The Bohr Atom

In the old particle description the model of the atom would be given by the statement of the conservation of total energy. For the coulomb force holding the atom together

the basic picture is that of total energy being conserved; kept constant. Energy is stored in the system of the electron-nucleus or atom. The picture that the wave aspect must retain, as it is the barest minimum statement we can possibly make, is the storage of energy in the atom. But of course this is no great problem, as we know that waves can store energy - we have seen this done in standing waves! We can allow both pictures to describe the situation if we allow the electron to be some sort of wave which, while the electron circles (? old picture) the nucleus, has a "wavelength" that exactly fits into the orbit length so that in successive orbits of the nucleus the "wave" superimposes with the wave of previous orbits to set up a standing wave, and permanently store the energy which the electron possesses. If the "wavelength" doesn't have the right relation to the orbit (i.e. that the circumference is an integral number of wavelengths long) then successive waves would quite quickly add up in such a way that they will cancel, and no energy could be stored, so that the two pictures couldn't be consistent with one another, which therefore must not happen. If there is no standing wave

(wave language) storing energy there cannot be an electron orbiting the nucleus (particle language).

In order to make a more quantitative combined picture, we must find some relation of particle "wavelength" and energy. We must at least have some rudimentary idea of what a particle "wavelength" is. In fact there existed a relation in the known properties of radiation which could be thought of as connecting these two seemingly unrelated properties. This relation was used by De Broglie to correct the two aspects together for particles in a very intuitive way strongly reminiscent of the "generation" of Newton's third law. Both were obtained by an appeal to symmetry.

It had been known (deduced from the basic electric force law) that electromagnetic waves carry <u>momentum</u>. In fact the relation between the energy stored in an electromagnetic wave and its momentum is

$$E = \text{momentum} \times c \quad \text{speed of light}$$

De Broglie was aware of the photoeffect, and its statement that

$$E = h\nu$$

The combination of these two statements leads to the conclusion that

$$h\nu = \text{momentum} \times c$$

for light, and through the relation of wavelength to frequency

$$\frac{h}{\lambda} = \text{momentum.}$$

Now; this is a relation of a wave property (wavelength) to a particle property (momentum). True, it is only correct as obtained, for light. But why should it only be true for light? Light is a wave that acts like a particle at certain times. What is the difference between this and a particle that acts as a wave at times? Why can't the relation be <u>equally</u> true for waves <u>and</u> for particles? After all, there is nothing in the formula which says that you must start looking at the equation on the side describing the wave property (wavelength) and end up finding out the particle property (momentum). Surely you can equally correctly start with a particle aspect (momentum) and transform it into a wave property (wavelength). Therefore this should be a true relation for anything which has both a wave and a particle aspect to it.

If this is so, then there is a restriction put on the possible speeds that the electron can have as it circles the nucleus. Not only must it conserve energy, but it must be in a standing wave, which means that not all energies will be possible, only those which correspond to the right electron momentum for the generation of the standing wave.

This basic picture was worked out quantitatively by N. Bohr, although he didn't have the benefit of the de Broglie argument, as it had not yet been thought of. He had to "cheat" and used the same formula by seeing that it was necessary to make things work. His basic picture was to try to fit in both the classical energy equation for an atom and the standing wave condition. When he did this, he arrived at a solution for the total energy of the atom that gave

$$TE = -\frac{E.}{N^2} \quad \text{basic constant (scale factor)}$$

where N (called a quantum number) could be any whole number (i.e. 1, 2, 3, 4 ..... etc.) from 1 to 8. In effect the number N, or the quantum number (quantum is Latin for low mid) tells us how much energy there is. These energies satisfy the standing wave requirement, and in terms of our previous picture, all the possible energies a bound atom can have are

Only these energies for the atom are allowed by his solution: no others can exist. From the form of the equation he arrived at we can see that for very small numbers N, the allowed energies are widely separated, but have relatively large (negative) values. (The negative value simply means that the electron cannot escape from the atom.) For large values of N, both the separation between allowed energies and the energies themselves are very small. Also, there is no larger (negative) energy than that for N = 1.

