A Dispersion-Engineered Josephson Travelling Wave Parametric Amplifier with Periodic Impedance Perturbation

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***Abstract*—We present a new approach to develop a Josephson Junction Travelling Wave Parametric Amplifier (JTWPA) that could potentially minimise the gain-ripple effect. Our design consists of a 50 Ω superconducting niobium coplanar waveguide (CPW) periodically loaded with series of Josephson junctions (JJs) to provide the non-linearity required for wave mixing. The embedded JJs alter the characteristic impedance of the transmission line abruptly, therefore creating the stopbands in the transmission (S_{21}) for the suppression of higher harmonics and provide the means for dispersion engineering required to achieve exponential gain. The simulated gain profile of the amplifier shows that we can obtain a minimum of 15 dB gain from 4–12 GHz, close to a 100% bandwidth performance. More importantly, the characteristic impedance of the main linear transmission line remains unaffected by the strong pump, therefore ensuring the TWPA remains impedance matched to the input and output ports. This can reduce the unwanted gain undulation that inflict the optimal performance of the TWPA.**

I. INTRODUCTION

The development of cryogenic low-noise, broadband amplifiers is motivated by wide ranging technologies that require ultra-sensitive detection of weak microwave signals, such as radio astronomy [1], [2], [3] and quantum computing [4], [5], [6]. These applications generally use a high electron mobility transistor (HEMT) amplifier to amplify the weak microwave signals. However, the added noise of HEMT amplifiers is still 5–10 times the quantum limit, and their heat dissipation is high. In recent years, there has been an intense interest in developing an amplifier with quantum-limited noise performance that can achieve similar high gain over a broad bandwidth. The superconducting travelling wave parametric amplifier (TWPA) is an ideal candidate for realising such a device. TWPAs achieve high gain by wave mixing which transfer energy from a strong pump tone to the detected signal tone in a very long, ultra-low loss non-linear transmission line. The non-linearity is typically introduced by either the kinetic inductance of a superconducting wire (KITWPA), or by Josephson junctions embedded along the line. An alternative approach to realise a JTWPA is to use superconducting quantum interference devices (SQUIDs). This type of amplifier is generally referred to as SQUID-based TWPA (STWPA).

To achieve exponential gain, the power dependent phase difference between all the propagating tones needs to be minimised. This can be done by engineering the dispersion relation of the transmission line. In KITWPAs, the impedance can be periodically varied along the line to achieve phase matching, termed as the periodic loading scheme [7]. In JTWPAs, where the phase difference between the propagating waves is larger, resonators can be capacitively coupled to the transmission line [8], [9]. Dispersion engineering can also be achieved by the periodical modulation of the SQUID area in STWPAs [10]. However, this approach requires the careful control of the magnetic flux, which could complicate the experimental setup.

Generally, the non-linear inductance of a JJ is much larger than the kinetic inductance of a superconducting wire, hence JTWPAs and STWPAs require a shorter transmission line and lower pump power than KITWPAs for similar gain [8], [10]. This results in a better noise performance compared to KITWPAs, because even though the superconducting thin film displays only a minuscule surface resistivity, a high current passing through a very long line could still induce a non-negligible resistive loss in terms of I^2R.

Another challenge yet to overcome is matching the TWPA to a 50 Ω environment. Experimental gain profiles of TWPAs reported in the literature have been shown to suffer from gain ripples which arise from the reflections caused by the imperfect impedance matching at the input and output port of the amplifier [7], [8], [10].

Here, we present a new method to design a JTWPA that could potentially be better matched to a 50 Ω environment, while retaining its low noise performance. A unique feature of our TWPA design is that the main transmission of our device is made up of a linear passive circuit component in the form of a CPW, hence the characteristic impedance of the line will not be altered by the strong pump. This in turn ensures that the impedance between our device and the input/output 50 Ω interfaces is well-matched, minimising the gain-ripples caused by reflections. Furthermore, we use periodic loading instead of a large number of resonators to achieve phase matching, simplifying the design significantly. The loading sections, comprising of sets of JJs, are embedded along the centre conductor of the CPW at equal distances, locally altering the characteristic impedance, as well as serving as the non-linear medium for wave mixing. These periodic sections create stopbands in the transmission (S_{21}) and provide the required excursion in the otherwise linear dispersion relation. Eliminating the need for resonators not only greatly simplifies the fabrication process to increase yield, it also helps to reduce the footprint of the amplifier.

