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A MODEL FOR COLLIMATION OF SUPERSONIC BEAMS IN EXTRAGALACTIC RADIO SOURCES

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ABSTRACT

This paper describes a model for the dynamics of "heavy" supersonic beams in extragalactic radio sources and explores its consequences for interpreting data on the collimation properties of radio jets in galaxies and quasars. The model presumes that non-relativistic supersonic beams of hot fluid originate from the active central regions of radio galaxies and quasars, and describes the supersonic dynamics of these beams in a self-consistent fashion. In contrast with previous work on this problem, the beam cross-section is solved for dynamically, considering the effects on the transverse expansion of the beam of its internal pressure, of an external confining pressure, and of self-consistent magnetic fields in the flow.

Sections 1 and 2 of the paper review the radio-astronomical context of the problem. Sections 3 and 4 outline the fundamental principles of the model and discuss theoretical and observational constraints on the solutions to its basic equations. Section 5 gives graphical illustrations of the variety of collimation behaviour that is consistent with the precepts of the model, and identifies the dominant physical processes associated with each behaviour. Section 6 shows how the model may be used to interpret the observed behaviour of the radio jet in the galaxy NGC 315. Section 7 summarises the main physical results of the paper and Section 8 suggests directions for future work.

1. Introduction. A long-standing problem in extragalactic astrophysics concerns the origin and preservation of the collimation of the structures of large-scale (> few kpc) extragalactic radio sources. By collimation we

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mean that the major emitting regions of the sources subtend angles typically $\ll 20°$ at the centres of the associated optical galaxies or quasars. The two components of the characteristic double-lobed structure of powerful extragalactic radio sources cannot therefore be freely-expanding "plasmons" containing only the relativistic particles and magnetic fields responsible for their synchrotron emission – the sound speed in relativistic plasmons would be $c/\sqrt{3}$ while their velocity of ejection from the central object must be $<c$, so they would generally subtend angles $>\arctan(1/\sqrt{3})$, i.e. $>30°$, at the central objects.

The collimation and confinement problem was exacerbated by the discovery (Miley and Wade 1971; Hargrave and Ryle 1976; Readhead and Hewish 1976) that the radio lobes of many of the more powerful extragalactic sources contain bright "hot spots" of enhanced emission that are much smaller than the lobes themselves. The median angle subtended at the central object in a large sample of such "hot spots" is of order $0\overset{\circ}{.}6$ (Spangler 1979a). The presence of such compact substructure puts severe constraints on acceptable models for the confinement of the sources (see, for example, the review by Miley (1980)).

The luminosity function of extragalactic sources is inconsistent with the strong luminosity evolution that would be associated with adiabatic expansion of plasmons from the scales of the radio sources associated with the central objects to the sizes of the radio lobes, suggesting that such expansion has not, in fact, occurred (Longair *et al.* 1973). Studies of spectral-index distributions over the extended sources (Jenkins and Scheuer 1976; Willis and Strom 1978; Burch 1979a; Strom and Willis 1980) have also shown that many sources lack the spectral-index gradients that would be produced by synchrotron losses in ageing populations of relativistic particles. Instead, the sources present the appearance of having fresh relativistic particles continuously injected into their radio lobes, possibly at or around the "hot spots".

Seminal ideas about the mechanisms for source collimation and for efficient and continuous delivery of the relativistic particles to the lobes have come from the work of Martin Rees (Rees 1971; Blandford and Rees 1974, 1978; Rees 1976). His basic proposal is that the active central objects in radio galaxies and quasars can emit directed supersonic flows (beams) of hot fluid containing relativistic particles (with or without associated magnetic fields). The directivity of the beams is presumed to arise either by confinement of an initially isotropic flow by the pressure of a flattened distribution of gas around the nuclear object (leading to the formation of twin "nozzles" at which the flows on each side of the object become supersonic) or, in an early variant of his model, by electromagnetic self-focussing. Once collimated near the central object, the proposed beams propagate with minimal expansion and dissipation towards the radio lobes. The "hot spots" in the lobes are interpreted as "working surfaces" at which the highly directive flows of fluid in the beams become disordered at an interface with shocked intergalactic material, so that a significant fraction of their bulk kinetic energy can be released into the lobes as freshly-accelerated relativistic particles.

The basic ingredients of this beam scenario have received strong support from the discoveries of bright, well-collimated radio jets linking the central radio sources in numerous galaxies and quasars to the brighter parts of their distant radio lobes. Figures 1 and 2 show observations made with the Very Large Array (VLA) of the radio jet/counterjet structures associated with the galaxies NGC315 and NGC383 (3C31). About thirty radio jets are now known; they have been found in sources ranging in intensity from weak ($\sim 10^{34}$ watt) radio galaxies such as 3C449 (Perley *et al.* 1979) to powerful ($\sim 10^{37}$ watt) quasars such as 4C32.69 (Potash and Wardle 1980). The radio jets are presumed to be inefficient (i.e. lossy) examples of the beam phenomenon visualised by Rees. The importance of such inefficient beams to the overall radio-source collimation problem is threefold. First, they demonstrate directly that the main problem of collimation of the extended radio structures can correctly be reduced to that of establishing and collimating directed beams emanating from the central objects in galaxies and quasars. Second, they show that in at least some cases the beams transport significant magnetic fields into the radio lobes. Third, studies of the intensity, spectrum and polarization distributions over the observable radio *jets* provide a meeting-ground for data and theoretical models of the dynamics and confinement of the underlying *beams*.

Our work has been motivated by recent observations from the VLA and from the Westerbork Synthesis Radio Telescope which show that the well-resolved radio jets in 3C31 (Bridle *et al.* 1980), in NGC315 (Bridle *et al.* 1979; Willis *et al.* 1981), and in the radio galaxy 0326 + 396 (Bridle *et al.* 1981) do not widen steadily with increasing distance from the central objects. Instead, they expand rapidly within a few kpc of the central object and then become more collimated at distances of order 20 to 100 kpc from it. A significant part of the collimation process is therefore taking place sufficiently far from the central object to be observable in nearby radio galaxies with these high-resolution radio interferometers. This paper describes a formalism by which the observed changes in width of the radio jets may be related to an MHD model of the underlying beam phenomena.

