z, this would probably indicate an essentially free jet with a constant internal sound speed, most plausibly due to shock dissipation of supersonic turbulence (e.g., Bicknell and Melrose 1982). But if the jet were confined by the surrounding medium (e.g., CH; BCH), then it would be reasonable to suppose that the turbulence would be inhibited relative to that of a free shear layer. Moreover, it is known (e.g., chap. 3 by J. P. Johnston in Bradshaw 1978) that the stability of laboratory internal flows is increased by walls with convex curvature toward the flow, and a denser region which flows towards the jet while compressing it might imitate this behavior. Equation (3) allows for this effect because the new energy $\propto(\Delta v)^2$ available to the turbulent cascade is thereby seen to go to zero with dR/dZ. However, we must recall that the existing turbulence will decay only in about a local convective time (taken where $dR/dZ \approx 0$), so that we may expect some lag between the actual level of turbulence and that predicted by equation (3). The convective transport both of turbulence and particles should be considered for a strict computation of the local emissivity of the jet. Unless the particle lifetime is less than a convective time at $dR/dZ \approx 0$, particle convection will normally be the dominant effect, and we allow for this qualitatively in our applications (§ III) in this paper.

We recall that the Kolmogorov velocity spectrum is (i refers to the scale at the upper end of the inertial range)

$$\Delta v_l \sim \Delta v_i(l/l_i)^{1/3} , \quad (4)$$

and that the effective internal viscosity of the jet is determined when the Reynolds number is fixed as

$$\nu_{\text{eff}} \sim \Delta v(R/\text{Re}) . \quad (5)$$

Moreover, the Taylor microscale for homogeneous turbulence is given by (e.g., Bradshaw 1978)

$$l_T \sim R(15/\text{Re})^{1/2} , \quad (6a)$$

from which follows

$$l_T \sim (15)^{1/2} l_K \, \text{Re}^{1/4} , \quad (6b)$$

on recalling equation (2). This last equation also allows us to write $l_T/l_K \sim 10^3\{\bar{B}(-5)R(21)/[\gamma_e(5)f\mu]\}^{1/3}$, which ratio is not very sensitive to our various uncertainties.

We adopt the hydrodynamic view (e.g., Falco 1977; see also Cantwell 1981) that the *Kolmogorov equilibrium cascade exists between* l_T *and* l_K as our first approximation to the complex interactions expected in the regime, so that we may set $l_i = l_T$ and $\Delta v_i = \Delta v$ in equation (4). It is this range of eddy scales that we expect to interact with the particle spectrum by way of Lighthill radiation. Depending principally on the effective value of $f\mu$ (see below), this range is always $\lesssim 10^4$ in scale and thus also in particle energy.

The Lighthill formulation of radiating eddies as developed for the MHD case by Kulsrud (1955), Parker (1964), and especially Kato (1968) is summarized by Stein (1981). In the presence of a magnetic field, the Alfvén waves (A) and the slow magnetosonic waves (S) are emitted predominantly in a monopole mode. The power emitted in this mode from an eddy of scale l is

$$P_{A,S} = (\eta_{A,S} \cdot l/\lambda)\rho \cdot (\Delta v_l)^3/l \,, \tag{7}$$

where λ is the wavelength of the radiated wave, and we have written the emission efficiency as $\eta \cdot l/\lambda$. The fast magnetosonic waves (F) are emitted either in a dipole mode (confined jet) or in a quadrupole mode (free jet) with the power ($n = 1$, dipole; $n = 2$, quadrupole):

$$P_F \sim \eta_F \cdot (l/\lambda)^{(2n+1)}\rho \cdot (\Delta v_l)^3/l \,, \quad n = 1, 2 \,. \tag{8}$$

In these expressions we should take $\overline{(\Delta v_l)^3/l} = (\Delta v)^3/R$, as this last expression is the source of the available energy. The time average is over times long compared to the Taylor "turnover time." It would be incorrect to use $(\Delta v)^3/l_T$ because this power can only be maintained sporadically in "bursts" of duration $(l_T/R)(R/\Delta v) = l_T/\Delta v$.

The principal factor in the emission efficiency is determined from

$$l/\lambda \sim \Delta v_l/\bar{c} \equiv f \,, \tag{9}$$

where $\bar{c}$ is the large scale Alfvén speed, $\bar{c}_A$, for A or F waves, and the mean sound speed, $\bar{c}_s$, for S waves. The factor f, introduced in equation (1), is identified here as the eddy scale in units of the wavelength of its principal radiated wave.

Equations (4) and (9) allow us to write that, in fact,

$$\Delta v_l/\bar{c} \sim (\Delta v/\bar{c})^{3/2}(\lambda/l_T)^{1/2} \equiv f \,. \tag{10}$$

Moreover we can *estimate* $\Delta v/\bar{c}_A$ from the results of CH, Bridle *et al.* (1980), and BCH, which show that the quantity $\epsilon_B \sim (\bar{c}_A/V_{Z\infty})^2$ must be $\lesssim 10^{-2}$ in order to prevent large-amplitude pinching oscillations (largely unobserved) in the regime where $\bar{B}$ is predominantly circumferential. Hence, using equation (3) and $\bar{c}_A/v_Z \lesssim 10^{-1}$, we obtain $\Delta v/\bar{c}_A \gtrsim 10(dR/dZ) = 0(1)$. Thus $l/\lambda \sim 1$ on the Taylor microscale, and it declines like $l^{1/3}$ or $\lambda^{1/2}$ (see eqs. [4] and [10] and recall eq. [9]) toward the smaller scales. We have already argued that $\Delta v/\bar{c}_s \lesssim 1$, so that we may conclude that the monopole emission of A and S waves will generally dominate the F wave emission ($l < l_T$). We see also that the emission efficiency $\eta_{A,S}\, l/\lambda$ decreases as $\eta_{A,S}(l/l_T)^{1/3} \approx \eta_{A,S}[f(l)]^{1/3}$, by equation (4). From this expression for $f(l)$ and the expression for l_K/l_T, we may estimate $f_K \sim 10^{-9/8}\{\mu\gamma_e(5)/[B(-5)R(21)]\}^{1/8} \sim 10^{-9/8}$. Consequently, on inserting this value in equations (1), (2), and (6b), we find $l_K \sim 10^{13}[\mu\gamma_e(5)/\bar{B}(-5)]$ cm, $\mathrm{Re} \sim 10^{10.8}\{\bar{B}(-5)R(21)/[\mu\gamma_e(5)]\}^{4/3}$, and $l_T/l_K \sim 10^{3.4}\{\bar{B}(-5)R(21)/[\mu\gamma_e(5)]\}^{3/8}$. It should be remembered that $\bar{B}R \sim$ const. in these formulae (e.g., CH; BCH), so that the numerical values are reasonably well known [(l_K, l_T) will scale with R].

