GEOMETRY IN THE SKY - THE COSMOLOGY OF THE GREEKS

Cosmology is the term used to describe the attempt to discover and interpret orderliness in what Man perceives to be the principal features of the Universe. In the previous chapter we saw some of the orderliness of the celestial cycles that were discovered in the early days of recorded human history. We shall study other ancient observations, particularly those of the planets, later. At this point we will look at early cosmological interpretations of the observations described so far; these will provide a starting-point from which to explore the path along which the scientific world-view has evolved in more modern times. We will examine only the cosmologies of the Greek-dominated Mediterranean culture, not because the Eastern world was uninformed or disinterested at this time, but because the evolution of cosmology from the Greek ideas of 2300 years ago to our modern view shows particularly clearly the interdependence of progress in astronomy, mathematics and physics.

1. Heavenly Perfection

We receive no impression of depth or perspective as we view the starry sky. Wherever we travel on Earth, no constellation changes size or position relative to the others so as to suggest that it is appreciably nearer or further than the rest. Are the stars therefore all so remote that differences in their distances from Earth are imperceptible or are they simply all at the same distance from us? The second possibility attracted the Greek thinkers from around 500 B.C. onwards because it harmonised with their concept of perfection in the heavens.

The Greeks knew that the layout of stars in the major constellations had not changed in recorded time; this encouraged the concept of basic immutability - immortality - of that stellar layout. The precise repetition of the daily cycle of the stars, in fact far more accurate than any timing device then available, was an impressive example of a perfectly regular motion. In contrast, the motions of terrestrial objects, whether motions brought about by Man or by natural forces, were transient - subject to friction and other imperfections. The apparent motions of Sun, Moon and stars around the Earth also traced out circles on the sky, and circles were considered to be the perfect geometrical curves on a plane. The Sun and Moon also appeared circular in outline, providing another example of geometrical perfection in celestial bodies.

If the circle was the perfect curve in two dimensions, then the perfect form in three dimensions was the sphere. From the sixth century B.C. onwards the cosmology of the geometrically-adept Greeks tended inexorably towards the concept of the sphere and the circle as the basic elements of organisation in the Universe. If the stars were points of light firmly embedded on the interior of a huge sphere over Man's head, this would produce the observed lack of depth or perspective in the stellar sky. Endless, regular rotation of the unchanging star sphere around the Earth would then give the appearance of the daily cycle of the stars.

The Sun and Moon were expected to be spherical bodies travelling at different rates around circular paths centred on the Earth. As well as this being consistent with the overall circular appearance of both bodies, the phases of the Moon could then be explained in terms of the lunar sphere being illuminated by light from the Sun.

The change in size of the circumpolar region of sky with changing position on Earth suggested that Earth itself might be basically spherical. The Greeks noted that Earth's shadow cast on the Moon at the time of a lunar eclipse showed a curved outline, also supporting the idea of a spherical Earth. (Evidence which seems whimsical today was also adduced - for example the fact that such remarkable beasts as elephants were known both to the South-East (India) and the South-West (Africa) of Greece, suggesting that South-East and South-West both led to the same strange region of Earth, as they would on a sphere.)

The cosmological significance of the sphere, embodying the ultimate in geometrical perfection, was firmly entrenched in Greek thought by the time of Aristotle (384-323 B.C.).

2. The Terrestrial Sphere

In contrast with the perfection and immutability of the heavens, Earth was seen as an arena of imperfection and transience. Even if the Earth were basically spherical, its surface was obviously marred in detail by mountains and valleys - deviations from the perfect scheme. Man-made motion was transient, requiring the continual expenditure of effort (force) to maintain it; for example, although wheels on simple bearings turned with a motion mimicing that of the sky overhead, they would cease turning because of friction unless something were done to maintain their motion. All movements and changes associated with the

Earthly arena of the Universe were impermanent and imperfect compared with the changelessness and regularity of the heavens.