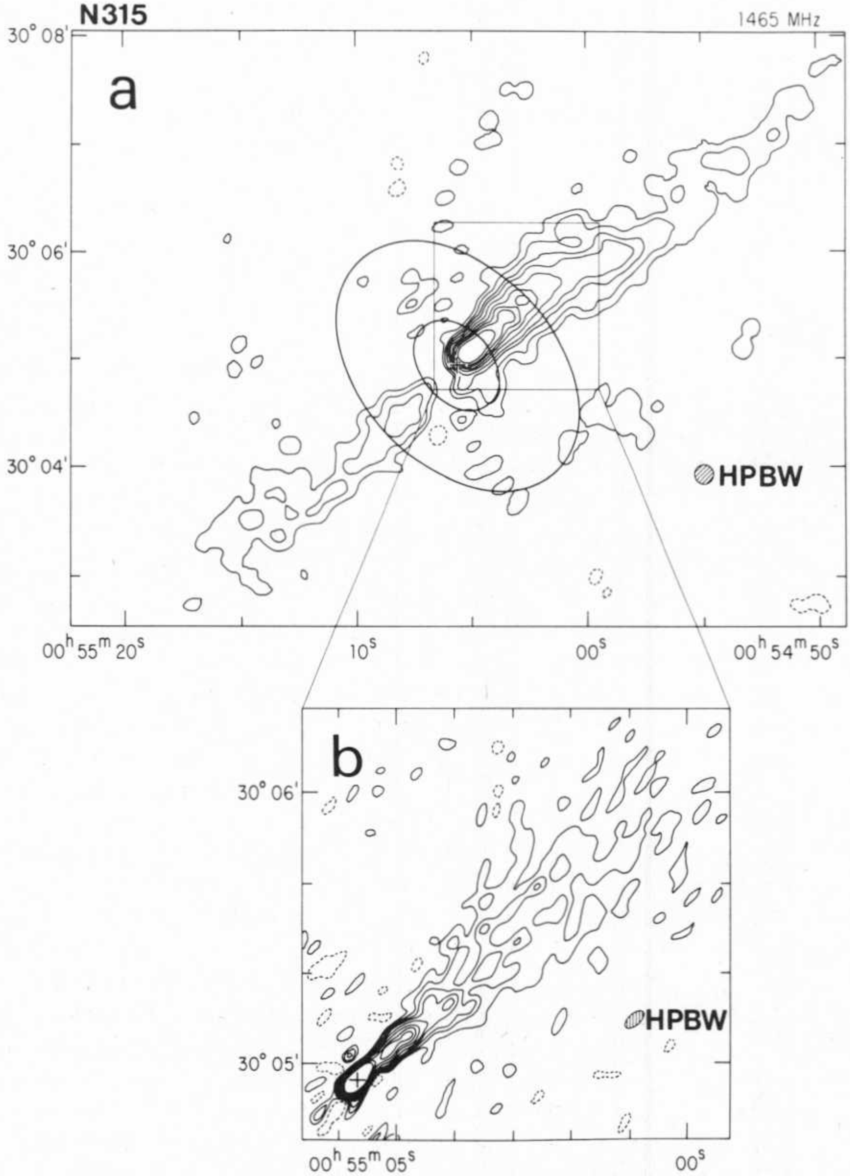

FIG. 1—(a) VLA 1465-MHz map of the inner 9 arc min of the jet/counterjet structure associated with NGC315, at 11 arc sec resolution (the HPBW is shown by the hatched circle). Contours are drawn at −2 (dashed), 2, 4, 15, 30, 40, 50, 70 and 90 mJy per beam. A 420-mJy unresolved radio core has been subtracted from the map at the position of the cross, which

2. Evidence for Particle Acceleration in the Jets. We discuss a "heavy" beam model in what follows because there is growing evidence that some form of relativistic particle acceleration and/or magnetic field amplification is taking place in the beams. The luminosity of an ensemble of electrons with a power-law energy distribution radiating by the incoherent synchrotron mechanism at frequency ν is

$$P(\nu) = c(x) N_0 B_{\perp}^{(x+1)/2} \nu^{-(x-1)/2}$$

where $N(E, E + dE) = N_0 E^{-x} dE$ is the number of electrons with energies between E and $E + dE$ per unit volume per steradian moving towards the observer and $c(x)$ is a numerical factor, e.g. Pacholczyk (1970). The slowest possible decline of emissivity with radius in a freely-expanding beam occurs in the ballistic case (in which the particle energies $E =$ constant). The emissivity cannot fall off more slowly with increasing beam radius R than it would due to the slowest possible flux-conserving decay of a field component (which is $B \sim 1/R$ for transverse or circumferential components of $\boldsymbol{B}$). Unless some energy reservoir in the beams can be tapped to reaccelerate particles (i.e. to maintain or increase the value of N_0) or to amplify magnetic fields, P should therefore decay with R at least as rapidly as $1/R^{(x+1)/2}$. The radio spectra of most of the observed jets imply that their electron spectra have $x \gtrsim 2$ so their emissivities should decay as $\sim R^{-1.5}$ or faster, whereas the emissivities of some radio jets with $x \sim 2$ have been observed to decline more slowly than this (e.g. Burch 1979b; Fomalont *et al.* 1980). The continuous re-energising of the relativistic (radiating) particles and fields that is implied by such slow emissivity variations is supported by the lack of spectral steepening due to synchrotron losses along the jets (Bridle *et al.* 1979; Burch 1979b; Willis *et al.* 1981). These results seem to us easiest to understand if the beams are not "light" flows of relativistic particles and fields alone, but are energetically dominated by "heavy", i.e. nonrelativistic and essentially nonradiating, material whose bulk kinetic energy can act as a reservoir from which the relativistic particle energies can be replenished. We therefore consider such a "heavy" beam in what follows.

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coincides within errors with the optical centre of NGC315. The ellipses show the boundaries of the inner overexposed core and the outer envelope of the image of NGC315 on the red-sensitive print of the Palomar Sky Atlas. (b) VLA 1465-MHz map of the inner 80 arc sec of the brighter jet at 2.5 by 5.0 arc sec resolution (the HPBW is shown by the hatched ellipse). The contours are at −4 (dashed), 4, 7, 10, 15, 20, 30, 40, 50 and 70 mJy per beam. The base of the jet is barely resolved, but it rapidly widens as it leaves the radio core, which has not been subtracted from this map. The observations shown in these maps have been described in detail by Bridle *et al.* (1979).

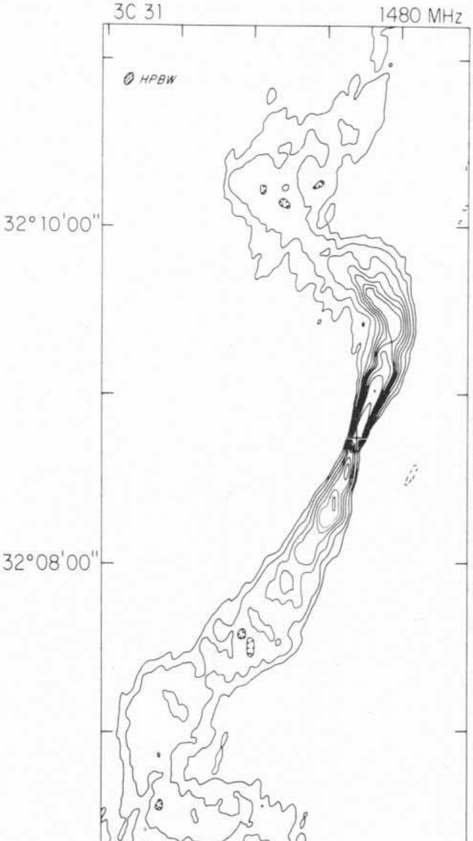

FIG. 2—VLA 1480-MHz map of the inner 6 arc min of the jet/counterjet structure associated with NGC383 (the radio source 3C31), at 2.5 by 5.0 arc sec resolution (the HPBW is shown by the hatched ellipse). Contours are drawn at −1.5 (dashed), 1.5, 3, 6, 9, 12, 15, 18, 24, 30 and 45 mJy per beam. The cross marks the position of the centre of NGC383. The jet and counterjet widen at decreasing rates as they leave a bright radio core that coincides with the centre of the galaxy. The observations have been described in detail by Fomalont *et al*. (1980).

