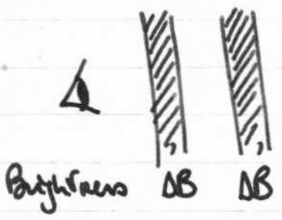
THERMODYNAMICS and EMISSION

SCHEUER

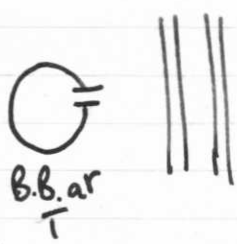
Synchrotron Radn. Panofsky & Phillips
Jackson
LeRoux, Annales d'Astrophysique **24**, 2 or 72 (1961)

All emission mechanisms must stem from accelerating charges. Mechanisms differ solely in their way of doing the accelerating.

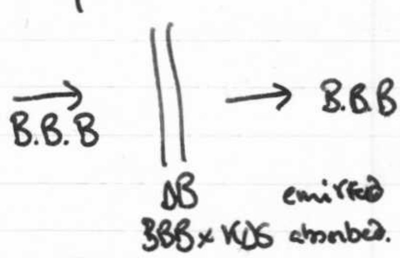


n slabs → nΔB

Suppose each slab contains material in thermodynamic ≡ⁿ at definite temp. T. (If really true, must isolate from rest of world, black-body radⁿ inside & no loss to us). Forgetting this → exchange with outside world much slower than internal processes. Then can make $n \to \infty$, $n\Delta B >$ Black Body at temp T.



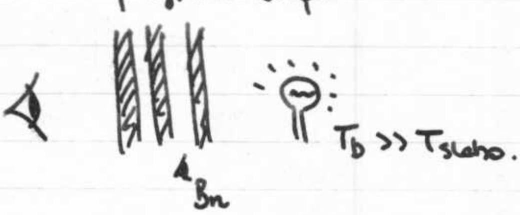
Then B.B. at T could be warmed by radn. from region at temp. T. 2LT forbids. ∴ must have forgotten absorption



→ B.B.B. + ΔB − BBB × KΔS

For thermodynamics to be sat., ΔB must ∴ = BBB × KΔS

T_b, "≡ⁿ. black body temp", "brightness temp". Implies nothing about mechanism or "physical temp".



$$B_{n+1} = B_n - K\Delta S . B_n = (1 - K\Delta S) B_n$$
$$= (1 - K\Delta S)^2 B_{n-1}$$

If slabs thin, so KΔS small

$$B_{n+1} = (1 - K\Delta S)^n B_1$$
$$= \left\{ (1 - K\Delta S)^{1/K\Delta S} \right\}^{nK\Delta S} B_1$$
$$= e^{-nK\Delta S} B_1 .$$

$$\frac{B_{out}}{B_{in}} = e^{-nK\Delta S} = e^{-KS}$$

Distribution of material matters not. What does matter is the total thickness.

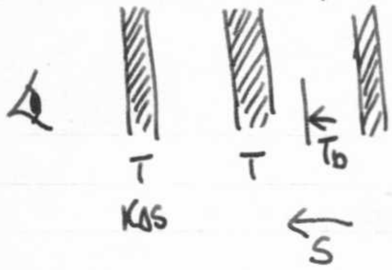
$\tau = \sum K \Delta S \to \int K dS$ for cont's medium. Optical depth

$$\frac{B_{out}}{B_{in}} = e^{-\Sigma K dS} = e^{-\tau}$$

Absorption = τ in nepers.
= $4.6 .. \; \tau$ in dB
= $\frac{4.6 ..}{2.5} \; \tau$ in magnitudes. etc.

BASIC ELECTRODYNAMICAL FORMULAE

SCHEUER



$dT_b = K\,ds\,T$ ∵ this from thermodynamics is what is radiated from one slab.

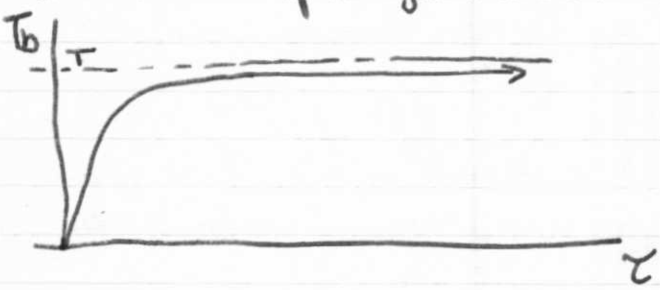
$\quad - K\,ds\,T_b$ ∵ of absorption of rad.ⁿ coming in.

$$\frac{dT_b}{ds} = K(T - T_b)$$

Emitter emits ~~K~~ $T\,d\tau$. Fraction reaching observer $=$ ~~K~~ $T\,d\tau.\,e^{-\tau}$

∴ For all of material $= \int_0 T e^{-\tau}\,d\tau = T(1-e^{-\tau})$

Distribution in space again irrelev. Optical depth all that matters.



Quite often the slabs are not in practice in thermodynamic ≡ⁿ Cosmic rays, u-v, etc. can cause excitations, & idea of temperature becomes a bit arbitrary. Can relate it to the actual emission process by some suitable temp.

$$\underset{\sim}{A} = \frac{\mu_0}{4\pi}\int \frac{[j]}{r}\,d\tau \qquad \phi = \frac{1}{4\pi\varepsilon_0}\int\frac{[\rho]}{r}\,d\tau \quad \text{—— (1)}$$

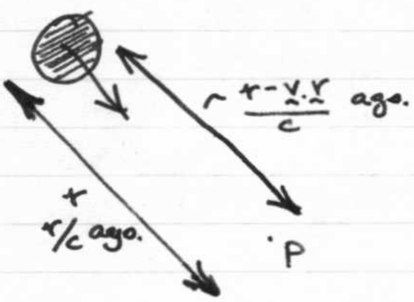
For moving electron, non-relativistically

$$\underset{\sim}{A} = \frac{\mu_0}{4\pi}\,\frac{e[v]}{r} \quad \text{—— (2)}$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\,\frac{e}{r}$$

should retard, but so large that vars. in it insignificant.

in a somewhat careless limiting case.



The effect integrate a charge density

Increases total charge observed

during the integration by a factor of $\beta = \dfrac{1}{1 - \frac{v.r}{|r|c}}$

$$\underset{\sim}{A} = \frac{\mu_0}{4\pi}\left[\frac{ev}{|r| - \frac{r.v}{c}}\right]$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\left[\frac{e}{|r| - \frac{r.v}{c}}\right]$$

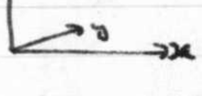
} Lienard Wiechert. (3)

L & L'
Pan & Ph.]

True for rel. ∵ come from Mⁱ.

$\underset{\sim}{A}$ not the most interesting object. Power over all directions is what is really interesting. Then power in given directions, or polarisation.

$$\underset{\sim}{B} = \underset{\sim}{\nabla} \wedge \underset{\sim}{A} \qquad \text{Plane wave}$$



Dirⁿ of propgⁿ

ELECTRODYNAMICS OF GENERALISED EMITTERS   SCHEUER

Put $\underset{\sim}{A} = \underset{\sim}{A_0}\, e^{i(\omega t - kx)}$ $\omega = 2\pi f$, $k = 2\pi/\lambda$

$B_x = \frac{\partial}{\partial y}(A_z) - \frac{\partial}{\partial z}(A_y) = 0$ ∵ $\partial/\partial y \cdot \partial/\partial z = 0$

$B_y = -\frac{\partial}{\partial x}(A_z) = ikA_z$

$B_z = -ikA_y$

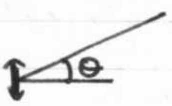
∴ only z-y cpts of A proj to zy plane of interest. Call projection $A_\perp$

- $B \perp A_\perp$ $|B| = k|A_\perp|$

Hence E via $i\omega\varepsilon_0 \underset{\sim}{E} = \dot{D} = \nabla \wedge \underset{\sim}{H}$.
Can do all this ∵ we know we will have plane wave at large distances.

Power out $= \left(\frac{1}{2}B.H + \frac{1}{2}D.E\right)c = \frac{B^2}{\mu_0}.c = Z_0\overline{B^2} = Z_0 k^2 \overline{A_\perp^2}$ — ④

- Hertz Dipole $|v| << c$. amp. of vib^n $<< \lambda$. (no φ-differences from diff. parts of oscillation)



$\underset{\sim}{A} = \frac{\mu_0}{4\pi} \cdot \frac{e[v]}{r}$

$= \frac{\mu_0}{4\pi} e v_0 e^{i\omega(t - r/c)} \cdot \frac{1}{r}$

$|A_\perp| = \frac{\mu_0}{4\pi} e \frac{v_0}{r} \cos\theta$

Total radiated power $= Z_0 k^2 \left(\frac{\mu_0}{4\pi}\frac{e v_0}{r}\cos\theta\right)^2$ / unit area.

4π Solid angle total $= \int_{-\pi/2}^{\pi/2} Z_0 k^2 \left(\quad\right)^2 . 2\pi r\cos\theta . r\,d\theta$

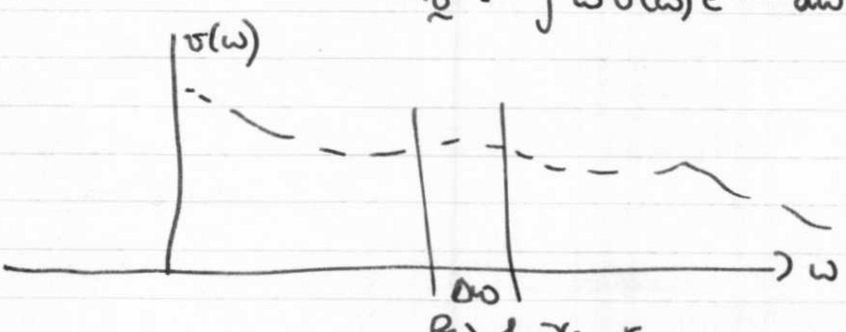
- $= \frac{Z_0 k^2 \mu_0^2 e^2 v_0^2}{8\pi} \int_{-\pi/2}^{\pi/2} \underbrace{\cos^3\theta\, d\theta}_{\downarrow\, 4/3}$

$= \frac{\omega^2 e^2 v_0^2}{6\pi\varepsilon_0 c^3} \equiv \frac{e^2 \dot{v}_0^2}{6\pi\varepsilon_0 c^3}$ ——— ⑤

$\left(= \frac{(e\dot{v}_0)^2}{6\pi\varepsilon_0 c^3} = \frac{(\ddot{m})^2}{6\pi\varepsilon_0 c^3}\right)$

- In general given $\underset{\sim}{v} = \int v(\omega) e^{i\omega t} d\omega$ Fourier analysis of some complicated motion.