But there occurred on Earth a pattern of natural movement that, while imperfect, seemed to be inexorable. There were four fundamental "elements" of the Greek description of terrestrial matter - Earth, Water, Air and Fire. Of these, Earth and Water were observed always to seek lower places on the Earth when set in motion by natural processes, while Air and Fire were perceived as leaping upwards. The Greeks interpreted this underlying organisation of natural motions in terms of an almost animalistic desire of the different forms of matter to seek their "proper place" in the arrangement of the terrestrial sphere. They supposed, in effect, that a rock at the top of an incline had a built-in urge to seek its "proper place" in the scheme of things. Provided with the least opportunity, such as the disturbance of soil near it by wind or rain, it would propel itself downhill to the lowest accessible position. The mass of human experience of natural motions near the Earth's surface was rationalised in terms of the innate desires of Earthy and Watery bodies to move downwards, while Airy and Fiery bodies strove to move upwards, perhaps towards the starry 'fires' overhead.

But what defined "up" and "down"? It was not enough to visualise innate tendencies towards motion in the different elements of matter; a guiding direction was also necessary. Here a crucial role was played by the underlying, if flawed, sphericity of the Earth. A sphere defines a unique point in space, its centre. If Earth were spherical, the naturally-occurring motions of the Earthy and Watery bodies could be explained as the desire of these elements to seek a <u>unique</u> single point in space - the geometrical centre of the Earth. Indeed Earth's (imperfect) spherical shape could have arisen from the competition among these elements to reach that ultimate "proper place", producing an essentially symmetric accumulation of matter around Earth's centre, with no large masses unduly remote from it.

This unique place at the centre of the Earth seemed also to be the centre of the star sphere and the centre of the apparent motions of the Sun and Moon. It thus came to be regarded as the centre of the Universe.