3. The Supersonic Dynamics of Heavy Magnetised Beams. (i) Principles. The fundamental principles of any nonrelativistic, dissipationless, isentropic, and steady MHD flow are embodied in the equations:

(Conservation of magnetic flux) $\nabla \times (\boldsymbol{V} \times \boldsymbol{B}) = 0$ (1)

(Conservation of mass) $\nabla \cdot (\rho \boldsymbol{V}) = 0$ (2)

(Conservation of momentum)

$$\nabla \frac{V^2}{2} + \frac{1}{\rho}\nabla p + \nabla\Phi = \boldsymbol{V} \times \nabla \times \boldsymbol{V} - \frac{1}{4\pi\rho}\boldsymbol{B} \times \nabla \times \boldsymbol{B} \tag{3}$$

(Equation of state on a stream line) $p = \text{const.}\, \rho^{\gamma}$ (4)

(Continuity of magnetic field lines) $\nabla \cdot \boldsymbol{B} = 0$ (5)

Here Φ is the gravitational potential and the other symbols have their conventional meanings. These general equations are difficult to solve even under the assumption of axial symmetry which it is reasonable to make when we are modelling a straight radio jet. They can however be simplified if gravity and pressure asymmetry are neglected and if the jet-like nature of the flow (evidenced by the radio data) is used to support the simplifying assumption of *transverse self-similarity* (homology) of the variations of the flow parameters.

(ii) Models with transverse self-similarity. In the belief that the variations of dynamical parameters transverse to the axis of the beam are relatively unimportant in comparison to their variation along it, we assume transverse homology of the flow. This assumption, which can be shown to be self-consistent (see below), allows us to focus attention on the longitudinal variations of the key dynamical parameters and also to study the collimation of the beam by examining the shape of a single reference stream line, taken as the boundary stream line. The solution of equations (1) to (5) outlined in this and in the following Section follows that given in detail by Chan and Henriksen (1980, henceforth CH); the reader should refer to their paper for more explicit theoretical discussion.

We adopt a cylindrical (r, ϕ, z) coordinate system where r (notated ϖ in CH) is the radial coordinate measured from the symmetry axis of the flow, ϕ is the azimuthal angle around this axis, and z is the coordinate along the axis (distance from the central object, where the hot fluid is presumed to be generated). The transverse homology is achieved by assuming that the magnetic field strength $\boldsymbol{B}$, flow velocity $\boldsymbol{V}$, specific angular momentum L (including both matter and field contributions) and pressure p in the fluid can be written in the forms:

$$\boldsymbol{B} = \left(b_r \frac{r}{R}, b_\phi \frac{r}{R}, b_z\right) \tag{6}$$

$$\boldsymbol{V} = \left(W_r \frac{r}{R}, W_\phi \frac{r}{R}, W_z\right) \tag{7}$$

$$L = \Omega R_A{}^2 \left(\frac{r}{R}\right)^2 \tag{8}$$

$$p = p_j - (p_j - p_e)\left(\frac{r}{R}\right)^2 \tag{9}$$

$$p_j = (\rho/\rho_s)^\gamma = p_{js} \cdot (\rho T/\rho_s T_s) \tag{10}$$

where $b_r, b_\phi, b_z, W_r, W_\phi, W_z$, the fluid pressure p_j on the axis of the beam, the pressure p_e of any external confining medium, the fluid density ρ and the beam temperature T are functions of z only, and subscript s denotes that a parameter takes its value at the sonic point. The variables $b_r, b_\phi, b_z, W_r, W_\phi, W_z$ specify the associated field quantities at the boundary between the beam and an external medium. This boundary occurs at a radius $r = R(z)$. The collimation properties of the beam are described by the form of $R(z)$ and our goal here is to relate this form to the physical conditions in the beam and its environs, for comparison with the observations of the width evolution of the radio jets. The quantity Ω is constant along a stream line in this self-similar model; it measures the azimuthal component of the drift angular velocity perpendicular to $\boldsymbol{B}$. The constant R_A is the radius of the beam boundary at the Alfvenic point in the flow (i.e. at the point where the longitudinal Alfvenic Mach number $M_A = \sqrt{(4\pi\rho)}V_z/B_z = 1$).

These self-similar forms are justified physically in CH in terms of the transverse incompressibility of paraxial jets. The forms required are also reasonable approximations to the core cross-sections of laboratory jets (e.g. Pai 1954).

By using (9) we have assumed that both the thermal pressure and the magnetic field pressure are continuous across $R(z)$. Such a match between the boundary properties of the beam and of the external medium would require viscous or magnetic entrainment of material from outside the beam, but we assume here that the direct effects of entrainment will be only of secondary influence on the dynamics of the beam. Interactions between beams and the surrounding media may in fact be indicated by recent VLA observations of two jets (NGC326 and 3C341 – Fomalont *et al.* 1981; Bridle *et al.* 1981) which appear to have faint, broad "cocoons" of emission around their bright ridges, but we defer discussion of this phenomenon to another paper.

On substituting the forms (6) to (9) into equations (1) to (5), it can be shown (see CH) that *these forms satisfy all equations save the Bernoulli integral*

$$E = \tfrac{1}{2}(V_r{}^2 + V_\phi{}^2 + V_z{}^2) + \int \frac{dp}{\rho} - \frac{B_\phi B_z}{4\pi\rho} \cdot \frac{r\Omega}{V_z}$$

Moreover, the Bernoulli integral can be split into longitudinal and transverse parts. The longitudinal part reduces to equation (12) below given our assumed forms. The transverse part will be negligible provided (1) that the magnetic field is not the dominant source of longitudinal acceleration in the beam, (2) that the stream lines are paraxial ($(W_r/W_z)^2$ and $(W_\phi/W_z)^2 \ll 1$) and (3) that there is pressure confinement (i.e. a "nozzle", as proposed by Blandford and Rees) at the base of the jet so that $(p_e - p_j) \ll p_j$ in the pressure-dominated region. These three assumptions define the problem to which the CH self-similar solution applies, and all three appear reasonable in the context of the observed radio jets.

(iii) Equations of the model. With the above assumptions, the theory of the transverse self-similar beam reduces to the solution of the following equations:

Radius of boundary stream line
$$\frac{dR}{dz} = \frac{W_r}{W_z} \tag{11}$$

Longitudinal Bernoulli equation
$$W_z^2 + \frac{2T}{\gamma - 1} = W_{z\infty}^2 = \frac{\gamma + 1}{\gamma - 1} \tag{12}$$

Transverse equation of motion
$$(1 - M_A^{-2})W_z \frac{dW_r}{dz} = \frac{1}{R}[2P + F_c - 2F_{\phi\phi} - F_{rz}] \tag{13}$$

where in (13) the quantities on the right describe the effects of:

Thermal pressure
$$P = \frac{T}{\gamma} - \left(\frac{p_{es}}{\gamma p_{js}}\right) p_e(z) T^{-1/(\gamma - 1)} \tag{14}$$

Centrifugal force
$$F_c = W_{z\infty}^2 \left(\frac{R}{R_1}\right)^2 \left[\frac{(R_A/R)^2 - M_A^{-2}}{1 - M_A^{-2}}\right]^2 \tag{15}$$

Magnetic pinching
$$F_{\phi\phi} = W_{z\infty}^2 \left(\frac{R}{R_1}\right)^2 M_A^{-2} \left[\frac{(R_A/R)^2 - 1}{1 - M_A^{-2}}\right]^2 \tag{16}$$

Magnetic surface tension
$$F_{rz} = W_r^2 M_A^{-2} \left[\frac{3W_z^2 - T}{W_z^2 - T}\right] \tag{17}$$

In equations (11) to (17), all variables are measured in units of their values at the sonic point of the longitudinal flow (i.e. at the location z_s of the "nozzle") except W_r, which is normalised to W_{zs}. The asymptotic value of the longitudinal flow velocity is $W_{z\infty}$, evaluated on the right hand side of (12), and we have introduced the quantity $R_1 = W_{z\infty}/\Omega$, whose physical significance is clarified in Section 4(iii) below.