The decline of the radiation efficiency of a turbulent eddy with $f(l)$ is offset somewhat, at least for the Alfvén waves (Kato 1968), by the tendency for η_A to rise with the approach of the magnetic field to equipartition (from 3.05 for a weak field to 15.5 for an equipartition magnetic field), which might be expected on the smaller turbulent scales (De Young 1980). Thus, we might expect strong Lighthill emission of MHD waves from eddies over the whole inertial range from l_T to l_K. We know moreover from the work of Lacombe (1977) and Eilek (1979, 1981) that such waves can be damped locally (10^3–$10^4\lambda$) by acceleration of suprathermal electrons. (We suppose sufficiently low thermal densities that heat losses do not dominate, as this gives reasonable estimates for the sources studied here. In general the "thermal efficiency" must be calculated from the various damping rates.)

We clearly are confronted at this point with a difficult and interesting calculation. We wish to know the self-consistent particle, MHD wave, and indeed (allowing for the back reaction) turbulent eddy energy spectra that are in equilibrium over the inertial range. We defer the proper solution of this problem to a subsequent investigation, but a useful estimate of the particle spectrum can be made by neglecting the back reaction on the eddies, as follows.

From equations (7) (time averaged), (9), and (10), we may compute for a unit volume the flux density of the "Lighthill-radiated" MHD waves, $w(k)$, as ($k \equiv 2\pi/\lambda$):

$$w(k) \equiv \left|\frac{dP(k)}{dk}\right| = \rho \cdot \frac{(\Delta v)^3}{R} \cdot \frac{\eta}{2}\left(\frac{\Delta v}{\bar{c}}\right)^{3/2} k_T^{1/2} k^{-3/2} \,, \tag{11}$$

where $k_T \equiv 2\pi/l_T$. Using the expression for the resonant wavelength in terms of the particle energy, namely $k = 1/(\mu r_{eg}) = (e\bar{B}/\mu)E^{-1}$, we also have

$$\frac{dP}{dE} = \frac{\eta}{2} \cdot \left(\frac{\mu}{e\bar{B}}\right)^{1/2} E^{-1/2}\left[\rho \cdot \frac{(\Delta v)^3}{R} \cdot \left(\frac{\Delta v}{\bar{c}}\right)^{3/2} k_T^{1/2}\right] . \tag{12}$$

We equate this damping power per unit particle energy to the synchrotron power dP_s/dE radiated by the system per unit particle energy in accord with the assumption that this radiation is the principle dissipative mechanism. We have then

$$\left|\frac{dP_s}{dE}\right| = \frac{2}{3}\left(\frac{dN}{dE}\right)\frac{(cr_e^2\bar{B}_\perp^2)}{m^2c^4} E^2 \,, \tag{13}$$

where (dN/dE) is the number of relativistic electrons per unit energy per unit volume with energy E, so that on equating equation (12) and equation (13), the particle spectrum is found to be of the form:

$$\frac{dN}{dE} \propto E^{-2.5} \tag{14}$$

for constant local conditions. This corresponds to a synchrotron emissivity $\epsilon_\nu \propto \nu^{-0.75}$, which is only a little steeper than the $\nu^{-0.6}$ spectrum observed for typical radio jets (Bridle 1981; Willis 1981). When the numerical coefficient in equation (14) is computed (by equating eqs. [12] and [13]) and the minimum particle energy is taken to be $\gamma_e \sim 10^3$, we deduce that the density of relativistic electrons $n_{er} \sim 10^{-6} n_e$ near the bases of the jets. We emphasize however that, although these estimates are plausible, the assumption of balanced input and output over the whole energy spectrum is arbitrary and none of the above can be considered proven until a careful calculation of the various spectra (including heat losses) has been completed.

Our estimated $\nu^{-0.75}$ spectrum is approached at higher frequencies in the bases of the jets in 3C 66B, 3C 31, and NGC 315, however, for which Butcher, van Breugel, and Miley (1980) determined "radio to blue" spectral indices of 0.70, 0.72, and >0.74 respectively. This is to be expected because particles radiating at optical frequencies will have shorter synchrotron lifetimes and are therefore more likely to be in equilibrium with the turbulent input. In fact, we might expect the spectral break to be at the particle energy for which the synchrotron lifetime $\tau_s(\gamma_e)$ is about equal to the turbulent "turnover time" or "convective time" $\tau_c \sim z/v_z \sim R/\Delta v$. This is because a family of electrons near this energy can radiate energy at just the rate at which it is both convected and delivered to the viscous subrange. From the relations $\tau_s \sim 10^{13.7} \gamma_e^{-1}(5) B^{-2}(5)$ and $\tau_c \sim 10^{13.5}[z(\text{kpc})/v_z(8)]$, we see that $\gamma_e(5) \sim 1$ (mm or far-infrared) would be so selected near the base of a jet. However, unless $v_z \propto 1/R$, this energy would increase as R after the magnetic field transition ($B_\perp^{-2} \propto R^2$ and $Z \propto R$).

We also observe that if collisionless shocks are to appear in our scheme then we would expect them to do so near the Taylor scale l_T because there the Lighthill waves are nonlinear. These might in turn lead to a fraction of the turbulent energy being diverted to shock-wave Fermi acceleration from direct resonant wave acceleration (e.g., Bicknell and Melrose 1982; Blandford and Ostriker 1978; Bell 1978*a*, *b*), but we are unable to estimate what effect this might have on the MHD wave or particle spectrum at present, and so we shall not consider it further here.

In summary, we propose that the turbulent energy available from the large-scale structure in the jet "mixing layer" is dissipated through the acceleration of relativistic particles or scales from l_K to l_T (much larger than the proton gyroradius and Debye length). The detailed interaction of turbulence, MHD waves, and thermal and nonthermal particles in this regime is left undiscussed, but a particle spectrum $\propto E^{-2.5}$ with a flattening below $E/m_e c^2 \sim 10^5 R(21)$ seems possible. We proceed now to deduce a singular observational consequence of this scheme and then to test it.

b) The Scaling Law for the Radio Surface Brightness

Given the suggestion that the synchrotron emission is the basic dissipation mechanism for the turbulence in the radio jets, we can write immediately that

$$\epsilon_\nu/\rho \sim e(\nu)(\Delta v)^3/R\ , \tag{15}$$

where the "jet radiation efficiency" $e(\nu)$ contains the prescription for the spectral distribution of the energy. In general there will be an additional factor for thermal efficiency, say $(1 - e_T)$, where $e_T(k)$ is the fraction of wave flux density $w(k)$ lost to heat and may be quite close to unity in some cases. However, we pursue our argument here with $e_T = 0$ for clarity. It is reassuring physically that an expression of the form of equation (15) may be calculated directly from equations (12), (13), and (14) [with the various coefficients collected and using the standard formula for the synchrotron emissivity of a power law: $e(\nu) \propto \nu^{-0.75}$], but, in view of the various uncertainties in that procedure, we prefer to base this section on simple scaling relations. We have also taken the radiation efficiency to be uniform in the jet. In fact one finds $e(\nu) \propto B^{-(0.25)}$ from equations (12) and (13) and the standard power-law formula (Pacholczyk 1970; eq. [3.50]) if $(k_T, B) \propto R^{-1}$, so that this assumption may not be too bad. Our basic idea does require (and the estimates of § II*a* support) us to take

$$\int_0^{\nu_{\max}} e(\nu) d\nu = \frac{1}{4\pi}\ , \tag{16}$$

neglecting losses to the thermal particles, and, if $e(\nu) \propto \nu^{-\alpha}$, then

$$e \equiv \int_0^{\nu} e(\nu) d\nu = \frac{1}{4\pi}\left(\frac{\nu}{\nu_{\max}}\right)^{(1-\alpha)}\ . \tag{17}$$