Let's see physically what this picture implies. To do this imagine we are in the process of putting together a hydrogen atom. We will start with an electron orbiting a proton at an almost infinite separation, but the electron weakly attracted to the proton. As the electron is accelerating around the proton it will be attracted to it, accelerate, and move closer to the proton giving off radiation (this is what we said would happen before). Now we say that the electron if initially in a state of motion described by

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TE where TE = $\frac{-E_o}{N^2}$ and N is very large. When it radiates off a small amount of energy it becomes more tightly bound, and it moves immediately into a <u>different state</u> of motion, described by $TE^1 = \frac{E_o}{N^{1\,2}}$ where $N^1 = N - 1$, or maybe N - 2, etc. These differences are very small, so that a <u>continuous</u> charge, continuous variation is effectively indistinguishable from our standing wave condition (i.e. N must be some integer). In fact the standing wave requirement puts no limitation on the possible energy of the atom at first, when N is very large. Now, however, as the electron <u>continues</u> to move towards the proton, the gaps between the allowed energy values becomes appreciable, and it is no longer possible for the electron to move continuously from one state (of motion) to the other. Now it is either in one state <u>or</u> the other, and <u>not</u> in between, because of our standing wave requirement. When it changes from one state to the other, it must do so <u>almost instantaneously</u> and in that time it has to get rid of the difference in energy between the two states - by emitting a sudden burst of radiation - sometimes called a photon. This means that the large energy jumps, the radiation ceases to be continuous but is emitted at <u>discreet times</u>; the energy of these jumps is relatively large, and so (because $E = h\nu$) should occur at high frequencies. Also we notice that there will become increasingly fewer jumps emissions of radiation possible at the high frequency and of the spectrum, because there are fewer allowed states, separated by large energy differences. Finally, we see that the electron will find itself in the state with N = 1. When it gets there <u>it can no longer emit any radiation</u>, as there is no other state with lower total energy that it can get to! It <u>cannot</u> radiate, therefore it has to stay in that state of motion forever. It is <u>stable</u>!

Let's see now if this new picture helps us out with our previous difficulties. First, the ultraviolet catastophe; here the classical picture correctly predicted the low frequency emission of radiation, and the Bohr picture leaves that unaffected. However Bohr predicts that, as there is a minimum total energy allowed for an atom, there is a maximum total energy that is possible to emit by radiation, hence an upper limit to the frequency spectrum. Of course this is what is observed. The Ultraviolet Catastrophe is resolved!

The radiation collapse of the atom is also solved. Again, we see that the classical picture of a spiralling, radiating atom is correct - at first, but finally the wave character of the electron takes over, and does not allow the electron to continue getting closer to the proton after it has reached the minimum standing wave condition.

The photoeffect, as we by now might guess, is also explained. At high frequencies, or large energy charges for the electron, the process of radiation is restricted to very short time spans since the electron cannot spend time "between" the allowed states, so that the radiation occurs in bursts - as was found necessary to explain the photoeffect. In addition, we can see how there should exist a definite threshold frequency (energy) for the ejection of electrons, since the electron, when it is in its stable energy state is bound by a single amount, and if the frequency (energy) of the radiation is not high enough, it cannot provide the electron with enough to become unbound.

Of course we cannot claim to have explained the electron scattering experiment, since we must admit that we have used it to obtain all the other explanations.

This new picture is capable of more than just a qualitative description of these problems. We remember that Planck, by using as a starting assumption the features we have incorporated into our picture, was able to obtain a quantitative agreement between the predicted and observed blackbody radiation spectrum. Also, Bohr was able to make numerical predictions about the radiation spectrum of hydrogen atoms (since the term E_o consisted entirely of basic known quantities like the electron charge and mass) and the total energy by which a stable hydrogen atom is bound together. These predictions were found to be in very close agreement with the observed quantities.

In fact his description of a series of discreet frequencies of emitted radiation give us the reason that we are able to identify clouds of luminescent hydrogen in space. It is because of this generally discreet nature of the emission of radiation from all atoms that we are able to ascertain the presence of various elements in the heavens!