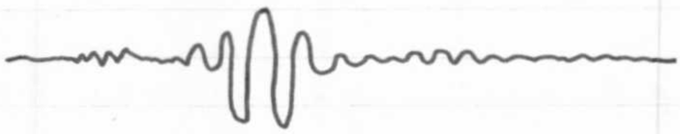
$\dot{\underset{\sim}{v}} = \int \omega v(\omega) e^{i\omega t} d\omega$



SPECTRUM ANALYSIS — SCHEUER

Look for total energy from aperiodic motion, not power. (Power for ∞ time → ∞ energy). 1st Fourier analyse excursion of electron
2nd Look at BW of interest.

Electron amp (time) : 

Spectrum : 

Power radiated $= \dfrac{e^2}{3\pi\varepsilon_0 c^3}\,\overline{\dot v^2}$ (note $\frac{1}{2}$ factor due to ‾)

Total energy radiated $= \displaystyle\int \frac{e^2}{3\pi\varepsilon_0 c^3}\,\overline{\dot v^2}\,dt$ — note that this is Fourier amp., not actual motion.

Parseval's Th: If $F(t) \rightleftharpoons f(\omega)$

$$\int |F|^2 dt = \int |f|^2 d\omega$$

∴ Total energy radiated $= \dfrac{e^2}{3\pi\varepsilon_0 c^3}\displaystyle\int_{\omega}^{\omega+\delta\omega} (\dot v_\omega)^2\, d\omega$

$= \dfrac{e^2}{3\pi\varepsilon_0 c^3}\,(\dot v_\omega)^2\,\Delta\omega$ (if $v(\omega)$ slowly var.^{n}) (6)

End of preliminaries.

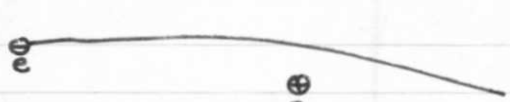
Need: i) something to excite electrons at $\nu \sim 10^{7-10}$ c/s.
ii) hard enough to → power observed

Compendium of Brightness Temps.

Galaxy	1° - 10°	1500 Mc/s	Milky Way photospheres ≃ 6000°
	300° - 1000°	150 Mc/s	Gaseous neb.^{ae} gas ~ 10^4 °K
	> 10^4 °	15 Mc/s	
Sources	up to 10^9 °K known		
	up to 10^6 ° K common		
Sun	Quiet - 10^6	100-1000 Mc/s	Corona ~ 10^6 °K (ionisation temp. Doppler broadening)
	Active - >10^{10}		
Orion,	~ 10^4		

Some hope of electrons in thermal eq.^{m} for quiet sun, orion nebula. Nowhere else.

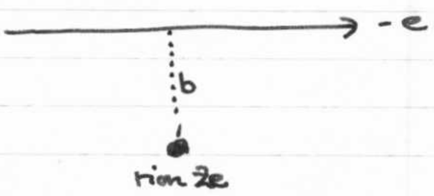
(6) can be rewritten Total power rad.^{d} $= \dfrac{2}{3}\cdot\dfrac{e^2\Delta\omega}{\varepsilon_0 c^3}\left|\dfrac{1}{2\pi}\displaystyle\int \dot v(t)\, e^{-i\omega t}\, dt\right|^2$



Hyperbolic path, Keplerian motion

$\therefore$ electron radiates, it deviates from hyperbolic path. Ignore this. Ignore recoil due to emission of photon, for RF photons. If want to deal with light $h\nu$, do have to consider recoil due to photon.

- i) Calculate motion of electron
  ii) Fourier analyse → acc^n appr. of frequency ν.



Collision parameter b, closest approach of undeviated electron.
Consider $\frac{1}{2}m_e v^2 \gg\gg Ze^2/4\pi\varepsilon_0 b$

- Time for collision $\simeq 2b/v$, $\ll$ period of RF radiation ($e \sim h\nu$ phase space). $\therefore$ treat acc^n as virtually a δ fn when Fourier-analysing it. Area under $\dot{v}_\perp(t)$ curve is $\int \dot{v}_\perp(t)\,dt = \Delta v_\perp$. For this to be true $2b/v \ll 1/\nu$, $b \ll v/2\nu$

$$\downarrow$$

$$\frac{Ze^2}{2\pi m \varepsilon_0 v^2} \ll b \ll \frac{v}{2\nu}$$

$$\text{Force } \perp^r \text{ path} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Ze^2}{4\pi\varepsilon_0 b^2}\cos^3\theta$$

$$\Delta p_\perp = \int F_\perp\,dt = \int_{-\pi/2}^{\pi/2} \frac{Ze^2}{4\pi\varepsilon_0 b^2}\cos^3\theta \,\frac{d(b\tan\theta)}{v}$$

$$= \int_{-\pi/2}^{\pi/2} \frac{Ze^2}{4\pi\varepsilon_0 b}\cos\theta\cdot\frac{d\theta}{v}$$

$$\Delta p_\perp = \frac{Ze^2}{2\pi\varepsilon_0 b v} = m\Delta v_\perp$$

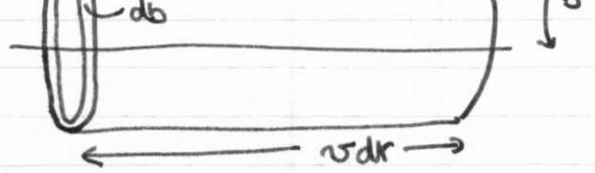
- $\therefore \int \dot{v}_\perp e^{i\omega t}\,dt \sim \Delta v_\perp$ if $e^{i\omega t}$ varies much more slowly than $\dot{v}_\perp$ in time of pulse.

$$\therefore \text{Total energy radiated (6)} = \frac{\cancel{Z}e^2\Delta\omega}{3\varepsilon_0 c^3}\left|\frac{1}{2\pi}\cdot\frac{Ze^2}{2\pi\varepsilon_0 b v m}\right|^2$$

- $$\Pi(b,v) = \frac{Z^2e^6\Delta\nu}{12\pi^3\varepsilon_0^3 c^3 b^2 v^2 m^2} \qquad (7)$$

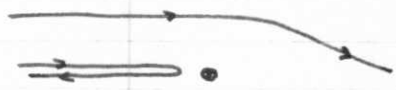
Power radiated from one collision. Must now sum over all poss. b's, v's in gas under consideration.

Summation of collision parameters.



No. of coll^{ns} for $b <$ coll. p. $< b+db$
$= v \cdot N_{ions} \cdot 2\pi b\,db$
$\therefore$ Mean power per. in $\Delta\nu$ range
$= \int \Pi(b,v)\cdot 2\pi N_i b v\,db$

$\therefore$ Mean power $= (\qquad\qquad)[\ln b] \rightarrow$ dgo. as $b \to \infty$, $b \to 0$. —— (8)



Now b_{min} was set by Δp.
$\Delta v_\perp$ cannot be $> 2v$
$b_{min} \sim Ze^2 / 2\pi\varepsilon_0 m v^2$

b_{max} set by pulse becoming comparable with $1/\nu$ looked for.
Rad^n cut off when $2b/v \sim 1/\nu$

$\rightarrow$ Mean power from one electron $= \dfrac{Z^2 e^6 N_i \Delta\nu}{6\pi^2\varepsilon_0^3 c^3 v m^2} \log\left\{\dfrac{2\pi\varepsilon_0 v^3 m}{2\nu Z e^2}\right\}$ —— (9)

Assume kinetic distrib^n: $\tfrac{1}{2}mv^2 = \tfrac{3}{2}KT$

$$v = \sqrt{\frac{3KT}{m}}$$

Mean power / unit vol. $= \dfrac{Z^2 e^6 N_i \Delta\nu N_e}{6\pi^2\varepsilon_0^3 c^3 m^2\sqrt{\frac{3KT}{m}}} \ln\left\{\dfrac{\pi\varepsilon_0 m}{Ze^2\nu}\left(\dfrac{3KT}{m}\right)^{3/2}\right\}$

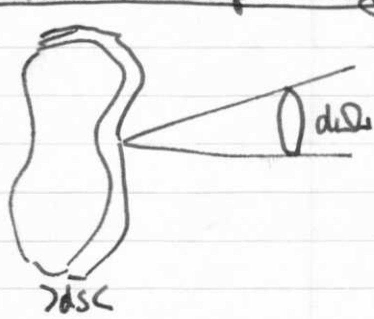
$$= \frac{Z^2 e^6 N_i N_e \Delta\nu}{(6\pi)^{3/2}\varepsilon_0^3 c^3 m\sqrt{2\pi m KT}} \ln\left\{\frac{27\pi^2\varepsilon_0^2 K^3 T^3}{Z^2 e^4 m \nu^2}\right\}$$

Ignoring scatter of electron vel^{y}s. is worst approx^n. Next best is to put in a Maxwell distrib^n & integrate over v's. Approx^n on b_{max}, b_{min} not so bad $\because$ $b_{max}/b_{min} \sim e^{20 \text{ or } 30}$.
If do proper Maxwellian calc^n, proper b_{min} & b_{max} calc^{n}s etc $\rightarrow$. —— (10)

Mean power /cc. (P) $= \dfrac{Z^2 e^6 N_i N_e \Delta\nu}{6\pi^2\varepsilon_0^3 c^3 m\sqrt{2\pi m KT}}\left\{\ln\left\{\dfrac{32K^3T^3\varepsilon_0^2}{Z^2e^4 m\nu^2}\right\} - 5\gamma\right\}$ $\gamma = .577\ldots$

ν only appears in log term only. Almost white noise. Approxns. of derivation breaks down first, before this ν-depce. becomes strong.

<u>Conversion to absorption coeff^t.</u>



Power into $d\Omega$
$=$ B.B. power into $d\Omega$ $\times$ $K\,ds$

also $= \dfrac{P \times \text{vol.}\, d\Omega}{4\pi}$, $= \dfrac{2K_BT\nu^2\Delta\nu}{c^2}$. Area. $K\,ds.\,d\Omega$

$$\therefore\ P = \frac{8\pi\nu^2 K_B T \Delta\nu K}{c^2}$$

$$\therefore K = \frac{Z^2 e^6 N_i N_e}{24\varepsilon_0^3 c\,(2\pi m K_B T)^{3/2}\nu^2}\left\{\ln\{\quad\} - 5\gamma\right\} \qquad (11)$$

$\therefore$ Absorption gets more & more important at low frequencies.