As the observed spectra of most radio jets are $S_\nu \propto \nu^{-0.6 \pm 0.2}$ (Bridle 1981; Willis 1981), we take the jet to be optically thin, so that the surface brightness $B(\nu) \sim \epsilon(\nu) \cdot s$, where s is the thickness of the jet along the line of sight [$(R^2 - \varpi^2)^{1/2}$ at a perpendicular distance ϖ from the centre of a circular jet]. This gives a geometric limb darkening for the jet such that, in the absence of intrinsic emissivity variations, the half-power points are on the circle $\varpi = [(3^{1/2}/2)]R$. Such limb darkening is consistent with the observed transverse intensity profiles in 3C 31 and NGC 315 (Fomalont *et al.* 1980). We now combine equations (3) and (15) with an equation for the mass flux, A, in the form (we consider a variable mass flux due to entrainment in § IV)

$$R^2 \rho v_z = A\ , \tag{18}$$

to write the surface brightness (ergs cm^{-2} s^{-1} Hz^{-1} sr^{-1}) as

$$B(\nu) \approx [1 - (\varpi/R)^2]^{1/2} e(\nu) A (v_z/R)^2 (dR/dZ)^3\ , \tag{19a}$$

that is

$$B(\nu) \approx [1 - (\varpi/R)^2]^{1/2} e(\nu) A v_\varpi^3/(R^2 v_z)\ . \tag{19b}$$

In practice, $B(\nu)$ is the quantity determined by observations of a well-resolved jet at a single frequency ν. However, although the *form* of the single frequency brightness variation should be given adequately by equation (19), the amplitude is more problematical because of the uncertainty in $e(\nu)$. We feel that the integrated result given by equation (16) is more reliable in this regard, so we will use the integrated radio brightness for amplitude determinations, namely

$$B \approx [1 - (\varpi/R)^2]^{1/2} eA(v_z/R)^2(dR/dZ)^3 , \quad (20)$$

where

$$e \approx 1/4\pi(\nu_{co}/\nu_{max})^{(1-\alpha)}$$

by equation (17). With the radio band "cut-off" at $\nu_{co} \sim 10^{10}$ Hz and $\nu_{max} \sim 4 \times 10^{11}(\gamma_e \lesssim 10^5)$, $4\pi e \approx 0.4$ for $\alpha = 0.75$, and ~ 0.16 for $\alpha = 0.5$. Even if the synchrotron spectrum extended into the X-ray band ($\gamma_e \gtrsim 10^8$), we would still have $\sim 1\%$ of the total turbulent power in the radio band for $\alpha = 0.75$. Which of these various energies actually obtains probably depends on the energy of injection of the particles at the base of the jet, but at least in the low-luminosity sources the integrated ratio brightness is likely to be a sensitive indication of the total turbulent power.

III. COMPARISON WITH OBSERVATIONS

In this section we compare the predictions of our basic scaling relations given in equations (19a) and (19b) with the observations of two well-resolved radio jets, those in NGC 315 and 3C 31. The variations of jet radius R with distance Z from the radio cores have already been modeled for these systems by Bridle *et al.* (1980) and by BCH; these models provide us with estimates of the variation of v_z with Z, essentially by requiring that the flows satisfy Bernouilli's equation.

a) NGC 315

A comprehensive study of NGC 315 at 610 MHz and 1415 MHz has recently been made by WSBF. Their Figure 11A shows the behavior of the 1415 MHz surface brightness $B(\nu)$ on the axis of the jet as a function of its FWHM. These data are replotted in Figure 1. The curve superposed on the data shows a prediction from our equation (19a) based on the BCH fit to the collimation variations of the jet. This procedure essentially adds the *form* of the variation of v_z, which is known from the BCH fit, to the smoothed values of dR/dZ and $R(Z)$ observed by WSBF. The brightness normalization can be considered arbitrary for the moment.

There is clearly excellent agreement between the shape of the curve predicted from equation (19a) and the observed brightness variation. This is encouraging because, in the absence of field or particle replenishment in the jet, the surface brightness would fall off as $R^{-3.4}$, as shown in Figure 1 by the dotted line. Furthermore, on this logarithmic scale, the fit is not very sensitive to details of the v_z variation determined from the BCH fit. As discussed by BCH, a variety of models with different

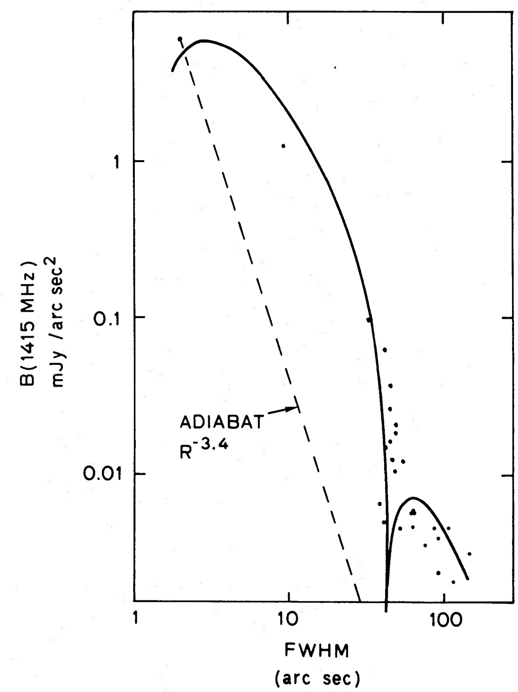

FIG. 1.—The 1415 MHz surface brightness data points (in mJy arcsec^{-2}) for NGC 315 are from Willis *et al.* 1981. The smooth curve is the prediction of equation (19a) with A constant, $\varpi = 0$, and v_z taken from the fit to the collimation data by BCH. The scaling quantity eA is the only free parameter. The adiabatic extrapolation from the initial brightness is also shown for comparison.

combinations of pressure and magnetic confinement could be fitted to the same $R(Z)$ data to within the observational errors; the goodness of fit in our Figure 1 is insensitive to this range of alternatives.

We note that equation (19a) successfully simulates the initial regime in NGC 315 wherein $B(\nu)$ falls off as $R^{-1.25 \pm 0.12}$ according to WSBF. Inspection of equation (19a) shows that when dR/dZ is approximately constant, and there is no significant entrainment, we expect the brightness to vary as $v_z^2 R^{-2}$, so that the slowest intensity decline in an unaccelerated regime would be R^{-2}. The ability of the model to reproduce the data therefore stems in part from the fact that the BCH fit to the collimation data required this to be an accelerated regime in which v_z increased by a factor of about 2.5. The success of the fit to this regime is therefore intimately connected with its proximity to the "sonic height" of 5″ (1.2 kpc, $H_0 = 100$ km s^{-1} Mpc^{-1}) in the BCH model for the collimation. Turbulent entrainment would provide an anternative explanation (see below), but the collimation data suggest a rapidly declining ambient density which would inhibit entrainment asymptotically. However, small variations in A can occur near the base even then (see eq. [25] in § IV).

