NEW LIMITS TO THE MAGNETIC FIELD STRENGTHS WITHIN SOME RADIO SOURCES

By A. H. Bridle

*Mullard Radio Astronomy Observatory, Cavendish Laboratory**

The flux densities of most non-thermal radio sources increase with decreasing frequency over the range of frequencies in which they have been measured, but those of a small number of sources exhibit maxima at low frequencies[1,2]. The occurrence of maxima is correlated with high radio brightness temperatures in these sources[1,3] and, if it is assumed that the sources radiate by the synchrotron mechanism, the maxima may most plausibly be interpreted as due to synchrotron self-absorption[4,5]. In this communication the low-frequency spectra of a number of sources are discussed in relation to this process, making use of newly available measurements of the flux densities of sources at 10·03 MHz[6] and of the angular diameters of sources at 178 MHz[7].

Synchrotron self-absorption produces a maximum in the flux density of a radio source at a frequency ν_m given in MHz by the equation

$$\nu_m^{2.5+\alpha_0} = 4\,F(\alpha_0)\,S_0\,\nu_0^{\alpha_0}\,(1+z)^{\frac{1}{2}}\,B^{\frac{1}{2}}\,\theta^{-2} \qquad (1)$$

where z is the redshift of the source, θ its angular diameter in seconds of arc at the frequency ν_m, and B the magnetic field strength in 10^{-6} gauss. S_0 is the flux density in 10^{-26} w m^{-2} Hz^{-1} at a frequency $\nu_0 > \nu_m$ where the spectral index α defined by $S(\nu) \propto \nu^{-\alpha}$ has the value α_0. $F(\alpha_0)$ is a numerical factor which is close to unity for $0·3 \leqslant \alpha_0 \leqslant 1$. This equation is derived with the assumption that the radio-emitting region is cubical with constant emissivity throughout its volume, and with one face perpendicular to the line of sight[4,5]. It may be shown[8] that the value of ν_m is not significantly different from that given by equation (1) if the source is in the form of a sphere, or of a spherical shell, provided the emissivity is constant throughout it. If the source is inhomogeneous, equation (1) gives the value of ν_m for each component region. The flux density of each component will decrease at frequencies less than the corresponding ν_m, and the spectral index α of the total radiation from the source will therefore decrease below each of the frequencies at which this occurs.

The equation may be used to derive an upper limit to the magnetic field strength within a source whose low-frequency spectrum and angular structure are known. If the observed spectrum of a source has a maximum at a frequency ν^*, $\nu_m = \nu^*$ if this maximum is caused by synchrotron self-absorption, and $\nu_m < \nu^*$ if it is caused by some other process[2]. In either case, ν^* provides an upper limit to ν_m, from which an upper limit to $B^{\frac{1}{2}}\theta^{-2}$ within the source may be derived from equation (1). If θ is known, an upper limit to B may therefore be found. From equation (1), the value of this upper limit depends on $\nu_m^{(5+2\alpha_0)}$ and θ^4. This sensitivity to ν_m makes the new observations at 10·03 MHz of particular value, as by extending radio source spectra to this low frequency we are able to place much stricter upper limits to B within some sources than was previously possible. The

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*Present address: Department of Physics, Queen's University, Kingston, Ontario, Canada.

