the current view[16,17] that it is quite different from NML Cygnus and is in fact a long-period variable. However, Low and Smith[5] thought that some T Tauri stars may also be pre-planetary systems and so high infra-red (and maybe visual) polarization would be expected in such stars. More polarization observations of NML Cygnus and other infra-red sources should yield more data and lead to the solution of this problem.

Since the rest of this paper was written a new observation has become available that adds weight to the idea that NML Cygnus is very similar to the T Tauri stars. Rubin, Ford and Christy[19] have observed emission-line nebulosity partly surrounding the infra-red star. This is extremely reminiscent in both form and nature to the nebulae that partly surround[20] T Tauri stars such as T Tauri itself and LkHα 120.

I am grateful to various colleagues for discussions on the contents of this paper, notably Dr. D. Lynden-Bell, Dr. R. G. Bingham and Mr. J. B. Alexander.

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NOTES FROM OBSERVATORIES

The Spectral Indices of Radio Sources

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Introduction

Several authors[1,2,3] have pointed out that if we take a sample of radio sources containing all the sources in a given area of sky with flux densities at a given frequency in excess of a specified value, then the relative numbers of sources with given spectral indices will depend on the frequency at which

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the sample is selected. A sample selected at a low frequency will include a greater proportion of sources with high values of the spectral index than will a similar sample selected at a high frequency.

We have examined the distribution of spectral indices in samples of sources selected at 10, 38, 178, 408, 610 and 1421 MHz, and find that the distribution varies with frequency in a way which implies that the spectral indices of the majority of sources follow a gaussian distribution with dispersion only $\pm 0{\cdot}11$.

Theory

For $\sim$80 per cent of radio sources the flux density $S(\nu)$ is related to the frequency ν by a power law $S(\nu) \propto \nu^{-\alpha}$ within the frequency range under consideration. If the sources radiate by the synchrotron mechanism, such a power law arises from a population of relativistic electrons with a distribution of energies, E, obeying the law

$$dn(E) = n_0 E^{-\gamma} dE$$

where $dn(E)$ is the number of electrons per unit volume with energies between E and $E+dE$, and $\gamma = 1+2\alpha$. The value of γ and the dispersion in this value among different radio sources are important parameters for theories of the physical processes giving rise to the sources.

It has been shown[1,2,3] that in the simple case where all sources have power law spectra and where the distribution of the spectral indices α in a complete sample of sources selected at a frequency ν_0 is gaussian with a mean spectral index α_0 and a standard deviation σ, the distribution of the spectral indices of sources selected at some other frequency, ν, will also be gaussian with standard deviation σ, but with a mean spectral index $\bar{\alpha}(\nu)$ given by

$$\bar{\alpha}(\nu) = \alpha_0 - \mu\sigma^2 \log_e(\nu/\nu_0)$$

In this expression μ is the exponent in the population law $N(S) \propto S^{-\mu}$ giving the number N of sources per steradian of the sky with flux densities in excess of a specified value S.

Distributions of spectral indices other than the gaussian give a non-linear relationship between $\bar{\alpha}(\nu)$ and $\log_e(\nu/\nu_0)$, although it may be difficult to detect the non-linearity over the range of frequencies considered in the present analysis.

Observations

Surveys of radio sources which are complete above a given limit of flux density over a selected area of the sky have now been carried out at several observatories at frequencies between 10 MHz and 1421 MHz. Table I indicates the limiting flux densities adopted and the areas of sky covered by each of these surveys. Many sources are observed in more than one of these surveys, and for these sources the spectral indices have been determined by plotting $\log[S(\nu)]$ against $\log[\nu]$ and using a least-squares technique to fit the best straight line to the observed points. It is possible to choose a limiting flux density at each frequency such that the spectral index is known for every source with a flux density greater than the limit adopted at that frequency; the sample of sources so obtained is then a "complete" sample in the sense used above. Because the uncertainties in determining spectral indices are greater for weaker sources, the limiting flux density at each frequency was chosen so that the estimated errors in the spectral indices did not broaden the distribution of α by more than 20 per cent. The flux density limits chosen at each frequency are indicated in Table II.