The other equations of the model are exactly integrable and yield:

$$b_z = b_{zs}/R^2 \tag{18}$$

$$b_r = b_z(W_r/W_z) \tag{19}$$

$$b_\phi = b_z(W_{z\infty}/W_z)(R/R_1)\left[\frac{(R_A/R)^2 - 1}{1 - M_A{}^{-2}}\right] \tag{20}$$

$$\rho = 1/W_z R^2 \tag{21}$$

$$W_\phi = W_{z\infty}(R/R_1)\left[\frac{(R_A/R)^2 - M_A{}^{-2}}{1 - M_A{}^{-2}}\right] \tag{22}$$

where $M_A{}^2 = 4\pi\rho V_z{}^2/B_z{}^2 = M_{As}{}^2 R^2 W_z$ from (18) and (21). Once equations (11), (12) and (13) are solved for $R(z)$, $W_z(z)$ and $W_r(z)$, the flow is therefore defined completely, including the run of beam temperature $T(z)$ which can be obtained from (21) by noting that $T/T_s = (\rho/\rho_s)^{(\gamma-1)}$. Given a prescription of the external confining pressure $p_e(z)$, and the conditions at the sonic point in the flow, the subsequent behaviour of the variable flow cross-section in the supersonic regime can be found self-consistently (in the assumed paraxial limit) using these equations.

It should be noted that the model describes the supersonic region of the beam without explicitly referring to the nature of the central object which supplies the beam's fluid. The entire central region of the source is replaced by initial conditions at the sonic point in the flow (i.e. at the "nozzle"). This replacement is possible because there is no feedback to these conditions from the supersonic, super-Alfvenic region. The nature of the central object enters the model only insofar as it determines the conditions at the sonic point ("nozzle"). This fact allows us to detach the description of the beam beyond the nozzle from the more difficult and speculative task of describing the object that ultimately produces it; this detachment means, of course, that observations of the collimation properties of large-scale radio jets will provide correspondingly little information about the central object (which may only be amenable to study by VLBI methods).

The procedure used in this paper to specify conditions at the nozzle does not select the "critical" solution that is smoothly trans-sonic at z_s. To do so requires assigning a non-zero value for dW_z/dz at z_s which is arbitrary in the absence of inner (subsonic) boundary conditions. We have reduced the number of model parameters by setting $dW_z/dz = 0$ at the sonic point in our present calculations because such solutions exist that are arbitrarily close to the critical solution for $z \gg z_s$, where comparison with the radio data is now possible. This procedure will require modification only for studies of details very close to the nozzle, or for studies of time-dependent jets (stability analyses).

4. Constraints on the Solutions. We must now specify constraints on the general flow problem outlined above, to investigate how the constraints influence the observable collimation properties of the radio jets.

(i) The Alfvenic radius R_A. The condition of transverse equilibrium at the nozzle determines (CH):

$$\frac{p_{es}}{\gamma p_{js}} = \frac{1}{\gamma} + \tfrac{1}{2}F_{cs} - F_{\phi\phi s} = \frac{1}{\gamma} + \frac{W_{z\infty}{}^2}{R_1{}^2(1 - M_{As}{}^{-2})^2} \times \left[\frac{(R_A{}^2 - M_{As}{}^{-2})^2}{2} - M_{As}{}^{-2}(R_A{}^2 - 1)^2\right] \quad (23)$$

In (23), $M_{As}{}^{-2}$ must be < 1, but R_A is an arbitrary boundary parameter. CH have argued however that it must be of order unity, and we set it to unity in what follows in order to restrict the parameter space to be explored; this reduces (23) to the simpler form

$$\frac{p_{es}}{p_{js}} \sim 1 + \frac{\gamma W_{z\infty}{}^2}{2R_1{}^2} = 1 + \frac{\Omega^2}{2}$$

and reduces the defining parameters of the problem to γ, R_1 and M_{As}, plus the form of the external confining pressure $p_e(z)$.

(ii) The adiabatic index γ. In the following discussion we will presume that the fluid pressure is dominated by the relativistic particles, so that we can put $\gamma = 4/3$ in the equation of state for the fluid (10). In doing this we are effectively assuming that the density of relativistic particles scales with the density of thermal particles. (Other assumptions can be justified, but we will not explore them here).

(iii) The magnetic field transition radius R_1. An observational constraint is available for R_1 if the beams are super-Alfvenic, so that $M_A{}^2 \gg 1$ and $(R_1/R_A)^2 \gg 1$. In this case, equations (18) and (20) show that $b_z \sim R^{-2}$ while $b_\phi \sim R^{-1}$ as W_z tends to $W_{z\infty}$. Under these conditions, confined beams described by our model show the same field evolution as the free beams described by Blandford and Rees (1978). A transition from dominant B_z to dominant B_ϕ has in fact been demonstrated from VLA observations of linear polarization in the jets of 3C31 and NGC315 (Fomalont *et al.* 1980). It has also been shown that in the B_ϕ-dominated far regimes of 3C449 (Perley *et al.* 1979) and NGC315 (Willis *et al.* 1981), the *equipartition* magnetic field varies as $\sim R^{-1}$. These field variations are consistent with our model if the flows are super-Alfvenic, in which case measurement of the beam radius at the distance where $B_\phi \sim B_z$ in a well-resolved radio jet will estimate the value of R_1, by means of (20). This has already been achieved for 3C31 and

NGC315 (R_1 = 300 and 825 pc respectively, using the data of Fomalont *et al.* 1980 and $H_0 = 100\,\mathrm{km.s^{-1}.Mpc^{-1}}$), and lower limits of 1.6 kpc and 2 kpc can be set for R_1 in the more powerful jets in 3C219 (Perley *et al.* 1980) and 4C32.69 (Potash and Wardle 1980), where the B_z-dominated regime extends over the entire jet.

It is sometimes stated (e.g. Miley 1980) that radio data which show that $B_z \sim R^{-2}$ and $B_\phi \sim R^{-1}$ support the hypothesis of a "free" beam. This is incorrect, as some confined beams (of which ours in its super-Alfvenic regime is an example) exhibit very similar behaviour; it will be necessary to examine both the longitudinal and transverse variations of B_z and B_ϕ in concert with intensity, spectral, and collimation data, before inferences about "freedom" for jets can properly be made.

As emphasized by CH, the asymptotic dynamical effects of the magnetic field depend mainly on the composite parameter

$$\varepsilon_B \equiv W_{z\infty}^{-1}(M_{As}R_1)^{-2} \sim b_\phi^2/4\pi\rho W_{z\infty}^2 \tag{24}$$

which measures the ratio of the energy in the circumferential (pinching) component of the magnetic field to the asymptotic bulk kinetic energy of the flow in the axial direction. We therefore use values of this composite parameter ε_B to characterise the models in the next Section.