CRITICISM OF FREE-FREE DERIVATION — SCHEUER

Spectral index for thermal source ~ 0.1.

<u>Difficulties in this derivation</u>

i) Electron - electron collisions.

Ion-ion collisions neg. ∵ ions hardly move.

E-E collisions have accns = & opposite for two electrons by N3L.

Only get rad^n if there is a lag bet. 2 electron positions (fields are = & opp.).
Then must have $b \sim \lambda$. Pulse length $\sim 2b/v \sim 2\lambda/v \sim 2c/\nu v$

∴ No rad^n if $c \gg v$, ∵ of b_{max} effect.

∴ only relativistic electrons can radiate like this. Too hot for known circumstances

Refces: Panofsky & Phillips
Jackson
G. Elwert, Z. f. Nat 3A, 477 (1948)
S.F. Smerd & Westfold, P.M. <u>40</u>, 831, (1949)
P.A.G.S., M.N., <u>120</u>, 231 (1960)
L. Oster, Rev. Mod. Phys. <u>33</u>, 525. (1961)

ii) value of b_{max}

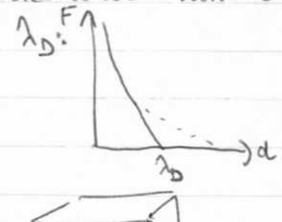
electron undeflected if passes bet. 2 protons symmetrically.

Power out $\propto$ (Acc^n of particle)^2 $= \left(\sum_{ions} \frac{e}{m} E_{ions}\right)^2$

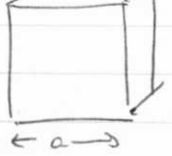
$$\left(\sum E_{ions}\right)^2 = \underset{\sim}{E}_1^2 + \underset{\sim}{E}_2^2 + \dots + \underbrace{2\underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2 \text{ etc}}_{\text{average out to zero}}$$

∴ Power out $\sim \frac{e^2}{m^2}\sum (E_{ion})^2$, i.e. sum of powers due to individual collisions. Independent of density of gas. ∴ <u>do</u> take length characterised by period of RF & electron velocity, <u>not</u> characteristic length of gas.

iii) Screening of +ve ions by average -ve e's about them. Pot^l of +ve ion is Coulombic near it, but falls off from this at larger distances. Characteristic reduction function of balance bet electrical & kinetic energies — $e^{-eV/KT}$ Coulomb potential not effective further away than λ_D:



Debye length → size of box of plasma inside which heat energy = electric energy



Na^3e: charge in the box → $\sqrt{Na^3}\,e$ fluctuations in charge

Elect/static energy $= \frac{1}{2}\frac{Q^2}{C} \doteq \frac{1}{2}\frac{Na^3e}{4\pi\varepsilon_0 a}$

Kinetic energy $\sim KT$

Biggest fluct^n allowable at temp. $T \sim KT$ ∴ put it =, $a^2 \leq \frac{8\pi\varepsilon_0 KT}{Ne^2}$

$$\lambda_D \sim \left(\frac{8\pi\varepsilon_0 KT}{Ne^2}\right)^{1/2}$$

$$\therefore \lambda_D \sim \frac{8\pi\varepsilon_0 KT}{\varepsilon_0 m \omega_p^2} \sim \frac{v_{thermal}}{\nu_p}$$

Never interested in $\nu < \nu_p$ ∵ these are evanescent in plasma. ∴ Never interested in $\lambda > \lambda_D$. Only relevant near ν_p.

iv) Power radiated from surface of refractive index $\mu = \mu^2\left(\frac{8\pi kT\nu^2 d\nu}{c^2}\right)$

Should we consider rad^n as rad^n from dipole embedded in μ? → emission($\mu\times$) absorption ($1/\mu\times$).

Or accelerated motion of one electron agitates its neighbours. But mutual effects of electrons are agreed cancelled ∵ accns. = & opp. Leaves only the protons. Should we then dig a mathematical hole round the accelerating proton & treat the rest of the medium as a cont's +ve charge? (c.f. Onsager). So far as he knows, no real resolution of ~~this~~ problem.

v) If charges are Ze, $N_i = 1/Z$, power up by $\times 2$
Charge concentrations are more effective at radiating. If they are $< \lambda_D$ in size, radiate $\nu < \nu_p$, forget them. Might have xs at ν near ν_p though ∵ of slight fluctuations with scale $\simeq \lambda_D$.

vi) Classical Mechanics.

Deflexion ϑ of electron $\sim \dfrac{\Delta p_\perp}{p} \sim \dfrac{2Ze^2}{4\pi\varepsilon_0 b v}\cdot\dfrac{1}{mv} = \dfrac{Ze^2}{4\pi\varepsilon_0 b v}\cdot\dfrac{1}{\frac{1}{2}mv^2} = \dfrac{V}{E}$

≡ to geometrical optics of de Broglie waves.
ϑ reasonably meaningful if b well-defined.

Diffraction of de Broglie wave through aperture $\sim b$.

$$\vartheta_{diff} \sim \frac{\lambda}{b} = \frac{h}{bp} = \frac{h}{bmv}$$

$$\therefore \frac{\vartheta_{diff.}}{\vartheta_{refr.}} \sim \frac{h}{\cancel{bmv}}\cdot\frac{\frac{1}{2}\cancel{m}v^2\cdot 4\pi\varepsilon_0 \cancel{bv}}{Ze^2} = \frac{1}{Z}\cdot\frac{v}{c}\left(\frac{4\pi\varepsilon_0 hc}{e^2}\right) = \frac{137v}{Zc}$$

∴ $\vartheta_{diff} \sim \vartheta_{refr}$ if $v \sim c/137$. How much intensity in diffracted beam?

$$\text{Depth of screen} \sim |\mu - 1|\cdot\frac{2b}{\lambda} = \left(1 - \sqrt{\frac{E-V}{E}}\right)\frac{2b}{\lambda} \sim \frac{1}{2}\frac{V}{E}\cdot\frac{2b}{\lambda} \quad \text{Bin}(\text{n})^n$$

$$\sim \frac{\vartheta_{ref}}{\vartheta_{diff}}$$

But just as ϑ_{diff} becomes important ∴ depth → small

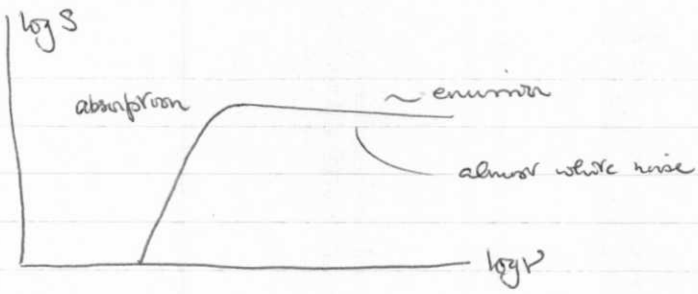
Electrons v. likely to have v's $\sim c/137$ ($T \sim 10^4$ °K)
Quantum calc^n done → b_{min} changes from $V = E$ to 1 Bohr radius.
For $T \sim 10^4$°K it works well enough for classical
$T \sim 10^6$°K (solar corona), really need quant. mech. approach.

Free-free transitions in general (not just the RF corner) Brussaard P.J. & v.de. Hulst
Rev. Mod. Phys <u>34</u>, 507 (1962)

Ready reckoner for radio astronomers.

$\tau = 1$ when $\nu \sim 20N\sqrt{L}$ Mc/s N = no. of electrons/cm^3, L = Diam of region in kpc
$T = 10^4$°K

Spectrum



SYNCHROTRON RADIATION et al.

- Cyclotron radiation. $\omega_g = eB/m$

  ω_g indepr. of v, ~ 2.8 Mc/s / gauss

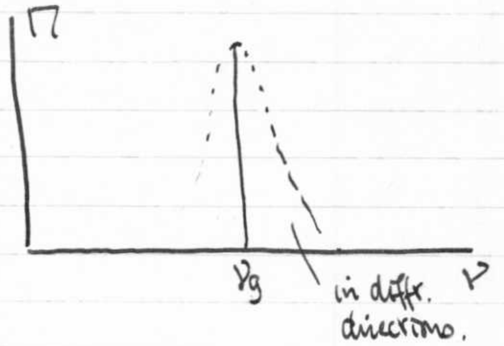
  If $v \ll c$, diameter of orbit $\ll \lambda_{\nu_g}$

  ∴ Electron radiates like 2 short dipoles $\pi/2$ out of phase.

  Hence $\Pi(\vartheta)$ $(\vartheta = \wedge B)$
  Poln.

  3 cases, $\underset{\sim}{v} \perp \underset{\sim}{B}$, $\underset{\sim}{v} \parallel \underset{\sim}{B} \neq 0$, $\underset{\sim}{v} \parallel \underset{\sim}{B} \overset{\text{not}}{\ll} c$.

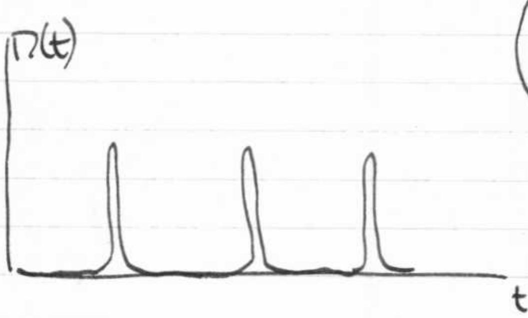
  Doppler-shifted.



Rel. trans. fields → polar peak biased forward.

Highly relativistic electrons

High field when electron nearly catches up with its own radn
∴ Nearly all radn then

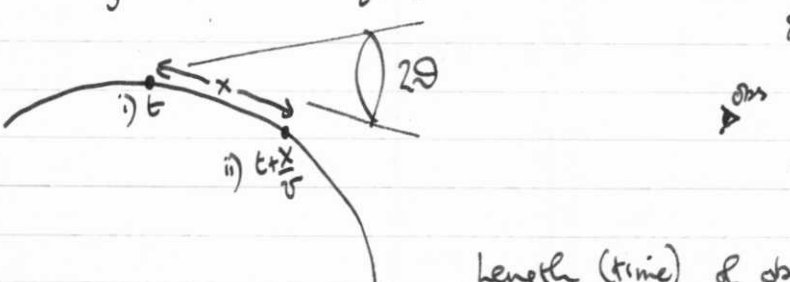


→ bursts of radn in given direction. ∴ Harmonics of the fund/l.