This initial regime is in marked contrast to the regime beyond $2R \sim 20''$ and extending to $2R \sim 45''$. Here, there is little variation in v_z in the BCH models, and the goodness of fit displayed in Figure 1 reflects the smoothed approach of dR/dZ to zero at $2R \sim 45''$ [yielding $B(\nu) \propto R^{-4.42 \pm 1.10}$ (see WSBF), hence $R \propto Z^{5/9}$ on this shoulder by eq. (19a)]. Subsequently, the predicted turbulent power drops to zero, but the observed brightness is sensibly constant over this whole flat region between 2'.5 and 7' from the nucleus (where $2R \sim 45''$).

In the outermost regime, the predicted turbulent power rises smoothly with dR/dZ to the correct observed level above the adiabat extrapolated from the plateau and then declines in step with the observations (approximately as R^{-2} [WSBF]). Once again there is no need to invoke efficient entrainment in this outer expansion, and the density law obtained from the collimation fit (BCH) would not predict it by equation (25) below. *Thus over the whole well-observed main jet in NGC 315, the predicted brightness tracks the observations remarkably well,* excepting only the collimation plateau. This has certain implications for the physical parameters of the jet as we now proceed to demonstrate.

Firstly, in order for the synchrotron emissivity to vary in proportion to the local turbulent power input as in Figure 1, the intermediate resonant waves must be strongly damped locally, primarily by acceleration of relativistic electrons. That is, the presumed acceleration must be both efficient and rapid (as a convective time scale). This requirement is in accord with the damping scales estimated by Eilek (1981) and with the dominant damping mode expected (suprathermal acceleration rather than heating; e.g., Lacombe 1977).

Secondly, "convective smearing" of the turbulence-dominated $B(R)$ relation will be avoided *only* if the lifetime of the radiating particles is shorter than the local convective or "turnover" time scale ($\tau_c \sim \Delta Z/v_z \approx R/\Delta v$). On the collimation plateau, the synchrotron lifetime of the electrons radiating at 1415 MHz in the equipartition field strength of $4 \mu G$ (WSBF rescaled to $H_0 = 100$ km s^{-1} Mpc^{-1}) and in the presence of the 2.7 K background would be $\tau_s \sim 7.6 \times 10^7$ years. The length of the shoulder region is $\Delta z \sim 65$ kpc, so that v_z need only be greater than $\Delta Z/\tau_s \sim 900$ km s^{-1} for convective "smearing" to occur. This is quite likely according to our estimates below, and so we expect the observed $B(R)$ to be flat in this region, as is observed.

In regions such as the inner and outer expansion regimes, where the brightness predicted by equation (19) is above that of the expansion adiabat, the local turbulent input will dominate as observed (Fig. 1) provided only that it is dissipated or converted on a convection time scale or less. The integrated (10 MHz to 10 GHz) radio brightness, assuming that the spectrum is $\nu^{-0.6}$ over this range (WSBF) should then be given by equation (20) with $4\pi e = \nu_{\max}^{-0.4}(10) \lesssim 1/4$. We now determine the scaling coefficient in equation (20) by fitting it to the absolute brightness of the jet in NGC 315.

We equate the observed and predicted brightnesses at $R = 2''$ (472 pc, $H_0 = 100$ km s^{-1} Mpc^{-1}) where the jet is well resolved by the VLA. Using equation (20) with $\varpi = 0$ then requires that $B/eA = 8.9 \times 10^{-45} v_s^2$, where v_s is the flow velocity at the "nozzle" in cm s^{-1}. The observed radio brightness at this part of the jet (interpolated from Table 3 of WSBF, renormalized to $H_0 = 100$) is 5.2×10^{-6} ergs cm^{-2} s^{-1} sr^{-1}, so that we require $eAv_s^2 = 5.9 \times 10^{38}$ cgs.

We now use $4\pi e \lesssim \nu_{\max}^{-0.4}(10)$ in accord with our earlier argument (the inequality reflecting our uncertainty about the energy lost to "heat"), together with equation (18), to obtain directly (and relatively firmly):

$$\rho v_z v_s^2 \gtrsim 3.5 \nu_{\max}^{0.4}(10) \times 10^{-3} \text{ cgs} , \qquad (21)$$

where ρ and v_z are the density and longitudinal velocity of the flow at the distance from the nucleus where the jet radius is $R = 472$ pc. These parameters can be replaced by parameters at the "nozzle" using the BCH fit to the collimation behavior of the jet, which gives $v_z = 2.1 v_s$ and $\rho = \rho_s/15$. Consequently, recalling also that $v_s^2 = \gamma p_s/\rho_s$ and taking $\gamma = 4/3$, relation (26) can be written in terms of the pressure p_s and jet velocity v_s at the "nozzle":

$$p_s v_s \gtrsim 1.9 \times 10^{-2} \nu_{\max}^{0.4}(10) \text{ ergs cm}^{-2} \text{ s}^{-1} ,$$

One sees immediately from this estimate of the energy flux in the jet together with the BCH value for the jet radius at the nozzle ($R_s = 180$ pc) that the lobe luminosity $L \sim p_s v_s(\pi R_s^2) \gtrsim 2 \nu_{\max}^{+0.4}(10) 10^{40}$ ergs s^{-1}. This value ($L\nu_{\max}^{-0.4}$) is satisfactorily close to the observed radio luminosity of each of the north-preceding and south-following *lobes* (Bridle *et al.* 1976), but this cannot be regarded too seriously in view of the various uncertainties (especially in v_z, R_s, and e).

We may also use the BCH fit to scale p_s from the equipartition calculation of WSBF. This gives (see, e.g., Perley, Willis and Scott 1979 for brightness dependence) $p_s \sim 1.5 \times 10^{-10} \nu_{\max}^{+0.23}(10)$ dynes cm^{-2} so that

$$v_s \gtrsim 1300 \nu_{\max}^{+0.17}(10) \text{ km s}^{-1} ,$$

and (22)

$$\rho_s \lesssim 1.1 \times 10^{-26} \nu_{\max}^{-0.34}(10) \text{ cm}^{-3} .$$

With the BCH value for R_s (180 pc), these values yield a mass flux of $2 \times 10^{-2} \nu_{\max}^{-0.17}(10)$ $M_\odot$ yr^{-1}, which is reasonable.