(iv) The external pressure law $p_e(z)$. The specification of the form of an external confining pressure $p_e(z)$ introduces parameters of the atmosphere of a given galaxy or quasar that are not fundamental to the beam model itself. That hot ($T \sim 10^7$K), dense ($n \sim 10^{-2}\,\mathrm{cm^{-3}}$) media exist in and around at least some radio galaxies is clear from the Einstein Observatory results on Centaurus A (Fabbiano *et al.* 1979), M87 (Fabricant *et al.* 1978, 1979) and Cygnus A (Fabbiano *et al.* 1979). The pressures p_e implied by the favoured thermal bremsstrahlung interpretations of these X-ray data are quantitatively similar to the beam pressures p_j which can be estimated for the observed radio-galaxy jets making the conventional (but as yet unsupported) assumption of energy equipartition between the magnetic fields and synchrotron emitting particles in the jets. It is therefore reasonable to assume the *existence* of external confining pressures p_e which may influence jet dynamics.

The X-ray data are not yet of sufficient quality to tell us what pressure distributions $p_e(z)$ are most commonly observed in radio galaxies, however, and there is even less information available for quasars. For this reason, present fits of the beam model to particular radio jets must contain *ad hoc* pressure-specification parameters which should at some (possibly not too distant) time be replaceable with well-defined parameters fitted to the X-ray observations of the parent objects. For the present, we take the external pressure p_e at $z > z_s$ in the two-component form

$$p_e(z) = \frac{f}{1 + (f - 1)(z/z_s)^m} + \frac{(1 - z_s/z)(z_s/z_e)^m}{1 + (z/H)^{m'}} \cdot \frac{f}{f - 1} \tag{25}$$

where

$$f = \left[\frac{\gamma + 1}{2}\right]^{\gamma/(\gamma - 1)}$$

At small z, this form describes a power-law pressure variation of index m, which we use to represent the "nuclear" atmosphere close to the centre of the galaxy or quasar. Above a certain height z_e however this pressure term is supplanted by a more slowly-varying term of index m' and scale H. We will choose $z_s \ll z_e \ll H$ in order to simulate the transition from a "nuclear" atmosphere such as that suggested by the X-ray data on Centaurus A to a more extensive "halo" atmosphere such as those detected around M87 and Cygnus A. The form of the pressure amplitudes in (25) is chosen so that extrapolation of the first term back to $z = 0$ would specify a central pressure $p_e(0)$ consistent with a subsonic flow satisfying the Bernoulli equation (12) and the equation of state (10). This constrains f to the form given; the coefficient of the second term in (25) is then determined by requiring it to equal the first near $z = z_e$.

5. *Numerical Examples*. In this section we show representative examples of the collimation properties of our models in the form of $R(z)$ curves computed from equations (11), (12) and (13). The curves shown have been chosen to display the *range* of collimation behaviour implicit in the physics, rather than to illustrate what may be *typical* behaviour in the astrophysical systems. Section 6 discusses a fit to the collimation data on a well-studied radio jet.

(i) Beams whose magnetic fields are dynamically unimportant. Figure 3 displays $R(z)$ curves for models in which the collimation of the beam is derived entirely from confinement by the external pressure p_e. In all of these models $\varepsilon_B = 0$ and $M_A = \infty$, so R_1 has no dynamical significance. Curve (A) shows a case where only the "nuclear" atmosphere in (25) is used, with sonic height $z_s = 10$ (horizontal axis), sonic radius $R_s = 1$ (vertical axis), and pressure index $m = 4$. In this and all other models shown, the adiabatic index $\gamma = 4/3$, so that the Mach number of the flow along the axis tends to $W_{z\infty} = 2.65$, from (12). This beam is accelerated by the internal pressure gradient to 90 per cent of its asymptotic flow velocity by $z = 56$ (i.e., 5.6 nozzle heights).

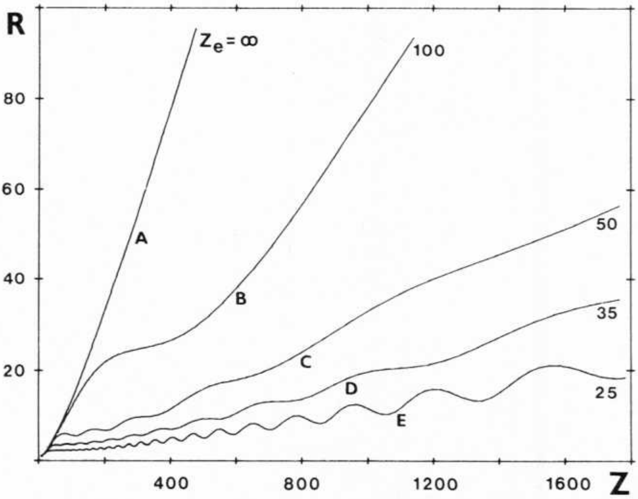

FIG. 3—Curves of beam radius R against distance z from the centre of the galaxy, for beams collimated entirely by external confining pressures. In all models the radius of the beam at the sonic point is set at $R_s = 1$, the height of the sonic point is set at $z_s = 10$ (defining the arbitrary length scale for the diagram), and the nuclear atmosphere contributes a pressure which declines with z according to the first term in (25), with $m = 4$. (A) the curve obtained when there is no halo pressure ($z_e = \infty$). (B) the curve obtained when a halo atmosphere with scale height $H = 200$ and pressure index $m' = 3$ is added using the second term in (25), with transition height $z_e = 100$. (C) as in (B), but with transition height $z_e = 50$. (D) as in (B), but with transition height $z_e = 35$. (E) as in (B), but with transition height $z_e = 25$.

In all such models with single power-law pressures, R increases monotonically with z. These flows become ballistic as the internal and external pressures decline, so they open at constant cone angles far from the central object. In the case shown the asymptotic cone angle is ~30°, so such a beam provides only about the same collimation as would the free-plasmon expansion referred to in Section 1. To obtain highly-collimated radio lobes with such models one would have to postulate either flat ($m \ll 3$) pressure laws extending from the nozzles to the radio lobes (which would be very unrealistic), or very narrow nozzle angles arctan (R_s/z_s) (Table 1 of CH documents the relation between asymptotic cone angle, m, and nozzle angle in this class of model). The significance to this discussion of such models with $m - 4$ is that the observed rates of expansion at the bases of the jets in 3C31 (Bridle *et al.* 1980) and NGC315 (Willis *et al.* 1981) are similar to those at the base of curve (A). This curve therefore serves as a useful benchmark for comparisons with models in which significant additional collimation occurs further down the flow.

Curves (B) to (E) of Fig. 3 show the effect of introducing a "halo" atmosphere with a flatter power law $m' = 3$ and a longer scale length $H = 200$, via the second term in (25), the other parameters being the same as in curve (A). The curves show the effect of decreasing the height z_e at which this halo pressure equals the "nuclear" pressure, from $z_e = 100$(B) to 50(C), 35(D) and 25(E). Two major effects are seen as z_e approaches z_s. First, where z becomes several times z_e, a collimation "shoulder" occurs at which the $R(z)$ curve flattens significantly. The onset of this shoulder occurs at lower z, and its length is greater, the closer z_e is to z_s. Second, the beam initially adapts to the pressure decline in the nuclear atmosphere, but as z_e enters the accelerating part of the flow an overshoot occurs which excites oscillations of the beam radius $R(z)$. These oscillations will be damped if $m' > 2\gamma$ (see CH), as is marginally the case in the examples shown here. Their amplitudes are greater and they persist for longer the closer z_e is to z_s. For all values of z_e, the wavelength of the oscillations increases with increasing z. If such oscillations grew sufficiently in a real beam, they would ultimately dilute its momentum flux sufficiently for it to be destroyed by its interactions with the circumgalactic environment.