L.W. Pots. ~ $1/1-v_r/c$. Big when $v_r \sim c$. v_r is vely. rel to obs. ~ $v\cos\vartheta$

- Width of beam over which electron radiates $\vartheta^2 \sim 2(1-v/c) \sim (1+v/c)(1-v/c) \sim 1-v^2/c^2$
  (taking $\cos\vartheta \sim 1-\frac{1}{2}\vartheta^2$) $= 1/\beta^2$

  $\vartheta \sim 1/\beta$



i) $t + R/c$
ii) $t + x/v + R/c$

Length (time) of observed pulse?

$$\left(t + \frac{x}{v} + \frac{R-x}{c}\right) - \left(t + R/c\right) = x\left(\frac{1}{v} - \frac{1}{c}\right)$$

$X = v.\Delta t_e$ where Δt = time during which e emits towards obs

$\Delta t_e = \vartheta / (d\theta/dt)$

$$\therefore \Delta t_{observer} = \frac{\vartheta}{(d\theta/dt)}\left(1-v/c\right) = \frac{\vartheta}{\omega}\left(1-\frac{v}{c}\right) \sim \frac{1}{2\omega\beta}\left(1+v/c\right)\left(1-v/c\right)$$
$$\sim \frac{1}{2\beta^3\omega}$$

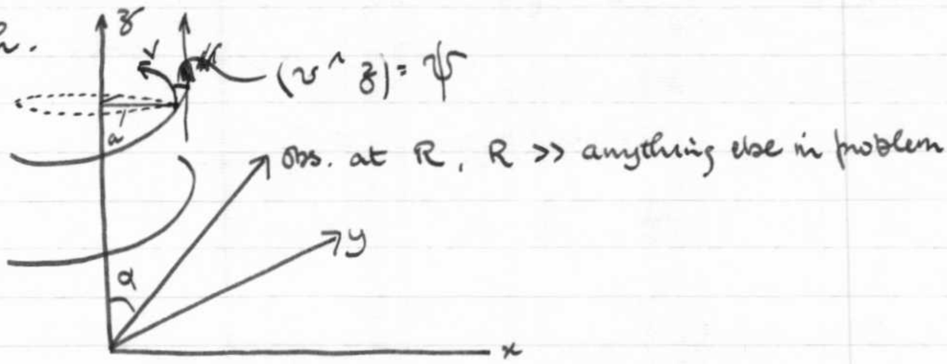
$\therefore$ Max freqcy $\sim \beta^3\omega$. Now $\omega = eB/m = \omega_g/\beta$
(1/Pulse length)

$$\downarrow$$
$$\nu_c \sim \beta^2\omega_g.$$

This is crit. freq. where synchrotron radⁿ begins to cut off.

1 Bev electron. $\beta \sim$ 1 Bev / 1 Mev ~ 1000. $\nu_c \sim 10^6\omega_g$. End of qualitative acct.

Geometry of Path.



$x = a\cos\omega t$, $y = a\sin\omega t$, $z = vt\cos\psi$ ——— (13)

$v\sin\psi = a\omega$. ——— (14)

$|r| = R - z\cos\alpha - x\sin\alpha$ ——— (15)

$\underset{\sim}{v} = \underset{\sim}{i}(-a\omega\sin\omega t) + \underset{\sim}{j}\, a\omega\cos\omega t + \underset{\sim}{k}\, v\cos\psi$

$\quad = -\underset{\sim}{i}\, v\sin\psi\sin\omega t + \underset{\sim}{j}\, v\sin\psi\cos\omega t + \underset{\sim}{k}\, v\cos\psi$

$t + \frac{r}{c} = t + R/c - \frac{v}{c}t\cos\psi\cos\alpha - \frac{a}{c}\cos\omega t\sin\alpha$

$\quad = \frac{R}{c} + t\left(1 - \frac{v}{c}\cos\psi\cos\alpha\right) - \frac{a}{c}\cos\omega t\sin\alpha$ ——— (16)

$$\left.\begin{aligned} v_y &= v\sin\psi\cos\omega t \\ v_{\perp y,R} &= v_x\cos\alpha - v_z\sin\alpha = -v\sin\psi\cos\alpha\sin\omega t - v\cos\psi\sin\alpha \\ v_R &= v_z\cos\alpha + v_x\sin\alpha = v\cos\psi\cos\alpha - v\sin\psi\sin\alpha\sin\omega t \end{aligned}\right\} \text{———} \quad (17)$$

$$\underset{\sim}{A} = \left[\frac{e\underset{\sim}{v}}{r(1-v_r/c)}\right]$$

But we only have $v(t)$, $r(t)$

want $v(t - r/c) = v\left(t - \frac{r\left(t - \frac{r\left(t - \frac{r(t-r\ldots}{c}\right)}{c}\right)}{c}\right)$

Now $[t] = t - [r]/c$

$t = [t] + [r]/c$ $\quad$ $[t]$ deps. on t.

We don't want (really) details of $\Pi(t)$ curve tho'. A frequency spectrum will do.

$a(p) = \int A(t)e^{ipt}\,dt \qquad dt = d[t] + \frac{d[r]}{c}$

$$d[r] = dr([t]) = \left[\frac{dr}{dt}\right] d[t] = -[v_r]\, d[t]$$

$$\therefore\ dt = d[t] - \frac{[v_r]}{c} d[t] \qquad dt = d[t]\left[1 - \frac{v_r}{c}\right]$$

$$\therefore \int_0^{\tau} A e^{ipr} dt = \int_0^{\tau} \frac{1}{R}\left[\frac{ev}{1 - v_r/c}\right] e^{ip[t + r/c]}\, d[t]\left[1 - \frac{v_r}{c}\right]$$

$$= \int_{1\text{ period}} \frac{1}{R}[e\underset{\sim}{v}]\, e^{ip[t + r/c]}\, d[t]$$

NOW: Everything is in $[t]$. All fns. of t are [fns.] of $[t]$.

$$\therefore \quad = \int \frac{1}{R} e\underset{\sim}{v}\, e^{ip(t + r/c)}\, dt$$ ($1/R \to$ amp. var^n, not so imp^t as phase)

Difficulty of ∞ series of $[\;]$ $\therefore$ overcome by looking for $a(p)$ cpts, not A in toto.

Observer $\quad \underset{\sim}{A} = \sum \underset{\sim}{A_n} e^{inw't}$ $\uparrow$ as seen by observer $\qquad \underset{\sim}{A_n} = \frac{1}{\tau_1}\int_0^{\tau_1} A e^{-inw't} dt$

τ' is period seen by observer.

$$\tau' \underset{\sim}{A_n} = \frac{e}{R}\int_0^{\tau} \underset{\sim}{v}\, e^{-inw'(t + r/c)}\, dt \qquad (18)$$

τ'

Don't have to change the ω' – it's just what we were after. $n\omega' = p$ above.

$$\tau' = \tau - \frac{\text{Dist. moved towards obs.}}{c} = \tau - \frac{v_y \cos\alpha}{c}$$

$$= \tau\left(1 - \frac{v}{c}\cos\psi\cos\alpha\right)$$

$$\frac{\omega'}{\omega} = \frac{1}{1 - v/c\,\cos\psi\cos\alpha} \qquad (19)$$

$$n\omega'(t + r/c) = n\omega'\frac{R}{c} + n\omega t - \frac{na\omega'}{c}\sin\alpha\cos\omega t$$

$\not$ y Pol^n: $$\tau' \underset{\sim}{A}_{y,n} = \frac{e}{R}\int_0^{\tau} v\sin\psi\cos\omega t\; e^{\frac{in\omega' R}{c}}\, e^{-in\omega t}\, e^{\frac{ina\omega'}{c}\sin\alpha\cos\omega t}\, dt$$

expand $\cos = \frac{1}{2}[e^{+} - e^{-}]$ $$= \frac{e}{2R} v\sin\psi\, e^{-\frac{in\omega' R}{c}}\left\{\int_0^{\tau} dt\, \exp\left[\frac{na\omega'}{c}\sin\alpha\cos\omega t - (n+1)\omega t\right] + \int_0^{\tau} dt.\exp[\;\cdots\; - (n-1)\omega t]\right\}$$

Put $\theta = \frac{\pi}{2} - \omega t$

$$= \frac{ev\sin\psi}{2R\omega} e^{in\omega R/c}\int_0^{2\pi} d\theta\, e^{i\{(n+1)\theta + \frac{na\omega'}{c}\sin\alpha\sin\theta\}}\, e^{\frac{1}{2}i\pi(n+1)} + \int_0^{2\pi} d\theta\, e^{i[(n-1)} \quad (n-1)$$

$$\text{cf.}\quad J_n(z) = \frac{1}{2\pi}\int_{\alpha}^{\alpha + 2\pi} e^{i(n\theta - z\sin\theta)}\, d\theta$$ Watson P.25