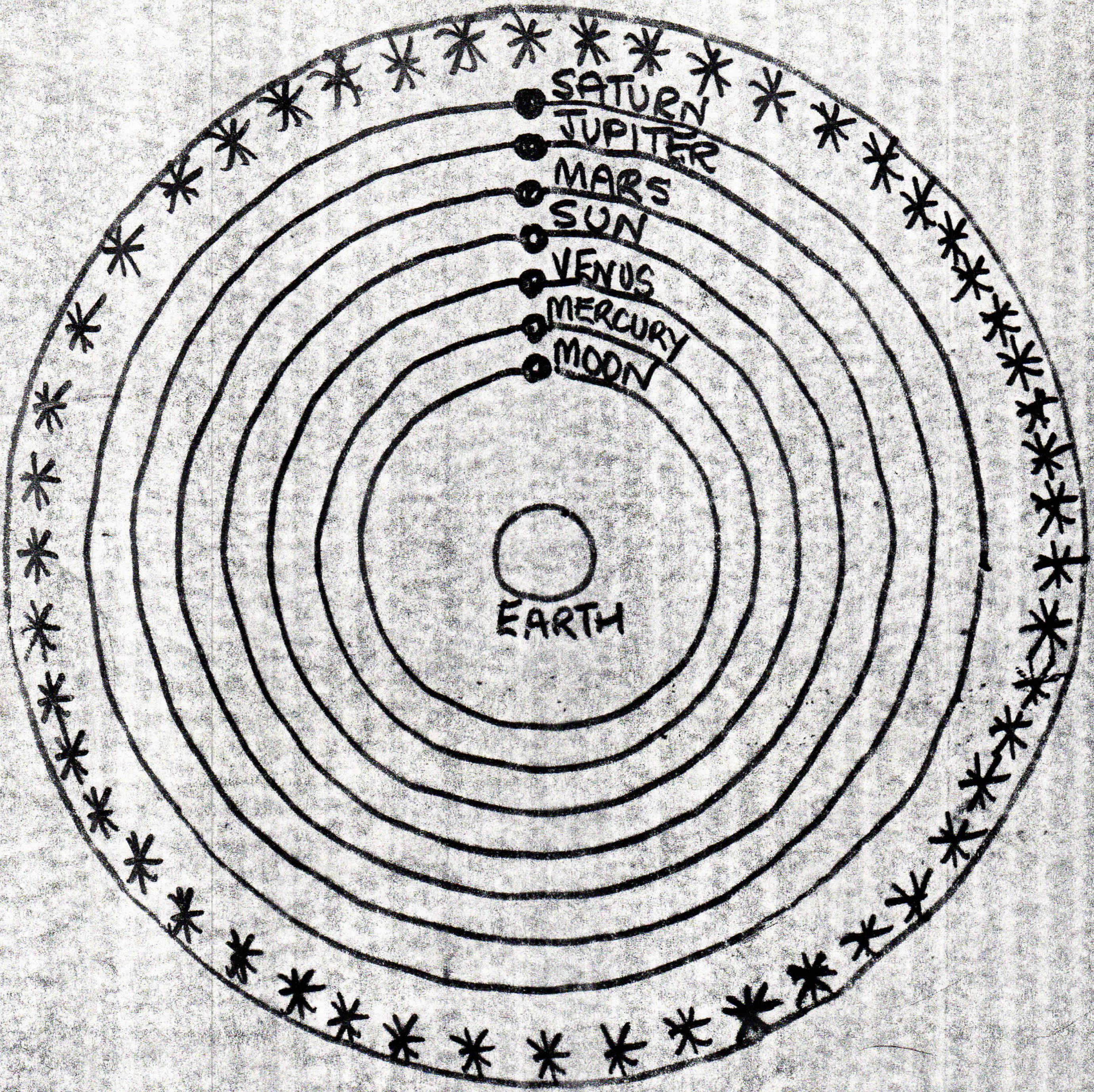
3. The World-model of Aristotle

The geometrical, astronomical, philosophical and physical thinking behind this cosmology evolved gradually over several centuries but was expressed particularly clearly by Aristotle, whose model of the Universe is illustrated schematically in Figure 1.

In Aristotle's model the Earth is a unique, somewhat flawed stationary sphere occupying the central region of the Universe. Motions of objects at the surface were dominated by the primal urge of matter to seek its proper place in the scheme - the elements Earth and Water seeking the centre of the Earth and the elements Air and Fire fleeing it. Around and including Earth was a spherical arena of imperfection and impermanence. The behaviour of the Airy and Fiery elements near the upper boundary of this realm of imperfection was thought to produce such transient sky phenomena as meteors ("shooting stars") and comets. Far above the surface of the Earth there occurred a transition to a fundamentally different realm of heavenly phenomena, beginning with the realm of the Moon, which was thought to be embedded in an invisible spherical shell of unknown material. From the transition region outwards the laws of motion were different from those on Earth; the various heavenly bodies did not seek the centre of the system but revolved endlessly and regularly around it in motions whose simplicity and perfection increased with distance from Earth. The motions of the spherical shells containing the Moon and Sun were imagined to be interlocked with the motion of the outermost shell, in which the stars were embedded. The ultimate perfection of the primal motion of the star sphere was imagined to be converted into the more complex but regular motions of the Sun and Moon as if through some arrangements of smoothly-operating gears.

An added complication of the model represented the five bright, apparently starlike objects which had been seen constantly changing position against the background of the stars, each in its own peculiar but regular pattern. These conspicuous "wanderers" (planets) were named Mercury, Venus, Mars, Jupiter and Saturn. Their motions were continuous but their cycles complex (as we shall see later); they were simply added as a factual detail, their significance perhaps a metter for debate. Their motions were clearly heavenly rather than terrestrial in character, so they were placed beyond the Moon in the general scheme.

This simple model of the Universe embodied some remarkable notions. It was built on a concept of mathematical perfection - the perfect symmetry of the sphere. It represented philosophical ideas of the uniqueness and centrality of the Earth and of finiteness in the Universe. It distinguished two main



CONCEPTION OF UNIVERSE IN TIME OF ARISTOTLE (ca. 350 B.C.)