Moreover, on the collimation plateau, (where $R \sim 22''$ [5.2 kpc] and $dR/dZ \sim 0$), WSBF concluded from their depolarization data that the thermal particle density must be $\lesssim 4 \times 10^{-4}$ cm^{-3}. Using equation (18), the above value for the mass flux ($\times 1/\pi$), the BCH value for v_z on the plateau ($\approx 2.55 v_s \gtrsim 3400$ km s^{-1} by eq. [27]), and the observed radius, we calculate $n_e \lesssim 3 \times 10^{-6} \nu_{\max}^{-0.34}(10)$ cm^{-3}. Such a low value is compatible with the measurements of WSBF in that their data were marginally consistent with no depolarization between 610 MHz and 1415 MHz. *Thus, the mass flux required to deliver the turbulent energy necessary for particle reacceleration in NGC 315 is well within available constraints on the thermal mass in the observed jet.* If a thermal density much larger than our estimate were to be confirmed, entrainment occurring just before the collimation plateau would be the most likely explanation (see § IV). For it is there that the collimating pressure profile (and therefore possibly the density profile) is flattest according to the CH fit given in BCH.

b) 3C 31 ≡ NGC 383

The distribution of intensity with distance from the radio core in the *northern* jet of 3C 31 has been observed at 4885 MHz and at 1480 MHz with the (incomplete) VLA by Fomalont *et al.* (1980), and there are several published CH fits to the collimation behavior (Bridle *et al.* 1980). Unfortunately, the jets in this source undergo large deflections ending in the lobe structure before reaching their first regime of $dR/dZ \approx 0$. Consequently, fits to the collimation properties using the CH-BCH formulation are not so tightly constrained as they are for NGC 315.

Figure 2 shows the variation of surface brightness, observed in the *northern* jet of 3C 31 at 4885 MHz, as a function of the FWHM of the jet. This plot is obtained by combining the data of Fomalont *et al.* (1980) with that of Bridle *et al.* (1980). Curves A and B are the predicted brightness variations using the collimation model (*a*) of Bridle *et al.* (1980) for A and a pressure-dominated CH fit (whose parameters are given in the figure caption) for B.

The most important variable affecting the goodness of fit to the observed brightness variation is the height of the sonic point. The best agreement between the predictions of equation (19) and the data for 3C 31 is obtained with sonic heights in the range $1''.5 < Z_z < 2''.5$ (0.35–0.67 kpc). This is comparable to the height inferred

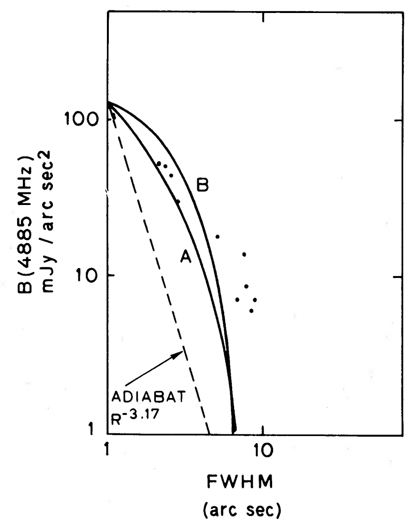

FIG. 2.—The 4885 MHz surface brightness data points (in mJy arcsec^{-2}) for 3C 31 are taken from Fomalont *et al.* 1980 and from Bridle *et al.* 1980. Curve A is the prediction of equation (19a) with A constant, $\varpi = 0$, and v_z taken from collimation model (*a*) of Bridle *et al.* 1980. Curve B is similar except that v_z is found from a pure pressure collimation model with (see, e.g., CH; BCH): $m = 5$, $m' = 3$, $R_s = 0''.21$, $z_s = 2''.3$, $z_e = 10.8$, $H = 20$. The adiabat appropriate for the spectral index of this source is also shown. Curves A and B have only the one free parameter, eA.

for the nozzle in NGC 315 by BCH (1.2 kpc) and is for both sources remarkably high. Figure 2 also shows, however, that the predicted $B(R)$ variations for 3C 31 begin to decline rapidly much sooner than the data, so that the predicted surface brightness in the more expanded regimes of the 3C 31 jet falls below that observed, by a factor ~ 10. Nevertheless, it is clear that our model is much closer to the observed behavior in the inner jet regime than is the adiabatic expansion law (shown by dashed line in the figure).

The regime of rapid decline in the predicted brightness relative to the observed brightness (and indeed to the adiabat) begins at $Z \sim 20''$ (corresponding to the FWHM $\sim 5''$). Just as in NGC 315, this is precisely where dR/dZ begins its decline (the point of inflexion $d^2R/dZ^2 \approx 0$; see, e.g., CH) to what would probably be a collimation plateau if the jet were to remain undisturbed (it may happen because the jet is disturbed at this distance if the disturbing effect implies a pressure plateau; see below). Thus we have in effect for 3C 31 only the first segment of the two-piece curve shown for NGC 315 in Figure 1, and it is dropping below the data just as does the initial segment of the NGC 315 profile.

The redshifts of NGC 315 (0.0167) and of 3C 31 (0.0169) are almost identical, so that we may compare the angular scales directly. On this basis, the convective time scale for 3C 31 on the shoulder of the plateau is about one-fifth that of NGC 315 (at the same jet speed), but the equipartition magnetic field is some 10 times greater ($\sim 4 \times 10^{-5}$ gauss), so that the synchrotron lifetime is 100 times smaller at the same energy (there is another multiplication factor ~ 1.7 in τ_s to allow for the difference in frequency). Therefore, unless the jet velocity in 3C 31 is some 12×900 km s^{-1} (see the argument for NGC 315), or unless the equipartition fields are inappropriate in this region, we should expect convective smearing of the turbulent power to be reduced in this source. In fact, the observed $B(R)$ variation (Fomalont *et al.* 1980) continues to be relatively flat beyond the shoulder. A relatively high jet velocity of $> 10^4$ km s^{-1} might not be implausible (see the estimates made by Perley, Willis, and Scott 1979 for 3C 449). Moreover, if there were a *density* plateau, and not merely a pressure plateau, associated with the slowing down of the lateral expansion of this jet, entrainment (§ IV) could prevent the turbulent power from dropping so rapidly with dR/dZ and thus lead to better agreement with the observations.

The 4885 MHz brightness at the base of the jet in 3C 31 has been measured to about 2″ from the radio core (Fomalont *et al.* 1980). Figure 3*a* of Fomalont *et al.* shows that the jet brightness initially *rises* to a broad maximum about 4″ from the core before the subsequent decline along the large-scale jet. This initial rise is fitted approximately by curves such as A in our Figure 2 when these curves are computed to smaller values of Z using the CH *models* for the jet collimation. (The actual variation of jet radius with Z in this regime is difficult to deduce as the jet is barely resolved in the transverse direction by the observations of Fomalont *et al.*). It is significant that this same initial rise and fall in brightness of the jet is also seen at optical frequencies (Butcher, van Breugel, and Miley 1980; van Breugel 1980), implying a $\nu^{-0.72}$ "radio to blue" spectrum. These observations in the optical band (which cannot be subject to convective smearing) *follow the turbulent power through a predicted rise and fall* immediately beyond the "nozzle." This is encouraging evidence for the basic principles of our model.