Now $v\sin\psi = a\omega$, $\therefore\ \tau' A_{y,n} = \frac{ea}{2R}\cdot 2\pi\, e^{\frac{in\omega' R}{c}}\left\{ i^{n+1} J_{n+1}\left(\frac{na\omega'\sin\alpha}{c}\right) + i^{n-1} J_{n-1}\left(\frac{na\omega'\sin\alpha}{c}\right)\right\}$

$$\tau' A_{y,n} = \frac{\pi e a}{R}\, i^{n-1} e^{\frac{in\omega' R}{c}} \left\{ J_{n-1}(\ ) - J_{n+1}(\ ) \right\}$$

via Watson P.45

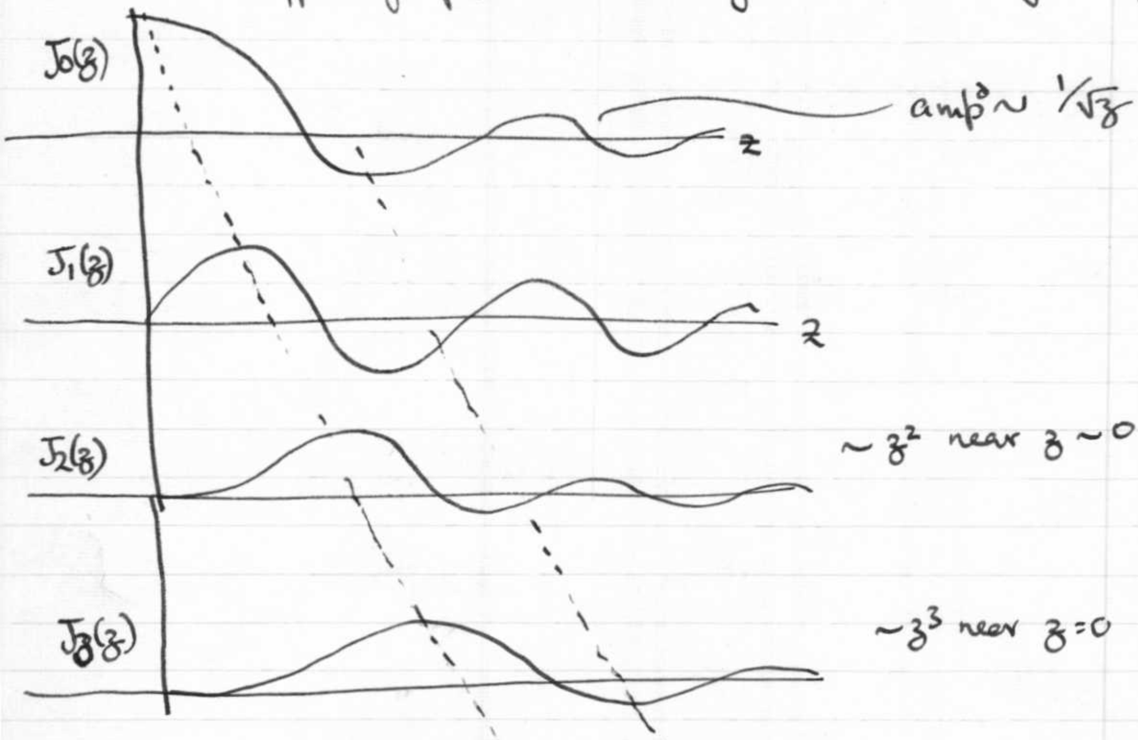
$$= \underbrace{\frac{\pi e a}{R}\, i^{n-1} e^{in\omega' R/c}}_{\text{Phase}} \left\{ 2 J_n'\left(\frac{na\omega'\sin\alpha}{c}\right)\right\} \qquad (20)$$

In other, $\perp y, r$ polⁿ:

$$\tau' A_{\perp y,r,n} = \frac{\pi e a}{R}\, i^{-n} e^{in\omega' R/c} \cdot \left[\frac{2(\cos\psi - \frac{c}{v}\cos\alpha)}{\sin\psi\sin\alpha}\right] J_n\left(\frac{na\omega'\sin\alpha}{c}\right)$$

Exact in so far as recoil bet. $h\nu$ + electron concerned negligible.

What we want is approxn. for power radiated at high n. J_n then large order, large argt. Difficult.



Now $\frac{a\omega'\sin\alpha}{c} = \frac{a\omega}{c}\sin\alpha \Big/ (1 - v/c\, \cos\psi \cos\alpha)$

$$= \frac{v}{c}\sin\psi\sin\alpha \Big/ 1 - \frac{v}{c}\cos\psi\cos\alpha$$

$$= 1 - \frac{1 - \frac{v}{c}\cos(\psi - \alpha)}{1 - v/c\,\cos\psi\cos\alpha}$$

always < 1
$v \sim c$, only just < 1

$\therefore$ As one looks at electron more or less head on, at high n $\longrightarrow$ very near the hill of $J_n(z)$, but not quite at best, then slip back down it again.

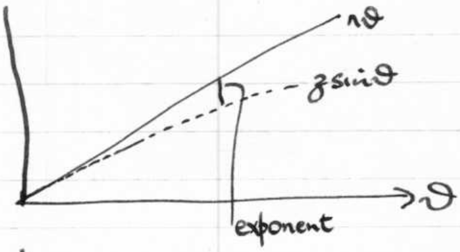
Appreciable radn. $\therefore$ only for $\psi \sim \alpha$ again.

Watson pp 248–250 tackles the approxⁿ. desired.

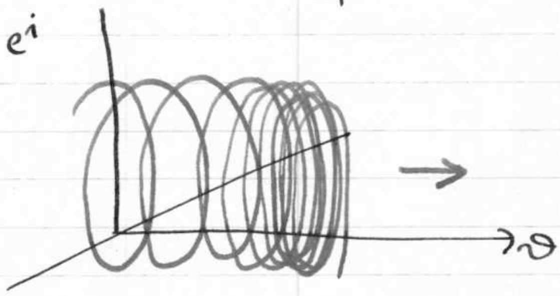
$J_n(z) = \frac{1}{2\pi}\int_{\alpha}^{\alpha+2\pi} e^{i(n\theta - z\sin\theta)}\,d\theta$ $\alpha = -\pi$ for convenience h/finite

$(n-z) \ll n, z$ $n, z \gg 1$ now.

$n\theta - z\sin\theta = (n-z)\theta + \frac{1}{6}z\theta^3$



Expr first linear
then cubic
then quintic, etc, as various
θ powers take over



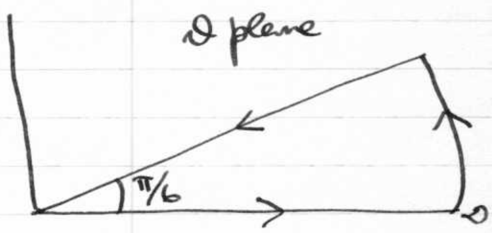
Rate of winding of spiral gets enormous

$J_n(z)$ ∴ → asympt. $\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i(n-z)\theta}\, e^{i\frac{1}{6}z\theta^3}\,d\theta$

doesn't matter which high power of θ we use to describe crushing together of the spiral.

$= \frac{2}{2\pi}\int_0^\infty \cos\left[(n-z)\theta + \frac{1}{6}z\theta^3\right]d\theta$

$= \frac{1}{\pi}\Re\int_0^\infty e^{i(n-z)\theta}\, e^{i\cdot\frac{1}{6}z\theta^3}\,d\theta$



No poles inside this path, ∴ $\int$ around it = 0.
Curved path integral → 0
∴ Can substitute $\theta = \vartheta e^{\frac{1}{6}\pi i}$

$= \frac{1}{\pi}\Re\int_0^\infty e^{i(n-z)e^{\frac{1}{6}\pi i}\cdot\vartheta - \frac{1}{6}z\vartheta^3}\, d(e^{\frac{1}{6}\pi i}\vartheta)$

Expand in terms of $\int\theta^n e^{-\frac{1}{6}z\theta^3}d\theta \to \int x^{n/3} e^{-\frac{1}{6}zx}\,d(x^{1/3})$
factorials

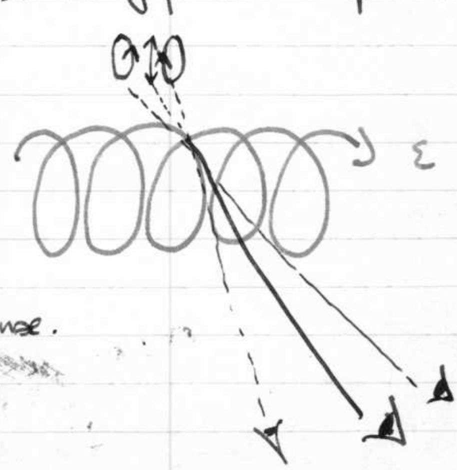
Then $J_n(z)$ $(n-z) \ll n, z$, $n, z \gg 1$

$\to \frac{1}{\pi}\left\{\frac{2(n-z)}{3z}\right\}^{1/2} K_{\frac{1}{3}}\left(\frac{2^{3/2}(n-z)^{3/2}}{3z^{1/2}}\right)$ $z < n$.

Poln. 2 A cpts have i differ. bet them — 90° out of phase – elliptical polⁿ

Near $\psi = \alpha$, amp^d. of ⊥ y, r changes sign.

∴ The radn is only y-polⁿ for observer
exactly in line



On each side of $\psi = \alpha$, elliptical polⁿ in differ. sense.

Mean power in unit $\delta\omega$ $= \varepsilon_0 k^2 |\overline{A_\perp^2}| \times \text{area} \times \dfrac{2\pi}{\omega'}$

$$= \frac{2\pi}{\omega'} \varepsilon_0 \frac{n^2\omega'^2}{c^2} \int |\overline{A_\perp}|^2 \, 2\pi R \sin\alpha \, R\, d\alpha$$

In principle can now do the $\int$ with the fns. on prev. page. Done in Leroux !!%*!

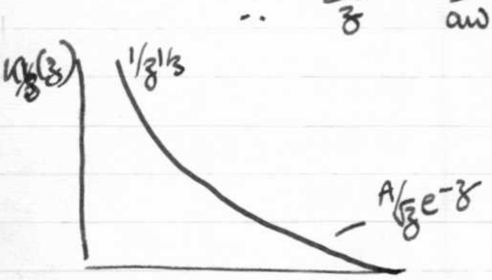
γ-shape comes purely from: $J_n^2\left(\dfrac{na\omega' \sin\alpha}{c}\right)$

$$n - z = n - \frac{na\omega'\sin\alpha}{c} = \frac{p}{\omega'} - \frac{pa}{c}\sin\alpha \qquad (p = n\omega')$$

$$= \frac{p}{\omega}\left(1 - \frac{v}{c}\cos\psi \sin\alpha - \frac{a\omega}{c}\sin\alpha\right)$$

$$= \frac{p}{\omega}\left(1 - \frac{v}{c}\cos(\psi - \alpha)\right)$$

$$\therefore \frac{n-z}{z} = \frac{c}{a\omega}\left(1 - \frac{v}{c}\cos(\psi-\alpha)\right) \qquad \because z = \frac{pa}{c}\sin\alpha$$



$$\therefore \operatorname{Arg}(K_{\frac{1}{3}}) = \frac{2^{3/2}}{3} \frac{p}{\omega}\left(1 - \frac{v}{c}\cos(\psi-\alpha)\right)\left(\frac{c}{a\omega}\left[1 - \frac{v}{c}\cos(\psi-\alpha)\right]\right)^{1/2}$$

At $\psi = \alpha$ $\quad = \dfrac{p}{3\omega} \cdot 2\left(1 - v/c\right)^{3/2} \sqrt{\dfrac{c}{a\omega}}$

$$\doteqdot \frac{p}{3\omega}\sqrt{\frac{c}{v\sin\psi}}\left(1 - v^2/c^2\right)^{3/2}$$

$$= \frac{p}{3\omega}\sqrt{\frac{c}{v\sin\psi}} \cdot \frac{1}{\beta^3}$$

$= 1$ then $p = 3\omega\sqrt{\dfrac{v\sin\psi}{c}}\,\beta^3 \sim 3\omega\sqrt{\sin\psi}\,\beta^3$

$\to$ again $p_{cr} \sim \omega\beta^3$ as on $\sim$ 4 pages earlier.