▤ HEAVENLY PERFECTION, ENDURING
▤ BASE IMPERFECTION, TRANSIENT

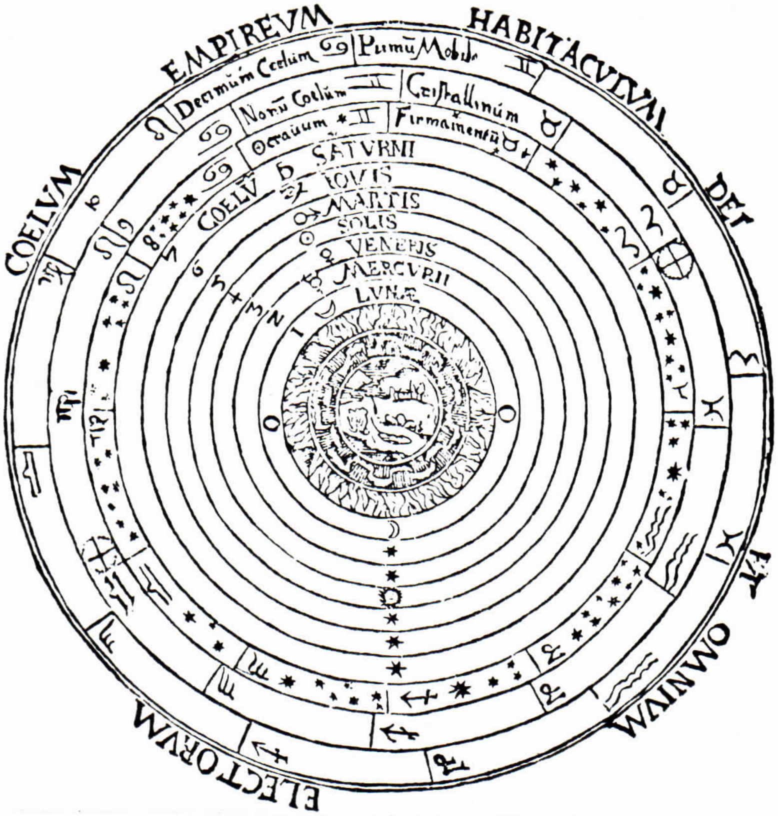


FIG. 36.—The celestial spheres. From Apian's *Cosmographia*.

arenas wherein different physical laws operated, characterised by the form of the natural motions that occurred in them. It was in satisfactory agreement with the astronomical facts available to observers 2300 years ago who were not equipped with precise instruments for measuring celestial configurations. It provided a harmonious basis for philosophy, theology, astronomy and physics which was competent to deal with most of the available information about the physical world. These positive aspects of the model, combined with the immense authority of its principal exponent - Aristotle, the then peerless investigator in physics, chemistry, astronomy, biology, medicine, politics, rhetoric, logic and metaphysics - gave it an influence on Western culture which was to last for about 2000 years. If you find it quaint and primitive, then you should try to identify the information in your own immediate experience (not reported for you by TV, or read in a textbook) which would be in conflict with it. Unless you have been an exceedingly astute observer of the natural world, it is unlikely that you would have incontrovertible evidence that the Aristotelian model is unsatisfactory based on your <u>own</u> observation. There was however an observer soon after Aristotle's time who became convinced that this scheme was quite wrong; his arguments failed to carry the day, but they are of interest as an early example of the importance to science of quantitative observation of carefully-selected phenomena.

4. Aristarchos of Samos

Aristarchos of Samos is thought to have lived from about 310 to about 230 B.C., i.e. approximately eighty years after Aristotle. The book in which he made his most remarkable contribution to cosmology has not survived to modern times, but Archimedes in 216 B.C. wrote in his "The Sand Reckoner" the following description of Aristarchos' teaching:

> You are aware that "Universe" is the name given by most astronomers to the sphere, the centre of which is the centre of the Earth, while its radius is equal to the straight line between the centre of the Sun and the centre of the Earth. This is the common account, as you have heard from astronomers. But Aristarchos brought out a book consisting of certain hypotheses wherein it appears, as a consequence of the assumptions made, that the Universe is many times greater than the "Universe" just mentioned. His hypotheses are that the fixed stars and the Sun remain unmoved, that the Earth revolves about the Sun in the circumference of a circle, the Sun lying in the middle of the orbit,

> and that the sphere of the fixed stars, situated about the same centre as the Sun, is so great that the circle in which he supposes the Earth to revolve bears such a proportion to the distance of the fixed stars as the centre of a sphere bears to its surface.