TABLE I

Surveys of Radio Sources

Frequency (MHz)	*Observatory*	*Limiting Flux Density* (10^{-26} w m^{-2} Hz^{-1})	*Area of Sky Covered*	*Ref.*
10·03	Dominion Radio Astrophysical Obs., Penticton, Canada.	150	R.A. 0^h to $16^h 30^m$ Dec. $+20°$ to $+60°$	(4)
38	Mullard Radio Astronomy Obs., Cambridge, U.K.	14 to 24 according to Dec.	R.A. 0^h to 24^h* Dec. $-10°$ to $+90°$	(5)
178	Mullard Radio Astronomy Obs., Cambridge, U.K.	2·0	R.A. 0^h to 24^h Dec. $-07°$ to $+80°$	(6, 7)
408	National Radio Astronomy Obs., Parkes, Australia.	3·5	R.A. 0^h to 24^h† Dec. $0°$ to $+20°$	(8)
610·5	Vermilion River Radio Obs., Illinois, U.S.	0·8	R.A. 0^h to 24^h Dec. $+40°$ to $+44°$	(9)
1421	Owens Valley Radio Obs., California, U.S.	1·2	R.A. 0^h to 24^h Dec. $+23°$ to $+30°$	(10)

*Except for certain regions near intense sources or near the galactic plane.
†Except for $|b^{II}| < 10°$.

Analysis of the Spectral Indices

The mean spectral index $\bar{\alpha}$ at each frequency was determined; the values obtained are listed in Table II. If these values are plotted against $\log_e[\nu]$, the relationship is found to be linear, and is best fitted by the expression

$$\bar{\alpha}[\nu] = 1{\cdot}005 - 0{\cdot}048 \log_e[\nu]$$

where ν is in MHz. The slope of this line gives the value $0{\cdot}048 \pm 0{\cdot}003$ for $\mu\sigma^2$, which for $\mu = 1{\cdot}8$ gives[14] $\sigma = 0{\cdot}16 \pm 0{\cdot}01$.

This relationship, which is similar to that obtained from more limited data by Williams and Stewart[13], accurately predicts the mean and standard deviation of the distribution of spectral indices of complete samples of sources selected at any frequency. If, however, we attempt to fit a gaussian form to the distribution actually observed at any frequency, the fit is poor for the following reasons:

1. The majority of sources observed at any frequency has spectral indices which are distributed in a gaussian manner with a standard deviation which is significantly less than 0·16.
2. At the highest frequencies a number of the sources selected has extremely low spectral indices. Many of these sources are optically identified with quasi-stellar objects and have radio angular diameters less than 1″ arc and spectra which deviate significantly from a power law. Some sources of this type have spectra which show a sharp cut-off

at low frequencies which is most probably due to synchrotron self-absorption[15]. Such a sharp cut-off may only occur in a source with a uniform brightness temperature, but it is also probable that low values of the spectral index may be caused by synchrotron self-absorption in sources which have a number of components with different brightness temperatures. In such sources the observed spectral index is determined partly by the energy spectrum of the relativistic electrons and partly by the distribution of the brightness temperature within the source. In order to study the energy spectrum of the relativistic electrons, we have therefore removed from the analysis all sources known to have angular diameters less than 1″ arc and whose spectra cannot be represented by a power law.

3. At the lowest frequencies a number of the sources observed has extremely high values of the spectral index. The spectra of the majority of these sources also deviate from a power law, with the spectral index increasing as the frequency decreases. Inspection of the histogram of spectral indices at 10 MHz suggests that the sources with $\alpha > 1{\cdot}2$ belong to a distinct population. These sources will be discussed in more detail in a later paper[16].

The sources discussed in (2) and (3) above were removed from the analysis, and the values of $\bar{\alpha}$ for the remaining sources were redetermined. The corrected values are given in Table III. This table also indicates the number of sources which were removed from the analysis for the reasons given above; the numbers of sources involved are small although their effect on the mean and standard deviation of the distribution is large. The data for 1421 MHz were not used in this second analysis as very few of the sources selected at this frequency have been observed in sufficient detail to enable those with small angular diameters and spectra which deviate from a power law to be removed.

Table II

The Mean Spectral Indices of Samples of Sources Selected at Different Frequencies

Frequency (MHz)	*Limiting Flux Density in the Sample* (10^{-26} w m^{-2} Hz^{-1})	*Mean Spectral Index* $\bar{\alpha}$	*Ref.*
10·03	150	0·89±0·03	
38	37	0·82±0·02	(11)
178	6·7	0·77±0·01	(11, 12)
408	5·8	0·72±0·01	(11)
610·5	3·0	0·70±0·02	(13)
1421	1·2	0·63±0·03	

All samples exclude sources with $|b^{II}| < 10°$.