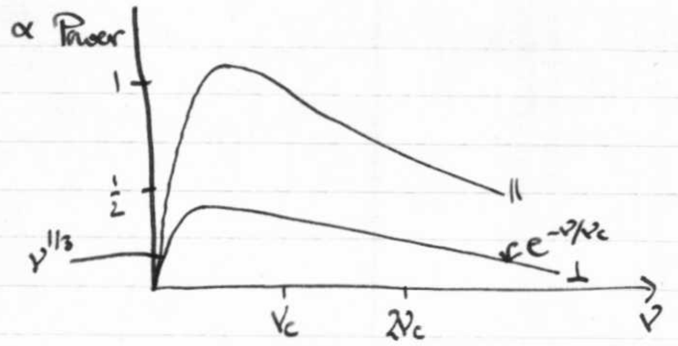
Leroux $\nu_c \sim \frac{1}{2\pi}\omega\beta^3\sin\psi = \beta^2 \nu_g \sin\alpha$ $\qquad$ ν_g = cyclotron freq.^{n} for non-rel. electron.

$n_c = (\beta\sin\alpha)^3$ (Order of harmonic depends on α)

Rad^n polarised $\perp r_z$.

$$\text{Power from 1 electron/Hz} = \frac{\varepsilon_0 e^2 \omega}{4\pi}\frac{\beta\sin\alpha}{\sqrt{3}}\left(\frac{\nu}{\nu_c}\right)\left\{K_{\frac{2}{3}}\left(\frac{2\nu}{3\nu_c}\right) - \int_{\frac{2\nu}{3\nu_c}}^{\infty} K_{\frac{1}{3}}(x)\,dx\right\}$$

$$\sim \frac{\varepsilon_0 e^2}{2}\nu_g \sin\alpha \begin{cases} 0.8\left(\nu/\nu_c\right)^{1/3} & \nu \ll \nu_c \\ 0.9\left(\nu_c/\nu\right)^{1/2} e^{-2\nu/3\nu_c} & \nu \gg \nu_c \end{cases}$$



On log scale, LH end more drawn out. Decline at LF really quite slow for radio astronomy!

$\leftarrow \perp z, \parallel z$

- For LF, Power $\sim 0.4 Z_0 e^2 (\nu_g \sin\alpha)^{2/3} \beta^{-2/3} \nu^{1/3}$

<u>z Poln</u> Power / electron / Hz $= \frac{Z_0 e^2 \omega}{4\pi} \frac{\beta \sin\alpha}{\sqrt{3}} \left(\frac{\nu}{\nu_c}\right) \left\{ K_{\frac{2}{3}}\left(\frac{2\nu}{3\nu_c}\right) - \int_{2\nu/3\nu_c}^{\infty} K_{\frac{1}{3}}(x)\,dx \right\}$

$$= \frac{Z_0 e^2}{2} \nu_g \sin\alpha \begin{cases} 2.4 (\nu/\nu_c)^{1/3}, & \nu \ll \nu_c \\ 1.78 (\nu/\nu_c)^{1/2} e^{-\frac{2\nu}{3\nu_c}}, & \nu \gg \nu_c \end{cases}$$

- At LF again, simplifies to $0.4 Z_0 e^2 (\nu_g \sin\alpha)^{2/3} \beta^{-2/3} \nu^{1/3} \times 3$

$\therefore$ At LF ratio of powers in 2 pol^n = 3:1.
HF extra ratio of ν between 2 polns.
Integrated over all ν's, $\because$ of this ν-depce, pol^n power ratio is 7:1.

Degradation rate? How fast does electron radiate over all ν's?
Approximate: use LF approx^n up to ν_c, but power = 0 after. (log scale remember)

Total radiated power $\perp z$ $= 0.4 Z_0 e^2 (\nu_g \sin\alpha)^{2/3} \beta^{-2/3} \int_0^{\nu_c} \nu^{1/3}\,d\nu$

$$= 0.4 Z_0 e^2 (\nu_g \sin\alpha)^{2/3} \beta^{-2/3} \cdot \frac{3}{4} \nu_c^{4/3}$$

- $$= 0.3 Z_0 e^2 (\nu_g \sin\alpha)^{2/3} \beta^{-2/3} \cdot \beta^{8/3} (\nu_g \sin\alpha)^{4/3}$$

$$= 0.3 Z_0 e^2 (\nu_g \sin\alpha)^2 \beta^2$$

Done better (via the K-approxns., $\rightarrow$ 0.33 instead of 0.3)
Other pol^n: $\rightarrow$ 0.9, done better $\rightarrow$ 7/3.

- $\therefore$ Total power radiated $= \frac{8}{3} Z_0 e^2 (\nu_g \sin\alpha)^2 \beta^2$

$\left.\begin{array}{l} \beta \propto \text{Energy} \\ \nu_g = eB/m. \end{array}\right]$ remember

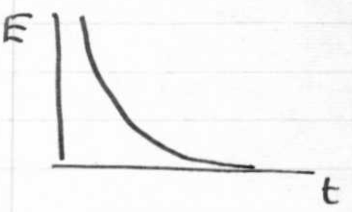
$$\frac{dE}{dt} = -\frac{8}{3} Z_0 e^2 (\nu_g \sin\alpha)^2 \beta^2 = -\frac{8}{3} Z_0 e^2 (\nu_g \sin\alpha)^2 \frac{E^2}{(m_0 c^2)^2}$$

$$\frac{dE}{E^2} = -\frac{8}{3} Z_0 e^2 (\nu_g \sin\alpha)^2 \frac{dt}{(m_0 c^2)^2}$$

$$\left[\frac{1}{E}\right] \cancel{\frac{dE}{E^2}} = +\frac{8}{3} Z_0 e^2 (\nu_g \sin\alpha)^2 \frac{t}{(m_0 c^2)^2}$$

If original E large ($E(0)$) → $\frac{1}{E} = \frac{8}{3}\frac{Z_0 e^2 (\nu_g \sin\alpha)^2 t}{m_0^2 c^4}$

$t \sim 1/B^2 E$ therefore.



All o.k. if B, α do not change during degradation.
Can't say anything about B, ∴ assume it's constant.
As all radn → dirn. of motion of electron (roughly), radn. reaction ~ straight back on electron, ∴ reasonably plausible that α constant during degradation.

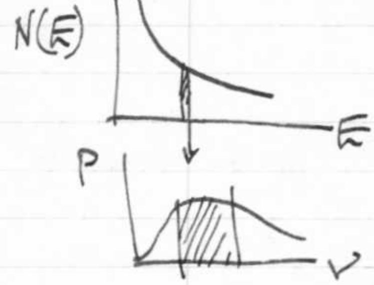
Energy distribn. $N(E)\,dE = K E^{-(2\alpha+1)} dE$ Say (!*!%!!!?!)

↓

Spectrum of emitted radn. Assume radn. of $\nu_c \to 2\nu_c$ produced according to relevant E

$$\nu_c = \frac{E^2}{(m_0c^2)^2}\nu_g \sin\alpha$$

$$d\nu_c = \frac{2E\,dE\,\nu_g \sin\alpha}{(m_0c^2)^2}$$



∴ $N(\nu_c)$ emitting $(\nu_c, \nu_c + d\nu_c)$
$$= \tfrac{1}{2} K E^{-(2\alpha+2)} 2E\,dE$$
$$= \tfrac{1}{2} K \left(\frac{\nu_c (m_0c^2)^2}{\nu_g \sin\alpha}\right)^{-(\alpha+1)} (m_0c^2)^2 \nu_g \sin\alpha\, d\nu_c$$
$$= \tfrac{1}{2} K (m_0c^2)^{-2\alpha} (\nu_g \sin\alpha)^{\alpha+2} \nu_c^{-(\alpha+1)} d\nu_c$$

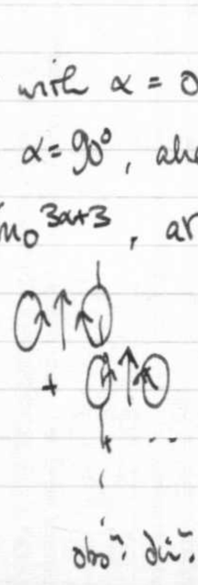
Remove the c's from ν_c ∵ take $\nu_c \sim \nu$.

Power radiated $P(\nu)$ in $(\nu, \nu+d\nu)$
$$= \tfrac{8}{3} Z_0 e^2 (\nu_g \sin\alpha)^2 \beta^2 N(\nu)$$
$$= \tfrac{8}{3} Z_0 e^2 \frac{\nu (m_0c^2)^2}{\nu_g \sin\alpha} \cdot \tfrac{1}{2} K (m_0c^2)^{-2\alpha-2} (\nu_g \sin\alpha)^{\alpha+4} \nu^{-\alpha-1} d\nu$$
$$= \tfrac{4}{3} Z_0 e^2 K (m_0c^2)^{-2\alpha} (\nu_g \sin\alpha)^{\alpha+3} \nu^{-\alpha} d\nu$$

Power / hz depends on ⟶ $\frac{1}{m_0^{3\alpha+3}}$ $B^{\alpha+3}$ $(\sin\psi)^{\alpha+3}$ $\nu^{-\alpha}$

∴ If field ½ed → 1/>10 in power with $\alpha = 0.7$
Sharp polar diagram 60° off $\alpha = 90°$, already 1/>10 down also!
No radn. from protons, ∵ $\sim 1/m_0^{3\alpha+3}$, at same energies.