(ii) Magnetic pinches with power-law decline of the external pressure. Figure 4 shows beams in which the collimation far from the nozzle is produced entirely by the pinch interaction between the circumferential magnetic field B_ϕ and the currents parallel to the beam axis. These models have a single power-law external pressure. The asymptotic behaviour of such models can be understood from equations (13), (14) and (16), which become

$$\frac{dW_r}{dz} \sim \frac{1}{RW_{z\infty}}\,[P - F_{\phi\phi}]$$

$$P \sim \frac{1}{\gamma}(1/W_{z\infty}R^2)^{(\gamma-1)}[1 - (p_{es}/p_{js})p_e(z)(W_{z\infty}R^2)^\gamma]$$

$$F_{\phi\phi} \sim {W_{z\infty}}^2\varepsilon_B$$

For any reasonable asymptotic pressure variation $p_e(z)$, the magnetic pinching term $F_{\phi\phi}$ must eventually dominate and so cause the beam to reconverge. Curve (A) in figure 4 repeats curve (A) from figure 3 for benchmark purposes. Curve (B) of figure 4 has the same sonic-point and pressure parameters but R_1 has been set equal to 5 (a plausible value for a weak radio galaxy, from the observations of 3C31 and NGC315 by Fomalont *et al.* (1980)), and M_{As} has been adjusted so that $\varepsilon_B = 0.0081$. The beam achieves a maximum radius $R \sim 84.5$ near $z = 1300$, and then recollapses to a radius $R \sim 12$ at $z \sim 2500$. The collapse is halted when the internal

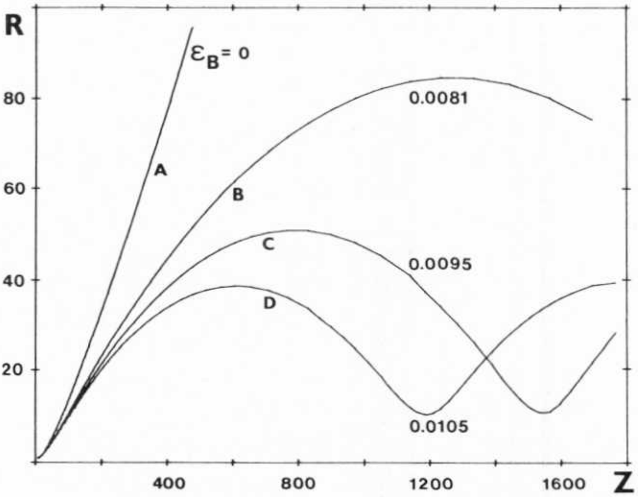

FIG. 4—Curves of beam radius R against distance z from the centre of the galaxy, for beams collimated by magnetic pinching in the presence of only a nuclear confining atmosphere. (A) the model shown in curve (A) of figure 3, repeated for comparison purposes. (B) the curve obtained when a magnetic field with $\varepsilon_B = 0.0081$ is added to the model in curve (A), the field transition radius (where $B_\phi = B_z$) being set to $R_1 = 5$. (C) as in (B), but with $\varepsilon_B = 0.0095$. (D) as in (B), but with $\varepsilon_B = 0.0105$.

pressure of the recompressed beam is again able to overcome the magnetic pinching. There is a substantial overshoot, and the beam radius continues to oscillate with a wavelength of $\sim$2500 in z and slowly growing amplitude (the high-z behaviour is not shown in this figure). Curves (C) and (D) show the effects of increasing ε_B to 0.0095 and 0.0105 respectively with R_1 held constant, the variation in ε_B being obtained by varying M_{As} (>1). As ε_B is increased, the first maximum of R is lower and is reached at smaller z, while the wavelength of the subsequent oscillations decreases.

These models prevent the free expansion of the beams at the expense of exciting large-amplitude and long-wavelength oscillations of the beam radius which would probably destroy a real beam. It is of interest that the effects of the magnetic pinching term become dominant below $z = 1000$ (100 sonic heights in these curves) when ε_B is as low as 0.01. The available VLA observations of radio jets probably extend to 100 sonic heights or more, and show very little sign of such large-scale oscillations of the jet radii. It is therefore unlikely that the observed jets are collimated entirely by magnetic pinches in which ε_B significantly exceeds 0.01. A possible exception is the well-known optical-radio jet in M87, which exhibits quasi-periodic oscillations in *intensity*; it is not yet clear however whether these correspond to oscillations in *radius*.

(iii) Magnetic pinches with two-component external pressure. The models illustrated in figure 4 oscillate through large amplitudes in R because the beams have expanded sufficiently for their internal pressures to have fallen to $\sim 10^{-5}$ of their values at the sonic point before the magnetic pinching forces overcome the initial expansions. The radial oscillations of magnetically-confined beams are less dramatic if their internal pressures are higher, and more nearly in balance with the magnetic pinching, where the beams first cease to expand. This condition can be achieved if the *external* pressure contains the second ("halo") term in (25). An interesting class of solution exists wherein this halo pressure does not by itself provide a long "collimation shoulder" (as in figures 3(C) to 3(E)), but merely slows the initial expansion sufficiently to allow magnetic collimation to occur without provoking large-amplitude oscillations.

As an example of this behaviour, we take as a benchmark (figure 5(A)) the pressure-slowed model which gave curve (B) of figure 3, and show the effects of adding a dynamically significant magnetic field of increasing ε_B from 0.0048 to 0.0143 in curves (B) to (E). These models are more strongly collimated at all distances from the central object, and exhibit magnetically-controlled radial oscillations of moderate amplitude; the wavelengths of these oscillations decrease with ε_B and increase with z in a given model. The oscillations grow slowly in amplitude and would eventually disrupt the beams at very large distances from the central objects.

Curve (E) is particularly interesting as its first maximum occurs at $R \sim 10$, well within the rapid-expansion regime of the benchmark curve (A), so this is truly a "magnetically-collimated" beam, yet the immediately subsequent radial oscillations are only of very modest amplitude (± 8 per cent of the average beam radius). Such models describe approximately cylindrical beams with well-defined average radii, in which the *adiabatic* loss problems of the sources would be virtually eliminated (although *radiative* losses would still occur).

It is tempting to speculate that if the motions of the radiating relativistic particles were sufficiently close to streaming along the field lines of such beams, so that the directive properties of synchrotron radiation confined most of their emission to angles near the jet axis and away from the observer, then these "magnetic cylinders" might represent the efficient (and invisible) energy pipelines hypothesised by Rees and others as the energy-transport mechanism in the highly luminous extended extragalactic sources with well-developed and compact "hot spots" in their lobes. We

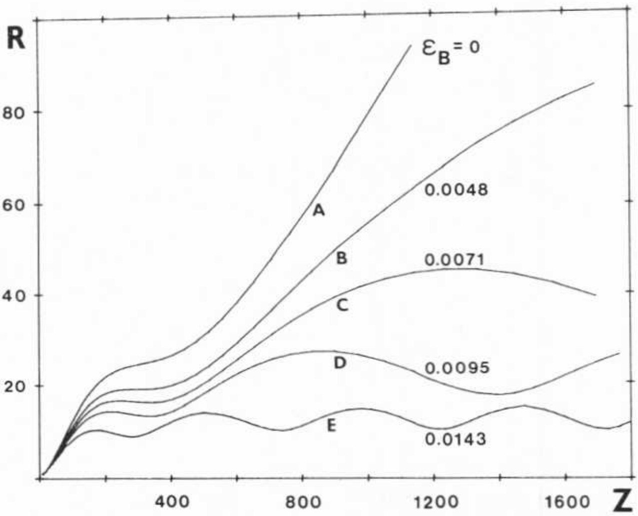

FIG. 5—Curves of beam radius R against distance z from the centre of the galaxy, for beams collimated by magnetic pinching and by two-component confining pressures. (A) the model shown in curve (B) of figure 3, repeated for comparison purposes. (B) the curve obtained when a magnetic field with $\varepsilon_B = 0.0048$ is added to the model in curve (A). (C) as in (B), but with $\varepsilon_B = 0.0071$. (D) as in (B), but with $\varepsilon_B = 0.0095$. (E) as in (B), but with $\varepsilon_B = 0.0143$.