IV. ENTRAINMENT AND THE MASS FLUX A

We have seen above that, principally because of the probably huge Reynolds numbers in the observed jets, *we expect that the observed radio jets are turbulent jets.* Moreover, this hypothesis leads directly to equation (19), which proves to be rather successful at fitting the $B(R)$ variation of at least some radio jets (§ III) with the mass flux A taken constant. But turbulent jets grow by continually entraining surrounding material into the turbulent region, effectively at the velocity of the largest eddies (see eq. [3] and following; also Cantwell 1981), and so to generalize equation (19) we must estimate the possible form of a variable mass flux $A(R)$.

We adopt a self-similar model of a compressible, turbulent jet in a nonuniform background. For simplicity we may suppose the jet to be nonrotating about its axis and to be nonmagnetic, although neither assumption is essential to the form of the solution. We model the turbulent jet by using the Navier-Stokes equations for the time-averaged mean quantities and a kinematic viscosity of the form

$$\nu = vr \sin\theta \,, \tag{23}$$

where v is a constant speed and $(r.\ \theta)$ are spherical coordinates having the jet axis as the polar axis. This simply states that the dominant viscosity is the "eddy viscosity" in a turbulent jet. An excellent estimate for the speed v is furnished by equation (3), namely $v = v_z\, dR/dZ$, where $v_z \equiv v_r(\theta = 0)$ and $R(Z)$ is, as usual, the radius of the jet at any section ($\propto Z$ for turbulent jet, Landau and Lifshitz 1959). The jet is supposed to be steady in the mean ($t > \tau_c \equiv r/v_z$).

This model yields the following form of solution to the equations of motion (Henriksen and Wang 1982):

$$\rho = \rho_s r^{-n} \rho(\theta)\,, \quad r \equiv r/r_s\,,$$
$$p = \rho v^2 P(\theta)\,,$$
$$v_r = v\, v_r(\theta)\,, \quad v_\theta = v\, v_\theta(\theta)\,, \tag{24}$$

where r_s and ρ_s are evaluated at a standard reference point; $\rho(\theta)$, $P(\theta)$, $v_r(\theta)$, and $v_\theta(\theta)$ are dimensionless functions to be found from the equations of motion (2), the continuity equation, and an energy equation [note that $kT(\theta)/(\mu_m m_{\rm H} v^2) \equiv P(\theta)$]. We are interested here only in the continuity equation which yields ($R \equiv r \sin\theta_j$, θ_j is jet half-angle)

$$A \equiv R^2 \rho v_z = \text{const.} - (\sin\theta_j)^{(n-2)} \frac{(R_s^2 \rho_s v)}{(2-n)} A(0) R^{(2-n)}\,,$$

or, for $n = 2$,

$$A \equiv R^2 \rho v_z = \text{const.} - (R_s^2 \rho_s v) A(0) \ln R\,,$$

where

$$A(0) \equiv \left\{\frac{1}{\sin\theta}\frac{d}{d\theta}[\rho(\theta) v_\theta(\theta) \sin\theta]\right\}_{\theta=0} < 0\,. \tag{25}$$

This last result gives a reasonable estimate of the forms of $A(R)$ to be used in equation (19) for an arbitrary ambient density power law (n). We may conclude that core entrainment will cause A in equation (19) to increase significantly with R if $n \lesssim 2$ ($A \propto R^{2-n}$ asymptotically with R). Otherwise, we see that A will soon tend to a constant with R. This result is important as it implies that a jet brightness that declines less rapidly than R^{-2} (see eq. [19a] when v_z and dR/dZ are constant) *may be associated with a radially flat ambient density profile* [$B(\nu) \propto R^{-1}$ by eqs. (19) and (25) if $n = 1$, $B(\nu) \sim$ const. if $\rho \sim$ const. (i.e., $n = 0$)] although this is without allowing for adiabatic losses in $e(\nu)$ which will steepen $B_\nu(R)$. Should this happen near the base of the jet, this mechanism would explain flat $B(R)$ variations without invoking a nozzle to produce an increasing v_z. The height usually attributed to the "nozzle" (CH, BCH) might in such cases simply be the onset of the "flat" density variation. Moreover, a jet which encountered density "clouds" would show considerable brightening or a "hot spot." It is also likely that vorticity and density irregularities will be entrained by the jet from ambient density "clouds" (also including the nozzle region) and convected outward. This will introduce a "hot spot" structure in the jet. For a sufficiently extensive density plateau, one might expect a periodic hot spot structure on an interval RdZ/dR.

We observe however that, for a given beam energy, the turbulent beam must develop a core-halo structure [$v_r(\theta)$, $v_\theta(\theta)$, and possibly $\rho(\theta)$ must be peaked sharply near $\theta = 0$ with broad "wings"] to compensate for the additional mass being added to the beam. The numerical demonstration of these conclusions must be left to a subsequent paper, but we note that our remarks here are generally in accord with the numerical results of Baan (1980), who used a similar turbulent viscosity.

Finally, in this section we give the analogs of equations (24) and (25) when there is an effective viscosity, ν, which is constant. These are:

$$\rho = \rho_s r^{-n} \rho(\theta)\,, \quad r \equiv r/r_s\,,$$
$$p = \rho_s \frac{\nu^2}{r_s^2} P(\theta) r^{-(n+2)}\,,$$
$$v_r = \frac{\nu}{r_s}\frac{v_r(\theta)}{r}\,, \quad v_\theta = \frac{\nu}{r_s}\frac{v_\theta(\theta)}{r}\,, \tag{26}$$

and

$$A = \text{const.} - \frac{\nu \rho_s R_s (\sin\theta_j)^n}{(1-n)} R^{(1-n)} A(0)\,,$$

and for $n = 1$,

$$A = \text{const.} - (\nu \rho_s R_s) \sin \theta_j (\ln R) A(0) .$$

We see that the core entrainment is less rapid for a given density profile than in the turbulent case [but the total entrainment may be greater due to a flatter $v_r(\theta)$ depending on the nature of ν: see also Baan 1980], but that the jet is effectively decelerated. This might be the fate of initially relativistic nuclear jets until they become turbulent, but the nature of the necessary viscosity is problematical (Baan).

V. CASCADING JETS

The nature of the connection between the nuclear (VLB) jets and the halo (VLA) jets must be addressed in view of the sources observed to have both. Early opinion (see, e.g., Miley 1980) seems to have considered the large-scale jet to be a simple extrapolation of the nuclear jet, but this leads to enormous nuclear sources of magnetic flux and possibly angular momentum (if the CH helical fields are supported). The resolution of this problem would seem to be the entrainment of mass, magnetic flux, and angular momentum by the beam as an inevitable part of the cascade to the larger scale (CH; De Young 1980; Baan 1980). One must, however, reconcile the similar position angles of the VLB and VLA jets that are observed in at least some cases (e.g., NGC 315 and NGC 6251) with the mechanism of this cascade. We attempt in this section to summarize the various possibilities.

Should the ambient pressure, p_a, in the circumnuclear material be such that $p_a \ll \rho_j v_{je}^2$ (ρ_j and v_{je} refer to emergent properties of the nuclear jet), then we would expect the jet to be relatively unrefracted (Henriksen, Vallée, and Bridle 1981). When $p_a \lesssim \rho_j v_{je}^2$, large deviations from the initial direction can occur however, unless this initial direction is accurately aligned with a symmetry axis of p_a. The case $p_a > \rho_j v_{je}^2$ will not arise in an existing jet because it would produce instead a confined core source.