If a distribn. of ψ present



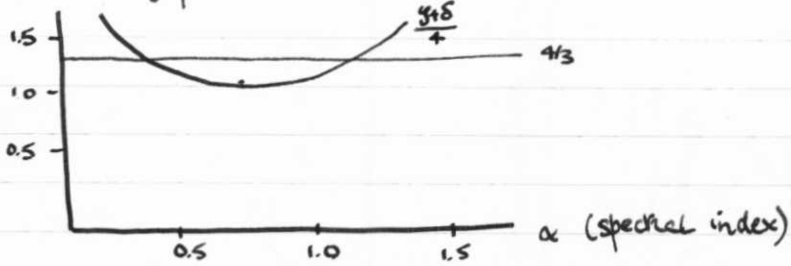
→ ↑ only. in a given dirn.

/ Lenoux' formulae differ by replacing $4/3$ by $\xi(2\alpha+1)$ for $\parallel_{\gamma,r}$

$\frac{1}{4}\delta(2\alpha+1)$ for $\perp_{\gamma,r}$

where $\delta(2\alpha+1) = \frac{2}{\sqrt{3}} \cdot \frac{1}{(3\alpha+3)} \int_0^\infty x^\alpha K_{\frac{2}{3}}\left(\frac{2x}{3}\right) dx$ and $\xi(2\alpha+1) = (3\alpha+4)\,\delta$

Then ratio of pol^{ns} is $3\alpha+4 : 1$ & $4/3 \rightarrow (\xi+\delta)/4$



Total power radiated = rate of loss of energy

$$\frac{dE}{dt} = -\frac{8}{3} r_0 e^2 (\nu_g \sin\psi)^2 \beta^2$$

$$m_0 c^2 \frac{d\beta}{dt} \longrightarrow \quad \therefore \frac{d\beta}{\beta^2} = (\qquad\qquad) dt$$

$$\frac{1}{\beta} - \frac{1}{\beta_{init}} = (\qquad\qquad) \tau$$

After time τ, $\beta_{max} = 1/(\quad)\tau$, E_{max} (Bev) $= 10^{10} / B^2 \sin^2\psi\, \tau$ (B: μgauss, τ: yrs.)

$\therefore$ Max. ν_c which ε's can have after τ, $(\nu_c)_{max} = 10^9 / B^3 \sin^3\psi\, \tau^2$ Mc/s (τ: Myrs.)

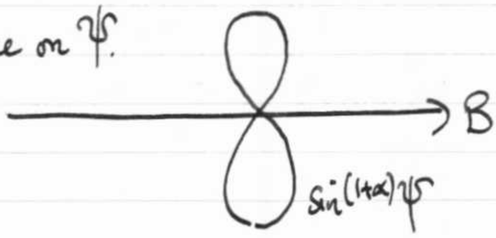
$(\nu_c)_{max}$ will have fallen to 10^3 Mc/s if i) no new supply of fast particles

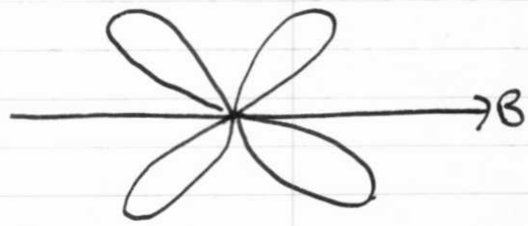
ii) $B \sim 10^{-6}$ gauss in $\tau = 10^9$ yrs

$B \sim 10^{-5}$ 3.10^7

$B \sim 10^{-4}$ 10^6

Dependence on ψ.





Isotropic distrib^n of ε's. Aged $\sin\psi = \left(\frac{10^9}{B^3_{\mu g}\tau^2\nu}\right)$

(Provided no restoration of isotropy by perturb^{ns})

<u>Crab.</u> Optical synchr. $\tau \sim 10^3$ yrs, $\nu_c > 10^{15}$ c/s, $\rightarrow B < 10^{-4}$ gauss.

To show that $\exists$ min. energy reqt. on B & ε for given flux.

Suppose source has uniform field B, electrons $N(E) = KE^{-(2\alpha+1)}$ $E > E_0$ (to prevent dgce)

Over vol. V, total no. of ε's $= KV\int_{E_0}^{\infty} E^{-(2\alpha+1)}\,dE$

$$= \frac{KV}{2\alpha}E_0^{-2\alpha} = N_{\text{cosm. ray. e}}$$

Electron energy $u_p = KV\int_{E_0}^{\infty} E.E^{-(2\alpha+1)}\,dE = \frac{KV}{2\alpha+1}E^{-(2\alpha+1)}$

Field energy $u_f = \frac{1}{2}\cdot\frac{1}{\mu_0}B^2 V$

Power $P(\nu)$ in $(\nu, \nu+d\nu)$ $= VK(m_0c^2)^{-2\alpha}\left(\frac{e}{2\pi m_0}\right)^{\alpha+1} z_0 e^2 (\sin\psi)^{\alpha+1} B^{\alpha+1}\nu^{-\alpha}\,d\nu$

$$= \text{const.}\ u_p.u_f^{(\alpha+1/2)}$$

Put $u = u_p + u_f$, $P = \text{const.}\ (u-u_f)u_f^{(\alpha+1/2)}$

$$\left(\frac{dP}{du_f}\right)_u = 0 \quad \text{when} \quad u_f = \frac{\alpha+1}{\alpha+3}u.$$

This gives max total power for given total energy, sim. min total energy for given power.
For $\alpha = 1$, $u_f = \frac{1}{2}u$, $u_f \sim u_p$.

$$\frac{P}{\nu(\sin\psi)^{\alpha+1}} = \frac{1+\alpha}{3+\alpha}\cdot\frac{1}{2\pi^2}\cdot\frac{e^2}{E_0 m_0 c^2}\cdot\left(\frac{u}{m_0 V\nu^2}\right)^{\frac{\alpha+1}{2}} u\left(\frac{4\alpha-2}{3+\alpha}\right)\cdot\frac{4}{3c}\cdot\frac{e^2}{E_0 m_0 c^2}\beta^{2\alpha+1}$$

i.e $P \sim u^{(\alpha+3/2)}$

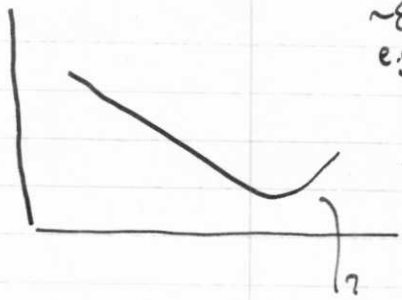
$\therefore$ Varn. in u between diffr. sources < varn. in P.

$$P \sim V^{-(\alpha+1)/2}$$

	α	V(cc.)	H(gauss) for u_{min}	u (ergs)
3C37	$-.7$	4.10^{69}	3.10^{-5}	3.10^{59}
3C40	$-.85$	10^{68}	4.10^{-5}	10^{58}
3C71	$-.25$	$<6.4\times10^{64}$	$>9.10^{-5}$	$<4.10^{55}$
Herc A	$-.9$	3.10^{70}	5.10^{-5}	5.10^{60}
3C295	$-.5$	5.10^{68}	2.10^{-4}	10^{60}

Spectral Types

(i)



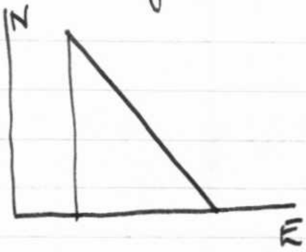
~8 sources
e.g. NGC 1275
(3C84)

van der Laan - ageing?

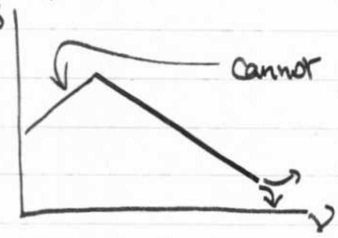
Average spectral index 0.75 ± ~ .2.
Total flux $\int S(\nu)\,d\nu \rightarrow \left[\frac{\nu^{1-\alpha}}{1-\alpha}\right]$

Does not diverge as $\nu \to 0$, altho' we saw that E_T does $[N(E) \to \infty$ as $E \to 0]$. Total energy of electrons < as we saw from assumed energy spectrum.

∴ Put in

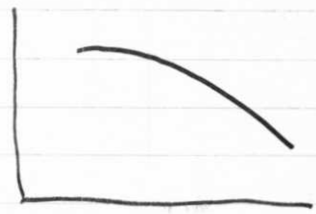


→



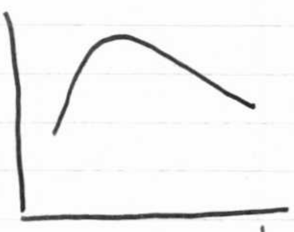
Cannot go as ~~>~~ $\alpha = \frac{1}{3}$.

Really expect smoother varn. than this. Observed spectra go as :



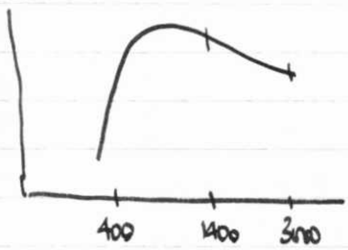
for v. large no. of sources.

3C 147



Low-ν fall-off faster than $\nu^{1/3}$.
∴ Something else must be happening.

Source observed at Parkes



Low-ν goes as ν^3 !!!

i) Sharp cut-off might be due to hydrogen abs? $\nu_{F\text{-}F} \sim 20 n\sqrt{L}$.
Are these sources inside associated visual regions, instead of the usual ◐ · ◐ ?
Known in many cases that dimensions ~ Galaxy. Hard to imagine that there is much HII outside galaxies. (note: Clusters of Galaxies & stability. Vely. dispersion > escape vely. calc. if mass is entirely in galaxies themselves. ∴ either clusters are unstable, or there is a lot of extra mass. Possibly also arguable from depolarisation of sources)

ii) $T_b^{\text{surf.}} <$ as $-(\alpha+2)$ if the source stays same size in sky. Particles doing the radiating have gyrofreq. energy $m_0c^2\left(\frac{\nu}{\nu_g}\right)^{1/2} \equiv 3kT$ for non-non relativistic electrons (no dens: ≡ part:)

Cannot have $T_b > T$ of above on th/dynamic grounds. What happens if $T_b > \frac{m_0c^2}{3k}\left(\frac{\nu}{\nu_g}\right)^{\frac{1}{2}}$?
Then source becomes optically thick to the synchrotron radn. itself.