If the data from Table III are analyzed as before it is found that the variation of the mean spectral index of the main population of sources with frequency satisfies the relationship

$$\bar{\alpha}(\nu) = 0{\cdot}870 - 0{\cdot}021 \log_e[\nu]$$

This expression gives $\mu\sigma^2 = 0{\cdot}021 \pm 0{\cdot}003$, which for $\mu = 1{\cdot}8$ gives

$\sigma = 0{\cdot}11 \pm 0{\cdot}01$. If we now attempt to fit the gaussian distribution predicted by this expression to the corrected distributions of the spectral indices at different frequencies, the fit is satisfactory (Fig. 1). In examining this figure it must be remembered that no attempt has been made to fit each individual gaussian curve to the histogram of the observed distribution, so that the goodness of fit at each frequency is an independent test of our interpretation of the data.

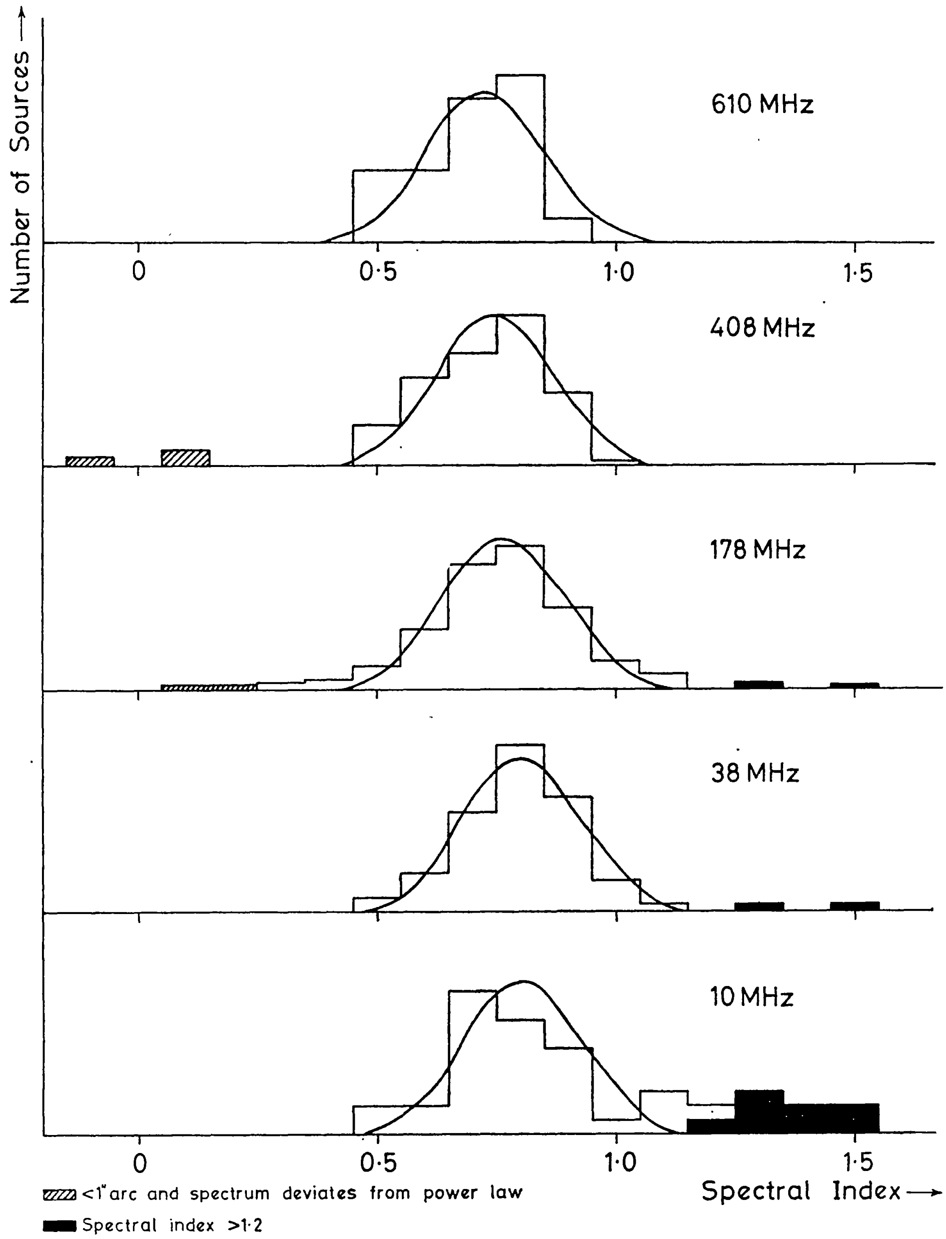

FIG. 1

Histograms of the distribution of spectral indices in complete samples of sources selected at several frequencies. The gaussian distribution predicted for the main population at each frequency is also shown.