have followed some of these models to very large distances from the central objects, to study the growth of the radial oscillations. The curve shown in figure 5(E) undergoes 4:1 amplitude oscillations by $z \sim 7000$ (700 sonic heights), and it is therefore likely that the beams would be disrupted in such a distant regime. Fits of our models to several radio jets (e.g. Section 6 below) suggest however that the sonic heights in radio galaxies may typically be ~ 1 kpc, so that the scales over which disruptive radial oscillations grow significantly in these ''magnetic cylinder'' models may in fact correspond to the scales in which the radio lobes are formed.

We again note that with the nozzle parameters adopted for these examples, the value of ε_B at which the magnetic collimation effects become dominant is ~ 0.01. Estimates of the density ρ and flow velocity V_z have been made for the jets in 3C31 (Blandford and Icke 1978) and 3C449 (Perley *et al.* 1979) from considerations of the curvature, depolarization, and other observed properties of these jets. On various such grounds it seems likely that we are dealing with densities $\rho \sim 10^{-22 \text{ to } -23}$ kg.m^{-3} and velocities $V_{z\infty} \sim 1000$ km.s^{-1}, in which case the condition $\varepsilon_B \sim 0.01$ corresponds to azimuthal magnetic fields B_ϕ in the range 4×10^{-6} to 4×10^{-5} gauss. Such field strengths are of the same order as those obtained for these jets under the conventional (but unproven) assumption that the magnetic fields and the relativistic particles responsible for the observed radio emission carry approximately equal energies. The examples shown here therefore suggest that such "equipartition" magnetic fields could be effective in collimating beams whose expansion parameters are similar to those of these models. Furthermore, field strengths significantly in excess of the equipartition values would probably be *too* effective at producing magnetic collimation, unless R_1 were very large (in which case it would be difficult (see equation 24) to make $\varepsilon_B \sim 0.01$ while keeping the flow super-Alfvenic and non-relativistic).

6. *Application to NGC315.* The jets in NGC315 are the longest observed in any nearby radio galaxy, and have been well resolved in the transverse direction with the Westerbork array at 20 cm (Willis *et al.* 1981), and the VLA at 20 cm and at 6 cm (Bridle *et al.* 1979). Their projected lengths are so large (240 kpc and 500 kpc, if $z = 0.0167$ and $H = 100\ \mathrm{km.s^{-1}.Mpc^{-1}}$) that we will assume they lie at right angles to the line of sight in what follows, to avoid implying still greater gigantism by projection. The transverse brightness profiles of the jets are approximately Gaussian in form, and we make the working assumption that the half widths between half maxima (HWHM) of these radio profiles measure the radius R of the underlying beam, to first order. This assumption might be incorrect if the distribution of pitch angles of the radiating particles is highly anisotropic, however, and numerical simulations are being made of the radiation patterns of relativistic particles moving in the field configurations likely to be present in these beams in order to evaluate this assumption more closely.

The filled circles in figure 6 show the observed development of the HWHM of the main jet in NGC315 at 20 cm, as a function of angular separation from the unresolved central radio core (located in the nucleus of NGC315). A regime of initial rapid expansion in the first 100 arc sec (24 kpc) is followed by a collimation "shoulder" from about 180 to 400 arc sec (42 to 95 kpc), then a slower re-expansion out to about 800 arc sec (190 kpc), where the jet begins to bend violently southwards and the collimation data therefore become unreliable. If these data indeed map the behaviour of the radius R of the beam as a function of z, magnetic pinches with a single power-law pressure are ruled out as the mechanism for collimation of this jet; in all such models, $R(z)$ decreases steadily beyond its first maximum at $z = z_{\max}$ until a deep minimum occurs near $z = 2z_{\max}$ (see fig. 4). Such behaviour is quite different from the re-expansion observed in NGC315.

Figure 5 suggests that magnetic pinches with a two-component external pressure could successfully simulate the NGC315 data, but the most es-

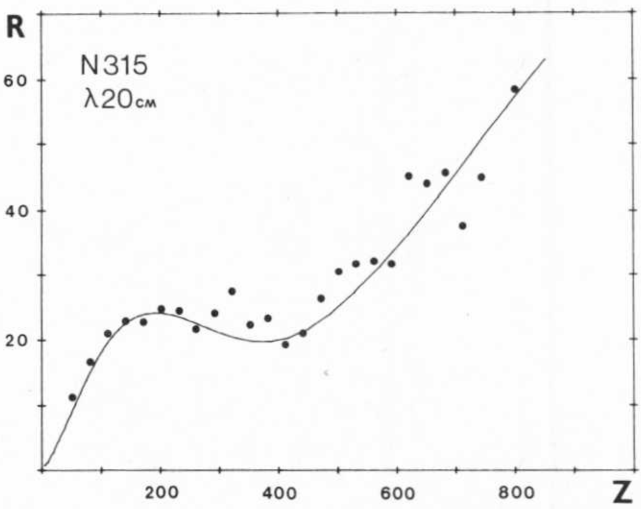

FIG. 6—The filled circles show the half-width between half maxima of the transverse intensity profiles of the brighter jet in NGC315 at 20 cm, plotted against angular separation from the radio core, both in arc sec. The curve shows a model with $\varepsilon_B = 0$ (no dynamical effects of the magnetic field), beam radius at the sonic point $R_s = 0.75$ arc sec, sonic height $z_s =$ 5 arc sec, initial pressure index $m = 4$, second pressure index $m' = 2.5$, second scale height H = 140 arc sec, and transition height $z_e = 52$ arc sec.

sentially *magnetic* characteristic of these models – the slow oscillation of the beam radius beyond the initial collimation shoulder – is not evident in the data. The jet in NGC315 may not expand at a *uniform* rate beyond $z \sim$ 400 arc sec, but there is no direct evidence for a radial *oscillation*. While acceptable models of this kind can be found, provided ε_B is less than ~ 0.01, the major features of the NGC315 data can be provided by models in which the magnetic fields (which must be present for us to detect the beam as a radio jet) have no significant effects on the beam *dynamics*.

The curve shown in figure 6 illustrates a fit to the data of a model with $\varepsilon_B = 0$, in which the collimation shoulder therefore results entirely from the two-component structure of the assumed external pressure. The fitted model is intermediate in character between those shown above in curves (B) and (C) of figure 3. The radius of the beam at the sonic point is 0.75 arc sec (180 pc), the sonic point is 5 arc sec (1.2 kpc) from the radio core, and the initial pressure index $m = 4$ (as in section 5). The scale height H of the second pressure is 140 arc sec (33 kpc), and its power law has $m' =$ 2.5. The two pressures are equal at $z_e = 52$ arc sec (12 kpc).