Expect $\nu_{\text{cut off}}$ to be given by $T_b(\text{High }\nu) = \frac{m_0c^2}{3k}\left(\frac{\nu_{co}}{\nu_g}\right)^{1/2}$

$$S = \frac{2k}{\lambda^2}\int T_b\,d\Omega = \frac{2kT_b}{\lambda^2}\Omega \text{ for uniform } T_b \sim \frac{2kT_b\vartheta^2}{\lambda^2}$$

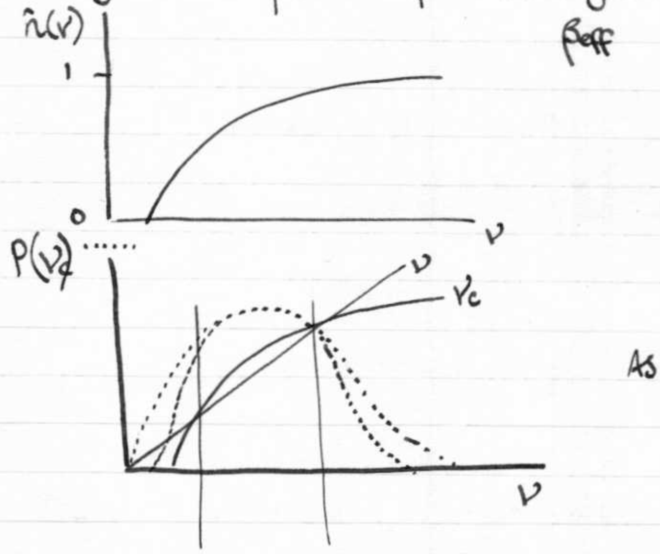
$$T_b = \frac{c^2 S}{\nu^2 2k\theta^2} = \frac{m_0c^2}{3k}\left(\frac{\nu}{\nu_g}\right)^{\frac{1}{2}}$$

$$= \frac{c^2 S_0 \nu_0^{\alpha} \nu_a^{-\alpha-2}}{2k\theta^2}$$

Then $\nu_{c.o}^{2\frac{1}{2}+\alpha} = A(c, m_0, c) S_0 \nu_0^{\alpha} \frac{\sqrt{B}}{\theta^2}$

Then for small θ, high $S_0 \rightarrow$ high $\nu_{c.o.}$ Bright, small sources.
See Williams, Nature

iii) Does synchrotron process stop in dense gas? Perhaps.



cut-off outside these lts. (cf. vacuo.)

As soon as $\nu_c > \nu_{observations}$, get radiation $\nu > \nu_c$, rad^n less than in vacuo.

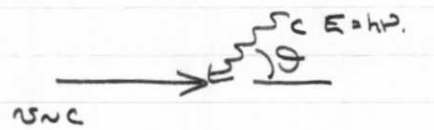
At what $O(\nu)$ is this significant?

$\frac{1}{2}\nu_p^2/\nu^2 \sim 1 - \frac{v}{c} \sim \frac{1}{2} m_0c^2/E^2$ $\left[|v - c_0| \sim |c_0 - c|\right]$

$\sim \frac{1}{2} \nu_g/\nu.$

$\nu \sim \nu_p^2/\nu_g.$

I.C.E.



What is ν of incoming photon as seen by O on electron?
To stationary obs., speed of wave crests is $c\sec\theta$.

Relative speed of electron & w. crests $= v + c\sec\theta$

$$\Delta t = \frac{\lambda\sec\theta}{v + c\sec\theta} = \frac{\lambda}{c + v\cos\theta}$$

Successive crests appear at $x = 0, t = 0$

$$t = \lambda/c + v\cos\theta, \quad x = v\lambda/c + v\cos\theta$$

These are successive crests as seen by O′ on electron, but time-interval will be diff.

i) at $t' = 0$

ii) at $t' = \beta[t - vx/c^2] = \dfrac{\lambda}{c + v\cos\theta}\cdot\dfrac{1}{\beta}$

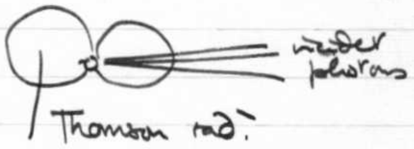
$\therefore$ ν' seen by obs. moving with electron $= \beta\nu\left(1 + \frac{v}{c}\cos\theta\right)$

For relativistic electron $v \sim c$, $\quad \nu' \sim \beta\nu(1 + v/c)$

For $\beta \sim 1000$, something looking like light to stationary obs. looks like soft X ray to 1 BeV electron.

Then do C.E. calc.^{n} Photon scattered from e by waggling e in its $\underset{\sim}{E}$ → dipole rad.^{n} from electron → probability of photon coming off in diff. directions & the cross-section. Must then transform back to the stationary. O- co-ordinates.

As seen from electron, almost all photons are straight-on. →



In going back, we get about another factor of β

Important thing is that narrow pencil of directions of photons is transformed into "random" re-rad.^{n} & we don't just invert the transformation when we go back. ∴ Get β^2 not 1.

$$\sigma = \frac{8\pi}{3}r_0^2 = 6.57 \times 10^{-25}$$

Unaffected by Lorentz ~~[illegible]~~ ∵ ⊥^{r} dim^s do not change on transf.^{n}

Electron virtually behaves like blob of this area then, losing $(\beta^2-1)h\nu$ to each $h\nu$ that this blob "hits". $\sim \beta^2 h\nu$

$$-\frac{\partial E}{\partial x} = \sigma N_{h\nu}\beta^2 h\nu = \sigma\beta^2 U_{light}$$

$$-\frac{\partial E}{\partial t} \text{ thru } c \rightarrow 1.97\times10^{-14}\, U_{light}\beta^2 \text{ cgs.}$$

Synchrotron $-\frac{\partial E}{\partial t} = \frac{8}{3} r_0 e^2 (H \sin\alpha)^2 \beta^2 = 10^{-13}\, U_{mag}\beta^2$ cgs.

Which is more important depends on U_{light} or U_{mag} v. much bigger.
Can think of Compt. scatt. as interaction of electron with $\underset{\sim}{H}$ of $h\nu$.

In 273 B U_{light} so great that fast electron cannot get thru it at all. Not a very important property of the average radio source tho. Could → γ ray flux from something like the halo by upgrading the light? Exercise: Calculate γ flux from i) halo, ii) 3C 273 B

Felton & Morrison P.R.Letts 10, 453 (1963) (See Hoyle lectures also).
Starlight + rel elect. density in halo from radio → X ray output of halo. F_h
cf. other places if assume densities.

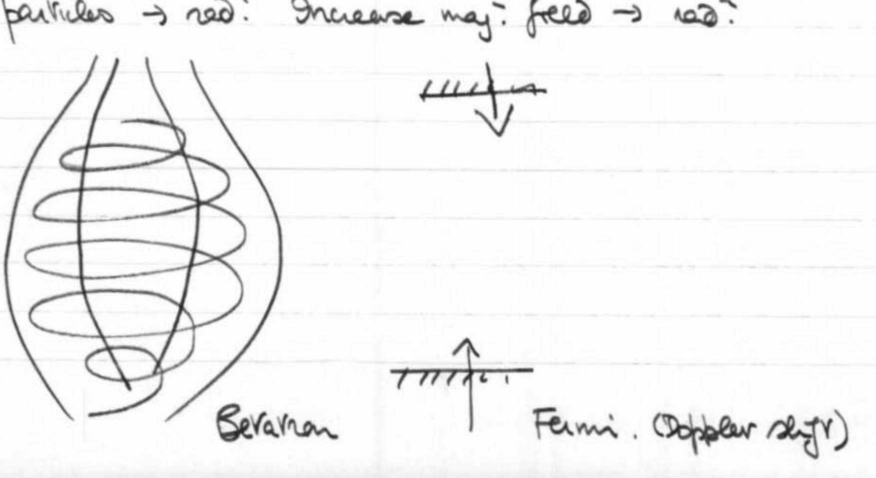
	N_e relative to halo	o/p of X ray
Halo	1	F_h
Halos of all external galaxies	0.5	$\sim 0.1\, F_h$
Local cluster	0.1	$\sim F_h$
All clusters in Universe	0.1	$6 F_h$
All space	1	$3\times10^4 F_h$
Observed X ray flux (roughly)	→	$300\, F_h$

→ upper limit on N_e of all space (universal rather than trapped). ∴ Fast electrons must be trapped, more or less, in the galaxies.

If more than $1/\beta^2$ of photons scattered by high enough density of particles, it snowballs. Just a little way short of this in the q.s.s's.

The Road to Spectra?
Have enough parameters to get anything you want.

Fast particles + fields.
Particles made in some factory which we don't understand. Squash particles + field, accelerate particles → rad? Increase mag. field → rad?



Suppose we have i) Source of low energy particles only
Exponential fall of S with ν $\quad \alpha = \infty$.

ii) All high energy particles, $\alpha = -1/3$
Rising spectrum.

iii) Steady state models. Almost certainly untrue, but regard as limiting cases of ?? sources perhaps. Suppose Energy loss $\propto E^x$

$$-\frac{\partial E}{\partial t} = cE^x \qquad \text{ICE } x = 2 \quad \text{Synch}$$

① No loss. Young source. Suppose factory $N \propto E^{-\delta}$ $\quad \alpha = \delta - 1/2$

② Loss by escape. Escape indep. of E. $\quad \alpha : \delta - 1/2$

Could easily have escape $\propto E$, dep. on size of Larmor radius for ex., high-E escape more.

③ $x = x$. Time spent in $dE \rightarrow$ spectrum of source.

$$= \frac{dE}{cE^x}$$

Particle then radiates at $\nu \sim \nu_c (\sim c'E^2)$ at rate $\sim E^2$. $= c''E^2$

$\therefore$ Energy radiated in $d\nu$ by 1 electron $= c''E^2 . dt_m(dE)$.

$$= c''E^2 . \frac{dE}{cE^x} = \frac{c''E^2}{cE^x} \frac{d\nu}{2c'E}$$

$$= c^* E^{(1-x)} d\nu$$

Now $E \sim \sqrt{\nu_c}$, $\qquad \therefore$ E rad in $d\nu$. $\qquad = c^{\sqrt{*}} \nu^{(1-x)/2} d\nu$

$\therefore$ Total power radiated in $d\nu$ is propl. to $N(> E^2 = \nu/c')$, $\quad \propto \int_{\sqrt{\nu/c'}}^{\infty} E^{-\delta} dE . c^{\sqrt{*}} \nu^{(1-x)/2} d\nu$

$$\propto \nu^{(1-\delta)/2} \nu^{(1-x)/2} d\nu$$

$$\propto \nu^{-(\delta + x - 1)/2}$$