TABLE I

Source	ν_m MHz	$B^{\frac{1}{2}}\theta^{-2}$	*Angular Diameter* θ	B (gauss)	*Remarks*
3C 2	< 38	< 42	At 178 MHz, 50% $< 0''\cdot1^{7}$	$<2\times10^{-7}$	QSO, $z=1\cdot037^{11}$ (*a*)
†3C 48	$\leqslant 70$	$\leqslant 120$	At 408 MHz, $0''\cdot4$ by $< 0''\cdot3^{12}$	$<10^{-4}$	QSO, $z=0\cdot367^{11}$
†3C 138	$\leqslant 80$	$\leqslant 620$	At 178 MHz, $\sim 0''\cdot15^{7}$	$\lesssim 2\times10^{-4}$	QSO, $z=0\cdot760^{11}$
†3C 147	$\leqslant 110$	$\leqslant 200$	At 408 MHz, $0''\cdot6$ by $< 0''\cdot4^{12}$	$<10^{-3}$	QSO, $z=0\cdot545^{11}$
3C 186	< 10	$< \cdot20$	At 178 MHz, $\sim 0''\cdot35^{7}$	$<8\times10^{-6}$	QSO, $z=1\cdot063^{11}$
†3C 190	< 10	$< \cdot48$	At 159 MHz, $\sim 1''\cdot6^{13}$	$<2\times10^{-6}$	QSO^{11}
3C 191	< 10	$< \cdot26$	At 178 MHz, 10% $< 0''\cdot25^{7}$ At 2·7 GHz, "point" double^{14}	$<3\times10^{-7}$	QSO, $z=1\cdot952^{11}$ (*b*)
3C 205	< 10	$< \cdot57$	At 159 MHz, $5'' \leqslant \theta \leqslant 11''^{13}$	$<7\times10^{-8}$	QSO^{11}
3C 216	$\leqslant 15$	$\leqslant 1\cdot2$	At 178 MHz, 20% $< 0''\cdot75$ or all $\sim 1''\cdot5^{7}$	$\lesssim 10^{-5}$	QSO^{11}
4C 14·31	< 10	$< \cdot54$	—	—	QSO, $z=0\cdot895^{11}$
3C 239	$\leqslant 15$	$\leqslant 15$	At 1·4 GHz, $< 10''^{15}$ At 159 MHz, $< 12''^{13}$	<2	QSO^{11} (*c*)
3C 245	< 10	$< \cdot83$	At 178 MHz, 30% $< 0''\cdot2^{7}$ At 410 MHz, components $< 0''\cdot5^{16}$	$<2\times10^{-6}$	QSO, $z=1\cdot029^{11}$ (*d*)
3C 277·1	< 10	$< \cdot53$	At 1·4 GHz, $< 10''^{15}$	$<3\times10^{-8}$	QSO, $z=0\cdot320^{11}$
†3C 286	< 38	(*e*)	At 178 MHz, all $< 0''\cdot8^{7}$ At 2·8 GHz, $< 0''\cdot05^{17}$	(*e*)	QSO, $z=0\cdot849^{11}$ (*e*)
†3C 295	$\leqslant 50$	(*f*)	At 159 MHz, components are $1''\cdot7$ by $< 1''^{13}$	(*f*)	galaxy, $z=0\cdot46^{18}$ (*f*)
†3C 298	$\leqslant 70$	$\leqslant 16$	At 178 MHz, 20% $< 0''\cdot6$ or all $\sim 1''^{7}$ At 1·4 GHz, components $< 0''\cdot6^{19}$	$<10^{-8}$	QSO, $z=1\cdot439^{11}$
3C 380	$\leqslant 18$	$\leqslant \cdot66$	At 159 MHz, $5'' \leqslant \theta \leqslant 11''^{13}$ At 2·8 GHz, $< 0''\cdot05^{17}$	$\leqslant 2\times10^{-8}$	QSO, $z=0\cdot692^{11}$ (*g*)
3C 433	$\leqslant 30$	$\leqslant 5$	At 1·4 GHz, $< 10''^{15}$	$< \cdot25$	galaxy ?^{18} (*h*)
3C 446	< 85	< 360	At 2·8 GHz, $< 0''\cdot05^{17}$	$<8\times10^{-7}$	QSO, $z=1\cdot403^{11}$ (*i*)
3C 459	< 38	< 23	At 178 MHz, $\sim 0''\cdot3^{7}$ At 2·7 GHz, components $< 3''\cdot5^{14}$	$\lesssim 4\times10^{-6}$	N galaxy, $z=0\cdot22^{18}$ (*j*)

Footnotes to Table I

(*a*) The spectral index α is constant from 2·8 GHz to 38 MHz, indicating that synchrotron self-absorption is unimportant throughout this frequency range. The value of B given in the table is that for the compact component, assuming its spectral index to be the same as that of the total radiation.