The minimum external pressures required by this model can be estimated from the usual equipartition calculations for the synchrotron jet; the calculations for NGC315 made by Willis *et al.* (1981) imply that nT products of order 10^4 to 7×10^2Kcm^{-3} in the halo of NGC315 would be sufficient to confine the jet laterally at distances of 15 to 200 kpc from the radio core. If the halo temperature were $\sim 2 \times 10^7$K, number densities n of 5×10^{-4} to 3×10^{-5} cm^{-3} would therefore be required at these two distances. Such densities are comfortably below those of several times 10^{-2} to 10^{-3} cm^{-3} deduced for the gas at comparable heights in a 3×10^7K isothermal X-ray halo of M87 by Mathews (1978). The confining medium required by this model for the collimation of NGC315 is therefore reasonable in view of present knowledge of the X-ray emission of other massive elliptical galaxies – the "nuclear" atmosphere required by the model may be an analogue of the extended X-ray emission near the nucleus of Centaurus A (Fabbiano *et al.* 1979) while the "halo" required by the model would be only a modest X-ray source in comparison with that associated with the halo of M87.

The model shown in figure 6 fits a sonic height of 5 arc sec to the collimation data, and this regime of the jet has been mapped by Bridle *et al.* (1979) at 4885 MHz. The emission from the jet at this frequency was too weak to be detected for the first ~5 arc sec from the radio core, then brightened significantly for the next ~5 arc sec of its length. We are encouraged by these observations, as the passage of the flow through the sonic point should generate shocks and turbulence which can be expected to accelerate particles and hence increase the synchrotron emissivity of the beam. The correspondence between the length of the "gap" at the base of the jet at 4885 MHz and the sonic height we have independently fitted to the collimation data may therefore be an indirect check on the validity of our description of this flow.

7. *Summary of Results*. We can summarise our physical results as follows:

(i) Beam confinement by a single-component external power-law pressure cannot produce the high degree of collimation observed in most extragalactic sources unless unrealistically flat pressure variations are assumed,

(ii) a single-component external pressure combined with an organized beam magnetic field whose energy is ~1 per cent of the beam kinetic energy can collimate the beam effectively by magnetic pinching, but would do so at the expense of exciting large-amplitude and long-wavelength oscillations of the beam radius which would probably destroy a real beam, and which are not seen in long radio jets observed at the VLA,

(iii) beam models with two-component external confining pressures can be made to fit the observed collimation properties of radio jets rather well. Even when such models do not contain dynamically important magnetic fields, they may exhibit "collimation shoulders" between regimes characterised by different expansion rates, as has been observed in NGC315. Amongst the models of this kind which do contain organised magnetic fields there is a class of "magnetic cylinder" models whose properties are attractive as pipelines for the energy transfer in highly collimated powerful sources. The condition $\varepsilon_B \sim 1$ per cent required for these models probably corresponds to field strengths near the "equipartition" values for observed radio jets,

(iv) the magnitudes of the external pressures required to confine the observed radio jets are (at least in low-luminosity radio galaxies) consistent with the existence of "nuclear" and "halo" atmospheres that are within the limits set by present X-ray observations of massive galaxies,

(v) the transition from dominant longitudinal to dominant transverse magnetic field that has been observed in several low-luminosity radio jets need not imply that these jets are freely-expanding.

8. Directions for future work. The model described here can evidently simulate important aspects of the observed collimation behaviour of radio jets, but many other observable properties of the jets remain to be described. Among the problems still to be treated are:

(i) The jet emissivity. This has two aspects – that of producing the population of relativistic electrons, and that of calculating their radiation pattern in the dynamically self-consistent magnetic field. Our description of the possible collimation mechanisms offers some solutions to the severe problem of adiabatic losses, especially in the "magnetic-cylinder" models, but the beams will still require replenishment of the energy lost to radiation by the particles moving at large pitch angles to the fields. The needed "reacceleration" may simply take the form of scattering of paraxial particles to higher pitch angles by the thermal component of the beam or by field irregularities (e.g. Spangler 1979b). Whatever the particle production and replenishment mechanism may be, calculations are required of the intensity and polarization of the synchrotron radiation from a population of relativistic particles with a general pitch angle distribution in the magnetic fields described by our equations. It would then be possible to compare the predictions of the models with the detailed intensity and polarization maps now obtainable with the VLA. In the models with large oscillations in beam radius, the effects of varying magnetic field strengths along the beam on the particle pitch-angle distributions may in fact be very significant.

(ii) The jet boundary. The pressure-balance boundary condition (9) in

our model probably defines the boundary of *streaming* relativistic particles. The finite gyro-radii and scattering of the particles will however cause diffusion out of the beams and may lead to the formation of low-intensity "sheaths" around them. Moreover, as discussed by Baan (1980), the same processes produce a viscous interaction between the beam material and its surroundings which could lead to significant entrainment of thermal material by the beam. In our model, where the magnetic field is supposed continuous at the beam boundary, entrainment will be enhanced due to shearing of the field (e.g. Henriksen 1970).

(iii) The flexibility of jets and beams. Some observations of radio jets show that the underlying beams can be stable to non-axisymmetric perturbations which result in "wiggles" or bends in the jets. We need to understand the mechanisms which impart such flexibility to the beams, and which can cause their gradual deflection without disruption. Given the emerging evidence for misaligned substructure (twists) within elliptical galaxies (e.g. Williams and Schwarzschild 1979; Schweizer 1980) it may be profitable to study the refraction of beams which propagate through pressure distributions whose components are misaligned. It might then be possible to obtain simultaneous constraints on "nuclear" and "halo" confining media in radio galaxies from data on the collimation and deflection of their radio jets.

(iv) The termination of the beams. We also need a description of the processes by which beams and jets terminate to form radio lobes, and the phenomena associated with Rees' proposed "working surface", which are also becoming amenable to observation with radio interferometers (e.g. Willis and Strom 1978, Burch 1979, Perley *et al.* 1980). A beam that is not disrupted by internal instabilities or by external pressure gradients must come to an end at a limiting distance z_L where

$$p_e(z_L) = \rho_L W_{zL}{}^2$$

Given that $R_L{}^2\rho_L W_{zL} = R_s{}^2\rho_s W_{zs}$ from (21), we should find that

$$W_{zL} \gtrsim R_L{}^2 p_e(z_L)/p_{es}$$

at the ends of any radio jets which do not wiggle or bend significantly along the path to their lobes. If radio observations can determine the terminal beam radius R_L of such a straight jet, this relation could be used as a consistency check for models of its collimation.

(v) The central regime. Finally, any acceptable beam model must ultimately be interfaced to a model of the central regime wherein the hot nuclear fluid and the magnetic fields that permit tracing of the beams are generated. Recent VLBI studies of compact radio cores in the radio

galaxies NGC6251 (Readhead *et al.* 1978) and 3C390.3 (Preuss *et al.* 1980) show that there are small-scale one-sided jets within a few parsecs of the active objects in these galaxies. The relationship of such small-scale jet structures to the emission gaps and one-sided bright features that are observed at the bases of the large-scale jets discussed here has still to be clarified. Chan and Henriksen (1980) have speculated that the heating mechanism for the fluid which supplies the large-scale jets may involve the entrainment of thermal material by small (pc-scale) relativistic jets in the radio cores. We are extending our treatment of heavy beams to include relativistic internal energies and relativistic bulk motions in order to study such possibilities.

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