In order for a high "nozzle" to form by the Blandford-Rees mechanism (CH; BCH; This paper) and for a memory of the "original" (after nuclear refraction as above) direction to be simultaneously preserved (as suggested in CH), the material entrained by the nuclear jet in any given interval must also be significantly heated. It is only then that a "bubble" of heated gas which retains some memory of the initial momentum, but which is also subsonic as required for a second nozzle, can be produced on the new spatial (and mass) scale. This clearly requires for a transverse spatial scale, l, that $l^2/\nu \approx l^2/a$, where a is the effective thermal diffusivity (we always assume that the jet is not radiating most of its energy). Hence, *the Prandtl number of the nuclear entrainment* $P_r \equiv \nu/a$ *should be* ≈ 1. If this ratio is much smaller than unity, then the expanded jet will not retain its original momentum sense, while if the ratio is much greater than unity the jet will not renozzle. We must also suppose that the cooling time of the heated gas is larger than l^2/a, in order that a "fire box" for the nozzle be in fact established.

Given that $P_r \approx 1$ and a sufficiently long cooling time, whether significant entrainment does in fact occur may be estimated in general from equations (25). However, a condition which by equation (25) is always sufficient (but usually not necessary) to ensure the jet's growth is simply $l^2/\nu \approx z/v_j$, or $\text{Re} \equiv l v_j/\nu \approx z/l \lesssim \cot \theta_j$.

We conclude therefore that $\nu \approx a \lesssim l v_j$ are sufficient conditions for an "upward jet cascade with memory." Moreover, *the easiest way of satisfying these conditions is by a turbulent transfer of heat and momentum from the jets to the ambient medium.* We are led then to a model in which the nuclear jet disperses its energy and momentum over a much larger mass scale by inducing vortical turbulence in the ambient nuclear atmosphere which results in efficient heat transport and entrainment. This nuclear "fire box" region can then renozzle to produce a jet of lower specific energy but carrying in exchange much more thermal material as well as angular momentum and magnetic field (the angular momentum is likely to be present in the ambient material, while the magnetic field can be produced in part by the induced turbulence [De Young 1980]). This "fire box" region may be X-ray bright as a thermal source but, without the energy input from a mean motion, as discussed in § II, we should not expect a strong inertial cascade to be maintained nor thus a strong synchrotron spectrum.

VI. DISCUSSION AND CONCLUSIONS

We attempted in § II of this paper to render plausible on physical grounds the proposal that the synchrotron radiation of relativistic electrons provides the effective viscosity for radio jets, and that these jets should be viewed as turbulent mixing regions. This proposal has still to be justified in detail theoretically, but it is shown here to lead to remarkable agreement with observations of the radio brightness in NGC 315 and 3C 31.

Our predicted brightness-radius variation given by equation (19), although subject to uncertainties (which we have discussed) regarding entrainment and spectral distribution, is based on very general considerations, essentially on scaling relations. It is therefore not expected to depend very sensitively on the details of our theory. In § III we have argued that the VLA and Westerbork Synthesis Radio Telescope data on NGC 315 and 3C 31 strongly support our prediction, as the expected surface brightness variation with jet radius fits the observations of NGC 315 well and fits that part of the observations of 3C 31 where it may best be expected to do so.

Our prediction relates the local synchrotron emissivity in a jet to the local turbulent input and does not apply in regions of the jet where the emissivity can be maintained by convection. Such regimes become apparent in both NGC 315 and 3C 31 where $dR/dZ \rightarrow 0$. Our models are self-consistent in that they do yield the convective time as less than the synchrotron lifetime in these very regions.

We have also made a preliminary fit to the absolute brightness of the NGC 315 jet, thereby determining the unknown amplitude coefficient $[e(v)A]$ in our equation (19). The deduced mass flux and velocity range in the jet appear very plausible. We have not found it necessary to include turbulent entrainment in our fits to NGC 315 or 3C 31. This has required substantial longitudinal acceleration of the flow near the base of the jets to fit the slow variation of brightness with jet radius there (regions of large dR/dZ). Our model for the brightness variation thus reinforces the concept of a "high nozzle" that was suggested independently (from collimation data) by CH (in their model and for 3C 449), by BCH for NGC 315, and by Bridle *et al.* (1980) for 3C 31. The rapid decline in ambient density required to produce the initial rapid expansions of these jets at their bases is also consistent with neglecting entrainment according to the estimates made here in § IV.

We cannot exclude an alternative model, however, wherein the jets have encountered density plateaus at their bases and efficiently entrain material as they grow (§ IV) according to equations (24) and (25). This entrainment would continue only until the ambient density variation changed its form drastically, such as during a steep decline from the nuclear atmosphere or in the collimation plateau ($dR/dZ \rightarrow 0$) region, thereby breaking the similarity. According to the ideas expressed in § V (elaborating those of CH), such a hot region might itself be the "cascade" or "nozzle" region wherein the nuclear jet is growing to the scale of the "VLA" jet.

The entrainment is also likely to be relevant to the existence of "hot spots" in the mean behavior of the beam. For should the jet encounter a density "cloud" in the average atmosphere (of sufficient extent) which it entrains, then the amplitude of equation (19) will increase anomalously and a "hot spot" will form.

In any event, both the $B(R)$ variations and the collimation behavior of NGC 315 and 3C 31 (and possibly 3C 449; CH) lead us to conclude that the "VLA" jets are influenced by an ambient medium at heights of 0.5–1 kpc from the galactic center, as though the "nozzle" envisaged by Blandford and Rees (1974) were there. This raises the question of the relation between the "VLB" jets and the "VLA" jets, particularly when they agree very nearly in position angle. The galaxy NGC 315 itself provides a clear example of the problem. Linfield (1981) has detected a jet about 0".003 (0.7 pc) long extending from the unresolved core of NGC 315. Its position angle agrees to within errors (and intrinsic oscillations) of $\pm 3°$ with the position angle of the base of the "VLA" jet. This "VLB" jet rapidly fades in brightness with increasing distance from the core, whereas the "VLA" jet rapidly increases in brightness at the very distance above the core modeled by BCH as its "sonic height" (Bridle *et al.* 1979; Bridle *et al.* 1982). These data taken together suggest that the collimation of the large-scale ("VLA") jet in NGC 315 has been at least a two-stage process. The first stage, or "trigger mechanism," occurs on scales less than 1 pc and produces a jet whose relativistic particles are rapidly cooled—the quickly fading "VLB" jet. We suggest (as did CH) that the interaction between this high-specific-energy (probably relativistic) jet and a surrounding gaseous atmosphere in the inner kpc of NGC 315 has permitted the formation of a *second* "nozzle" (as discussed in § V and above). Moreover, we maintain that this strong interaction has restarted a turbulent cascade capable of accelerating electrons to relativistic energies in a regime (the "VLA" jet) where the cooling time for the particles is now much longer.

We are making further observations of the brightness distributions at the bases of large-scale radio jets using the VLA to examine whether there is similar evidence for "second-stage" reacceleration in other radio galaxies on kiloparsec scales.