(*b*) The spectral index is constant within the error limits from 2·8 GHz to 10 MHz. A decrease in the index between 400 MHz and 100 MHz is also compatible with the data, however, and it is possible that this source is inhomogeneous, with a high-frequency component which cuts off near 300 MHz.

(*c*) The source does not exhibit interplanetary scintillation at 178 MHz, so the angular diameter at this frequency must be $\gtrsim 2''$; the magnetic field strength must therefore be $\gtrsim 10^{-3}$ gauss if the maximum in the spectrum is due to synchrotron self-absorption.

(*d*) The spectral index is constant from 1·4 GHz to 10 MHz, indicating that synchrotron self-absorption is unimportant throughout this frequency range. The interplanetary scintillation data indicate that the source contains more than one component, however, and the angular diameter measurements at 159 MHz and 410 MHz appear to conflict; further observations of the angular diameter at low frequencies are needed to resolve this discrepancy.

(*e*) The spectral index is $\alpha = 0{\cdot}5$ at frequencies above 1 GHz but decreases to 0·15 between 400 MHz and 38 MHz. The flattening of the spectrum near 400 MHz may be attributed to synchrotron self-absorption in the component observed by Palmer *et al.*[17] if the magnetic field strength is $\lesssim 2 \times 10^{-3}$ gauss, which is compatible with equipartition of energy in this component. There is probably a more extended component of the source which predominates at low frequencies.

(*f*) The variation of α near the frequency at which the flux density is a maximum is too gradual to be fitted by a homogeneous model of the source. The main component of the source probably has $\theta \sim 1''$ and $B \sim 10^{-3}$ gauss.

(*g*) The direct measurements of the angular diameter imply that the structure of the source changes between 2·8 GHz and 159 MHz. Only the low-frequency component of the source is considered here.

(*h*) The source does not exhibit interplanetary scintillation at 178 MHz, so the angular diameter at this frequency must be $\gtrsim 2''$; the magnetic-field strength must therefore be $\gtrsim 4 \times 10^{-4}$ gauss if the maximum in the spectrum is due to synchrotron self-absorption.

(*i*) The spectral index is constant from 2·7 GHz to 85 MHz, indicating that synchrotron self-absorption is unimportant throughout this frequency range.

(*j*) Based on a flux density measurement at 38 MHz supplied by Dr. P. J. S. Williams.

sensitivity to θ makes the analysis liable to error, however, if the frequency at which θ was measured is much higher than ν^*. The majority of measurements of angular diameters $< 1''$ has been made at frequencies ~ 1 GHz or higher where this angular resolution is relatively easier to obtain, and the extrapolation of angular structures measured at such frequencies to frequencies near 10 MHz must be treated with caution. It is possible that in some sources compact components detected at high frequencies will be affected by synchrotron self-absorption at lower frequencies, and that the observed low-frequency emission will come from other components of greater angular extent. In such cases the use of the high-frequency value of θ with the low-frequency spectral data in equation (1) will lead to anomalous results. As discussed above, the spectral index α should decrease near frequencies at which synchrotron self-absorption becomes important in such sources, so that if the spectral index of a source is accurately constant from frequencies near ν^* to frequencies above that at which θ was measured, it is reasonable to use this value of θ in equation (1). The recent measurements of small angular diameters at 178 MHz by the interplanetary scintillation method[7,9] are therefore of particular value.

The values of the upper limits to B which may be derived in this way are insensitive to the value of the redshift of the source, being proportional to $(1+z)^{-1}$. For sources whose redshifts are not known, the upper limit to B may be obtained from equation (1) by placing $z=0$. Finally, it may be noted that the values of B obtained are independent of the interpretation of the origin of the redshift.