TABLE III

Corrected Mean Spectral Indices at Different Frequencies

Frequency (MHz)	*Number of Sources Included in Reduced Sample*	*Number of Sources Excluded from Reduced Sample*	*Mean Spectral Index* $\bar{\alpha}$
10·03	33	*(*a*) 0 †(*b*) 9	0·78 ± 0·03
38	62	(*a*) 0 (*b*) 2	0·80 ± 0·02
178	229	(*a*) 4 (*b*) 2	0·76 ± 0·01
408	82	(*a*) 3 (*b*) 0	0·73 ± 0·02
610·5	20	(*a*) 0 (*b*) 0	0·70 ± 0·02

*(*a*) Denotes number of sources excluded because their spectral indices $\alpha > 1 \cdot 2$.

†(*b*) Denotes number of sources excluded because their radio angular diameters $< 1''$ arc and their spectra deviate from power law.

Conclusions

The results may be summarized as follows:

(*a*) The spectral indices of the majority of sources follow a gaussian distribution with a standard deviation of 0·11, implying a standard deviation of 0·22 in the distribution of γ. The narrowness of this distribution suggests that in the physical process producing relativistic electrons in a radio source the parameters which determine the relative numbers of electrons at different energies are very similar from source to source.

(*b*) Observations at the high frequencies indicate the presence of a population of sources with unusually low spectral indices. In these sources the spectral index of the radio radiation probably does not reflect the distribution of energies of the relativistic electrons but is partly determined by the process of self-absorption. If these sources represent an early stage in the evolution of the main population, the spectral index of their radiation will gradually increase until it assumes a value characteristic of this population.

(*c*) Observations at the low frequencies indicate a further group of sources with spectral indices $> 1 \cdot 2$. It is possible that in these sources also the radio spectral index does not reflect the energy spectrum of the relativistic particles as they are produced in the source. These sources may represent a late stage in the evolution of the main population in which the spectral index has been increased by radiation losses or by the escape of the most energetic particles from the source region.

Acknowledgments

One of us (A.H.B.) thanks the Science Research Council for the award of a Research Fellowship.

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The Variable Star HD 170682 in M25

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Mitchell[1] has announced that the star HD 170682 (=CPD −19° 6881), a member of the open cluster M25 (Johnson's[2] number 50) has recently brightened by $0^{m}\cdot20$. From mid-1955 to mid-1961 the star appeared to remain constant at about $V=7^{m}\cdot92$. However, from mid-1964 to mid-1966 the star was observed to have $V=7^{m}\cdot73$. No significant changes were observed in the $B-V$ or $U-B$ colours during this whole period. Spectroscopic observations of this star have been made by a number of investigators in the past. Hayford[3] classified the star as B6, Feast[4] as B6 V, Wallerstein[5] as B7 III, and Wampler[6] *et al.* as B5 III. The HD type is B8. No spectral peculiarities were noted by any of these investigators, whose work preceded the brightening reported by Mitchell. However, spectra obtained on 1967 May 3, 12 and 16 with the Cassegrain spectrograph of the 74-inch Radcliffe reflector (48 Å/mm at $H\gamma$) show the moderately broad ($V \sin i \sim 150$–200 km/sec) $H\beta$ absorption to have a strong central emission component. Such an emission component was absent on the earlier Radcliffe spectrum with the same camera (1955 July 27). Our 1967 spectra show $H\alpha$ to be a strong emission line. Unfortunately, so far as we are aware, none of the earlier spectroscopic observations covers the red spectral region, but the star does not appear to have been detected in any objective prism $H\alpha$ survey whilst three other stars in or close to M25 have been listed[7,8] as showing bright $H\alpha$. It seems justifiable therefore to assume that the brightening reported by Mitchell and the development of Be characteristics are related. In view of the fact that the star has, presumably, recently ejected a shell it is of interest that on our first plate (1967 May 3) the main $H\alpha$ emission line is accompanied by a sharp, fainter, bright line displaced about 950 km/sec to the blue. This spectrum was well widened and there seems little doubt that the line is