Finally, we note that the model for the brightness variations presented in this paper must eventually be reconciled with the observed linear polarization amplitude and structure. The amplitude has values exceeding 60% in the outer main jet of NGC 315 (WSBF). Both this striking fact and the polarization structure have been interpreted either in terms of large-scale helical fields (e.g., Fomalont *et al.* 1980; CH) or in terms of a tangled field which lacks a longitudinal component (Laing 1981). It is not immediately clear whether or not either of these mean field structures are compatible with the ring vortical turbulence proposed here. We are, however, somewhat hopeful because a mean (entrained) angular momentum coupled to the mean jet momentum will inevitably tend to produce a helical mean field (see, e.g., CH) provided only that the amplitude of an eddy magnetic field decreases sufficiently rapidly with its scale.

We are grateful to Drs. G. V. Bicknell, B. J. Cantwell, and J. A. Eilek for valuable discussion. R. N. H. thanks Prof. P. A. Sturrock for his hospitality while on sabbatical leave from Queen's University at Kingston, and gratefully acknowledges the support of NASA grant NGR 05-020-668. A. H. B. thanks the National Radio Astronomy Observatory and the University of New Mexico for the facilities they have provided during a sabbatical leave also from Queen's University. This research has been supported by grants to R. N. H. and A. H. B. from the Natural Sciences and Engineering Research Council of Canada (NSERC).

REFERENCES

Baan, W. A. 1980, *Ap. J.*, **239**, 433.
Bell, A. R. 1978*a*, *M.N.R.A.S.*, **182**, 147.
———. 1978*b*, *M.N.R.A.S.*, **182**, 443.
Benford, G., Ferrari, A., and Trussoni, E. 1980, *Ap. J.*, **241**, 98.
Bicknell, G. V., and Melrose, D. B. 1982, in preparation.
Birkinshaw, M., Laing, R. A., and Peacock, J. A. 1981, *M.N.R.A.S.*, **197**, 253.
Blandford, R. D., and Ostriker, J. P. 1978, *Ap. J.* (*Letters*), **221**, L29.

Blandford, R. D., and Rees, M. N. 1974, *M.N.R.A.S.*, **169**, 395.
Bradshaw, P., ed. 1978, *Topics in Applied Physics*, Vol. **12**, *Turbulence* (2d. ed; New York: Springer-Verlag).
Bridle, A. H. 1981, in *IAU Syyposium 97*, *Extragalactic Radio Sources*, ed. D. S. Heeschen and C. M. Wade (Dordrecht: Reidel), in press.
Bridle, A. H., Chan, K. L., and Henriksen, R. N. 1981, *J.R.A.S. Canada*, **75**, 69 (BCH).
Bridle, A. H., Davis, M. M., Fomalont, E. B., Willis, A. G., and Strom, R. G. 1979, *Ap. J.* (*Letters*), **228**, L9.
Bridle, A. H., Davis, M. M., Meloy, D. A., Fomalont, E. B., Strom, R. G., and Willis, A. G. 1976, *Nature*, **262**, 179.
Bridle, A. H., Fomalont, E. B., Palimaka, J. J., and Willis, A. G., 1981, *Ap. J.*, **248**, 499.
Bridle, A. H., Fomalont, E. B., Palimaka, J. J., and Henriksen, R. N. 1982, in preparation.
Bridle, A. H., Henriksen, R. N., Chan, K. L., Fomalont, E. B., Willis, A. G., and Perley, R. A. 1980, *Ap. J.* (*Letters*), **241**, L145.
Brown, G. L., and Roshko, A. 1974, *J. Fluid Mech.*, **64**, 775.
Brown, G. L., and Thomas, A. S. W. 1977, *Phys. Fluids*, **20**, 243.
Burch, S. F. 1979, *M.N.R.A.S.*, **187**, 187.
Butcher, H. R., van Breugel, W. L., and Miley, G. K. 1980, *Ap. J.*, **235**, 749.
Cantwell, B. J. 1981, *Ann. Rev. Fluid Mech.*, **13**, 457.
Chan, K. L., and Henriksen, R. N. 1980, *Ap. J.*, **241**, 534 (CH).
De Young, D. S. 1980, *Ap. J.*, **241**, 81.
Eilek, J. A. 1979, *Ap. J.*, **230**, 393.
———. 1981, preprint.
Falco, R. E. 1977, *Phys. Fluids*, **20**, 5124.
Fomalont, E. B., Bridle, A. H., Willis, A. G., and Perley, R. A. 1980, *Ap. J.*, **237**, 418.
Henriksen, R. N., Vallée, J. P., and Bridle, A. H. 1981, *Ap. J.*, **249**, 40.
Henriksen, R. N., and Wang, D-J 1982, in preparation.
Kato, S. 1968, *Pub. Astr. Soc. Japan*, **20**, 59.
Kolmogorov, A. N. 1968, *Soviet Phys. Uspekhi*, **10**, 734.
Kulsrud, R. M. 1955, *Ap. J.*, **121**, 461.
Lacombe, C. 1977, *Astr. Ap.*, **54**, 1.
Laing, R. A. 1981, *Ap. J.*, **248**, 87.
Landau, L. D., and Lifshitz, E. M. 1959, *Fluid Mechanics*, (New York: Pergamon).
Linfield, R. 1981, *Ap. J.*, **244**, 436.
Masson, C. R. 1979, *M.N.R.A.S.*, **187**, 253.
Miley, G. K. 1980, *Ann. Rev. Astr. Ap.*, **18**, 165.
Owen, F. N., Hardee, P. E., and Bignell, R. C. 1980, *Ap. J.* (*Letters*), **239**, L11.
Pacholczyk, A. G. 1970, *Radio Astrophysics* (San Francisco: Freeman).
Pacholczyk, A. G., and Scott, J. S. 1976, *Ap. J.*, **203**, 313.
Parker, E. N. 1964, *Ap. J.*, **140**, 1170.
Perley, R. A., Willis, A. G., and Scott, J. S. 1979, *Nature*, **281**, 437.
Stein, R. F. 1981, *Ap. J.*, **246**, 966.
Townsend, A. A. 1976, *The Structure of Turbulent Shear Flow* (2d ed.; Cambridge: Cambridge University Press).
van Breugel, W. L. 1980, Ph.D. thesis, Rijks-Universiteit, Leiden.
Willis, A. G. 1981, paper presented at Second ESO/ESA Workshop, "Optical Jets in Galaxies," ESO, Munich, 1981 February 18–19.
Willis, A. G., Perley, R. A., and Bridle, A. H. 1982, in preparation.
Willis, A. G., Strom, R. G., Bridle, A. H., and Fomalont, E. B. 1981, *Astr. Ap.*, **95**, 250 (WSBF).

A. H. Bridle: National Radio Astronomy Observatory, VLA Program, P.O. Box 0, Socorro, NM 87801

K. L. Chan: Applied Research and Systems, Landover, MD 20785

R. N. Henriksen: Institute for Plasma Research, Stanford University, Stanford, CA 94305