Remember that such a Sun-centred (heliocentric) cosmology did not displace the Artistotelian Earth-centred (geocentric) cosmology throughout Europe until the Seventeenth Century A.D. - well over 1800 years later! Yet Archimedes clearly says that Aristarchos proposed that the Earth moves around the Sun in a circular orbit, not the Sun around the Earth; furthermore that the stars must be <u>incomparably</u> more distant from the Earth than is the Sun - hence his analogy to the infinite proportion between the <u>centre</u> of a sphere (an infinitesimal point) and its <u>surface</u> (a finite area). The nature of Aristarchos' cosmology is also clear in this passage from Plutarch (in his "On the Face in the Moon's Disc"):

> Cleanthes ... thought it was the duty of Greeks to indict Aristarchos of Samos on the charge of impiety for putting in motion the Hearth of the Universe, this being the effect of his attempt to save the phenomena by supposing the heaven to remain at rest and the Earth to revolve in an oblique circle, while it rotates at the same time about its own axis.

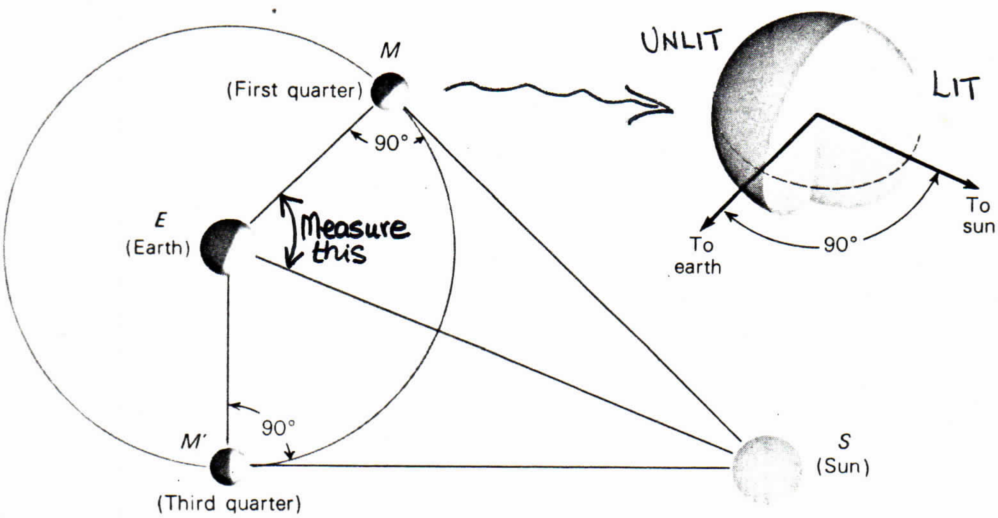
Clearly Aristarchos taught not only that the Earth revolved around the Sun (producing the appearance of the Sun's yearly cycle against the stars) but also that it rotated on its own axis (producing the daily cycle of the stars). Apart from his postulating a circular, not elliptical, orbit for the Earth Aristarchos' description parallels the modern view of the Solar System and its dimensions relative to interstellar spaces. The man Cleanthes died in 232 B.C., so we can be sure that Aristarchos had these ideas over 1750 years before the publication of the 'classic' treatise on the Sun-centred Universe by Copernicus in 1543 A.D. Why was Aristarchos so far ahead of his time, and why were his teachings ignored?

Lacking the most vital piece of evidence - Artistarchos' own book - we cannot be sure of the logic which brought him to his immensely prescient conclusions. But time has preserved his treatise "On the Sizes and Distances of the Sun and Moon", from which it is clear that he had deduced from visual observation that the Sun was much larger than the Earth. Aristarchos achieved this insight over two thousand years before the invention of space probes and radar measurements, by applying the branch of mathematics which the Greeks developed most - geometry - to the interpretation of carefully-chosen observations.