Table I gives the limits which the low-frequency spectral data place on ν_m within a number of radio sources. Sources marked with the symbol † have previously been discussed by other authors[1,2,10]. The corresponding limits to $B^{\frac{1}{2}}\theta^{-2}$ are given for B in 10^{-6} gauss and θ in seconds of arc, and where possible an observed value of θ is quoted, together with the frequency at which it was determined. The upper limit to B corresponding to this value of θ is given.

TABLE II

Source	B_{eq} (gauss)	U_{eq} (ergs)	U (ergs)
3C 2	$>2\times10^{-3}$	$>4\times10^{56}$	$>2\times10^{62}$
3C 48	$>2\times10^{-3}$	$>8\times10^{57}$	$>1\times10^{59}$
3C 138	$\geqslant4\times10^{-4}$	$\geqslant3\times10^{56}$	$>1\times10^{57}$
3C 147	$>1\times10^{-3}$	$>3\times10^{58}$	$>3\times10^{58}$
3C 186	$>1\times10^{-4}$	$>2\times10^{57}$	$>6\times10^{59}$
3C 191	$>2\times10^{-4}$	$>1\times10^{58}$	$>1\times10^{62}$
3C 245	$>4\times10^{-4}$	$>4\times10^{57}$	$>3\times10^{62}$
3C 277·1	$>1\times10^{-5}$	$>1\times10^{59}$	$>1\times10^{59}$
3C 298	$>1\times10^{-4}$	$>5\times10^{57}$	$>5\times10^{57}$
3C 380	$\geqslant7\times10^{-5}$	$\geqslant1\times10^{60}$	$\geqslant1\times10^{60}$
3C 446	$>8\times10^{-4}$	$>2\times10^{56}$	$>2\times10^{61}$
3C 459	$>3\times10^{-4}$	$>3\times10^{56}$	$>2\times10^{61}$

The magnetic field strength B_{eq} corresponding to equipartition of the total energy of the radio source between the magnetic field and the relativistic particles may be calculated for sources whose redshifts are known, if it is assumed that these redshifts are of cosmological origin. Values of B_{eq} are given in Table II. These were calculated using the distance-redshift relation appropriate to the Einstein-de Sitter model universe, and assuming that 1 per cent of the total particle energy of the source is contributed by the radiating electrons. The calculations are necessarily approximate as values must be assumed for the limiting frequencies of the synchrotron emission spectra; these have been taken as ν_m and 10 GHz in each source. For many sources only upper limits to ν_m and θ are available, so that the derived value of B_{eq} is only a lower limit to this quantity.

It is interesting to compare these values of B_{eq} with the upper limits to the magnetic-field strengths derived from the low-frequency spectra. In 3C 2, 3C 48, 3C 186, 3C 245, 3C 446 and 3C 459, and possibly also in 3C 191, B_{eq} is significantly greater than the upper limit to B implied by the spectra. The energy content of these sources must therefore be greater than that predicted on the assumption that it is in equipartition. Table II gives values of the total energy U_{eq} each source would contain if it were in equipartition, and of the total energy U implied by the spectral data.

All the sources listed above except 3C 459 are identified with quasi-stellar objects. These results imply that if the redshifts of these objects are of cosmological origin, their total energies are comparable with those of the powerful radio galaxies, providing support for the postulate that the two types of radio source may represent different stages in the evolution of the same physical system[20]. 3C 459 is identified with an N galaxy, and therefore provides an example of a system whose radio properties are similar to those of the quasi-stellar sources which are not in equipartition, and whose optical properties are intermediate between those of a quasi-stellar object and of a galaxy.

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THE HELIUM CONTENT OF GROOMBRIDGE 1830

By J. B. Alexander and S. D. Pepper
Royal Greenwich Observatory, Herstmonceux

Considerable importance attaches to the problem of finding the initial helium abundance of the very metal-deficient stars in the Galaxy. Faulkner[1,2,3] and Faulkner and Iben[4] have recently discussed this subject with particular reference to the properties of extreme subdwarfs in the general field and to the post-main sequence stages of the evolution of stars in globular clusters. For main-sequence stars, the helium abundance may,