Aristarchos had first noted that when the Moon is seen in the "Half Moon" phase the Sun's light must be falling on it at right angles to the line of sight from the Earth (Figure 2). His geometer's knowledge told him that if one other angle in the right-angled triangle formed by Sun, Moon and Earth at this instant could be measured, then he could determine the ratio of the lengths of two sides of the triangle. He accordingly estimated the angle between the Sun and the Moon in the sky at the time of the "Half Moon" phase. This angle is not at all easy to measure. The angle is very close to a right-angle (90^o), the Sun is intensely bright and half a degree across, and the exact time of "Half Moon", crucial to the geometry, is difficult to estimate. Aristarchos gauged this crucial angle to be "less than a quadrant by a thirtieth of a quadrant", i.e. 87^o, from which he inferred that "the distance of the Sun from the Earth is greater than eighteen times, but less than twenty times, the distance of the Moon". In fact the angle should have been $89^o51'$ (much too close to the right-angle for visual estimation) and the Sun 400 times more distant than the Moon, but Aristarchos' attempt had given him a conclusion that was correct in principle if not in detail - the Sun was much more remote from Earth than was the Moon.

He next observed that Sun and Moon subtend the same angle at the Earth when the Sun is totally eclipsed (Figure 3) and inferred from this that the ratio of their sizes must be the same as the ratio of their distances. Hence the Sun was (by his reckoning) between 18 and 20 times larger than the Moon.

It remained for him to estimate the relative sizes of Earth and Moon. Here again Aristarchos mixed geometrical ingenuity with shrewd observation. In essence, his argument was based on the observation that the Moon moves by $1/2^o$, its own apparent diameter, in about one hour relative to the stars. This fact could be related to the relative sizes of Earth and Moon by <u>timing</u> eclipses of the Moon (Figure 4). At an eclipse of the Moon, the Moon passes through the Earth's shadow. The time it spends fully in the shadow therefore measures the size of the shadow in relation to the size of the Moon. As the Sun is very distant from both, the Earth's shadow at the Moon is about the same size as the Earth. Depending on the exactness of the Sun-Earth-Moon alignment at a particular eclipse, the time spent by the Moon in Earth's shadow can vary, but it is longest when the alignment is most precise. The longest time that the Moon is observed to be fully eclipsed is $1^h\ 40^m$ (1 2/3 hours); inspection of Figure 4



Aristarchus's method of measuring the relative distances of the sun and moon.

Fig. 2

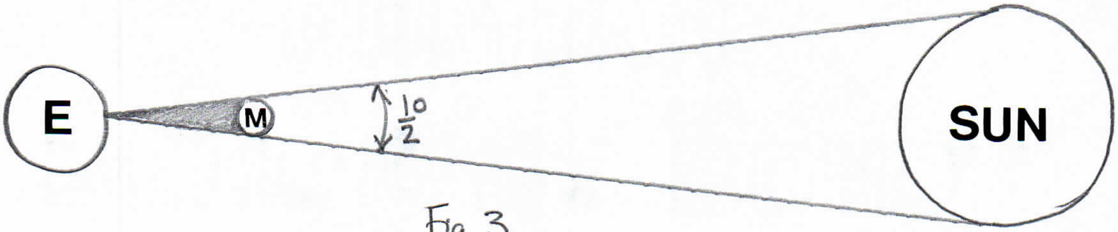


Fig. 3

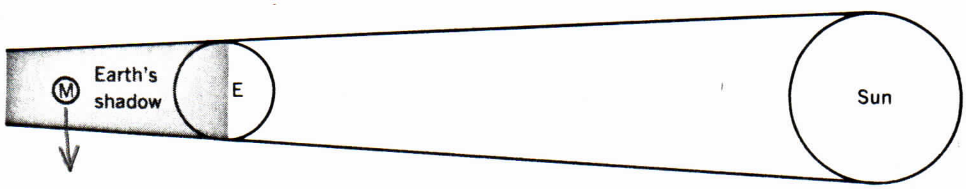


Fig. 4

should then convince you that, as the Moon moves through its own diameter in an hour, the diameter of the Earth's shadow at the Moon must be 2 2/3 times the diameter of the Moon. To a fair approximation this means that the diameter of the Earth itself must be 2 2/3 times that of the Moon. Aristarchos made such a calculation and combined it with his Sun-to-Moon ratios to conclude that the Sun was much larger than the Earth.

Presumably it then clashed with his intuitions to suppose that the greater body (Sun) revolved around the lesser (Earth). Once he assumed that the Sun lay at the centre of the system, he would have been forced to the conclusion, in conflict with "common sense", that the Earth moved - not only yearly in orbit around the Sun, but also daily around an axis through the two celestial poles. If the stars were not to show different perspectives when viewed from different places in Earth's orbit they must then be effectively infinitely remote compared with the distances to the Sun.

It is a tragedy in the history of cosmology that Aristarchos' written reaction to this awesome concept has been lost in the confusion of time.

5. Aristarchos Ignored

Aristarchos' insightful discussion seems to have had no lasting impact on Greek cosmology. He had virtually no followers, let alone important followers, in the centuries which followed. In the very "Sand Reckoner" in which Archimedes reported Aristarchos' theory he argued that Aristarchos cannot really have meant to utter such an absurdity as the infinite proportion of the distance to the stars.

If the effectively infinite distance to the stars was unacceptable, then even more so was the concept of a moving Earth. Among the arguments advanced against the daily motion of the Earth at this time were

a) Why did arrows fired vertically upwards not get left behind as the Earth turned, and so fall to the West of the spot from which they were launched?

b) Why were clouds that were loose in the atmosphere not always left behind to the West as the Earth turned?

c) Why would loose material on the surface of the Earth not fly off it like mud from the rim of a spinning wheel? Indeed why would the Earth itself not be disrupted into pieces by a constant rotation?

Against the yearly motion it was argued that this would leave no recognisable "proper place" to be the focus of the strivings of Earthly and Watery materials. The description of natural motions near Earth's surface required a unique centre for these materials to seek; how could this centre change constantly as the Earth moved around its alleged orbit? More generally, if the Earth moved with the planets in orbit around the Sun, then what distinguished Earthly from heavenly matter to produce the obvious differences between material motions near Earth and those seen in the heavens?

The remark attributed to Cleanthes about Aristarchos' "impiety" illustrates that still more was at stake. If Earth moved as a "heavenly" body around the Sun, was the imperfect terrestrial sphere then moved into the perfect heavenly regime to defile its order - or was the perfection of the heavens illusory and the whole Universe flawed?

With hindsight in these matters we can see what lay at the root of these difficulties - concepts of motion and its origins were fuzzy and based on limited experience of Man-made circular motions from which things tended to break away and of natural motions which appeared to be classifiable into two regimes. The circular motions of the heavenly bodies could not be explained - indeed their inexplicable perfection was a cornerstone of cosmology and philosophy. They needed no explanation, but were a manifestation of that which distinguished the outer regions of a finite Universe from the imperfection of what lay, with Man, at its core.

It is not as though the Aristotelian school had a complete description even of terrestrial motion. The path of a javelin could not be described satisfactorily - it should have plunged to Earth seeking its "proper place" as soon as it was released by the thrower, and elaborate assumptions about its displacement of the surrounding air were necessary to reconcile Aristotelian principles with the observation of its continued flight. To have accepted Aristarchos' cosmology would however have threatened what small understanding they had, putting nothing in its place to compensate the loss.

This was too great a price to pay merely to accommodate some few geometrical facts about the skies. Aristarchos' astronomical achievements were praised, but his cosmological deductions ignored. Although he had actually enunciated the major facts of a modern description of the layout of the local Universe, the path of science was to turn away from his insight for almost 2000 years. Cosmology here was like a fogbound traveller wandering on one side of a chasm, who stood next to the bridge leading to the other side yet could not see it; and who wandered on until the fog cleared, whereupon it became necessary to retrace some difficult steps which had been made while the fog still persisted.

By the time the nature of motion itself had been subjected to more careful investigation, cosmologists had more than the observations to contend with; they faced the new charge of "heresy" in the face of the dogma of powerful churches.