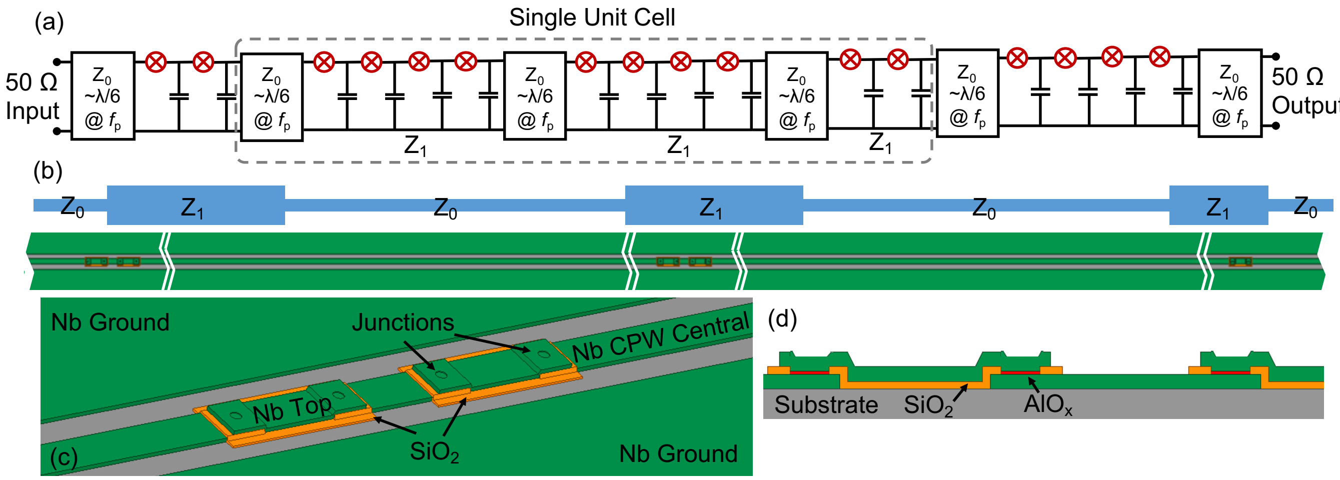

Fig. 1. (a) Circuit diagram: CPW is matched to the 50 Ω input/output. The unit cell consists of three CPW sections and three sets of JJs where one set has fewer (or more) junctions. The total length is half the wavelength of the central frequency. (b) Schematics of the amplifier: sets of JJs are periodically embedded in the centre conductor of a 50 Ω CPW, locally altering its characteristic impedance to Z_1 (c)&(d) Drawings show how the tunnel junction can be embedded along the CPW line. Gaps are created along the central conductor of the CPW, where thin layer of SiO_2 is deposited to protect the junction, before the top Nb layer is used to connect the CPW line.

II. Design Concept

There are two approaches to realise the periodic loading scheme with JJs. One option is to equally space the JJs along the transmission line, and periodically alter the junction size (thus the characteristic impedance) of a few of them to create the required stopbands and dispersion gap. However, this approach is hard to realise in reality. The transmission line is long compared to the junction size, thus the number of junctions required would be formidable from the fabrication point of view. To reduce the number of JJs, they could be distributed sparsely along the main transmission line, but in this case, they no longer behave as distributed elements, but rather as discrete lumped elements, and exponential parametric gain may not be possible.

A better solution is therefore to embed sets of JJs only at the loading sections. The junctions in a set are packed closely together to ensure that they behave as a distributed element. The spacing between the sets is chosen to be approximately one sixth of the wavelength (λ_c) of the central/pump frequency. This creates a large stopband at three times the pump frequency to suppress the unwanted third harmonic that may cause a shock wave in the amplifier. Additionally, every third of these sets have a reduced (or increased) number of junctions, to create the auxiliary stopbands and divergence in the wave vector near the pump frequency for exponential gain.

Fig. 1(a) and (b) illustrate the concept of our periodically loaded JTWPA. The low loss transmission line is expected to be realised as a niobium (Nb) CPW line, shown in green, in which the geometry will be optimised to obtain 50 Ω characteristic impedance. At the loading sections, junctions are embedded in the central conductor, altering the characteristic impedance from Z_0 to Z_1 at every $\sim \lambda_c/6$. The junction sections will be designed such that their dimensions match the width of the central conductor, as shown in Fig. 1(c). The single unit cell consists of three 50 Ω CPW sections, three sets of JJs where the third set has fewer (or more) junctions, as highlighted by the dashed box in Fig. 1(a). The length of such a unit cell corresponds to half the wavelength of the central frequency.

The tunnel junctions considered here comprise of niobium superconducting electrodes sandwiching an aluminium-oxide tunnelling barrier, as shown in Fig. 1(d). We expect to fabricate the junction array using the standard trilayer technology via Self-aligned Niobium Etch Process (SNEP) [11], [12], as shown in Fig 1(c) and (d). First, the Nb-AlO_x-Nb trilayer is formed across the entire wafer. Next, the trilayer is etched away to pattern the ground plane and the 'bottom' centre CPW strip line. The junctions are then defined by etching off the top Nb layer of this remaining trilayer apart from the area where the junctions are located. Before depositing the top Nb 'wiring' layer, a thin (few tens of nanometer) layer of SiO_2 is deposited around the junctions and in the gaps along the 'bottom' centre CPW strip line, to ensure that there is no electrical contact between the bottom and the top junction electrodes. Finally the 'top' Nb layer is deposited to complete the fabrication of the TWPA.

The choice of using Nb as the main superconducting material to form the transmission line and the JJs is mainly motivated by our aim to develop an ultra-low noise amplifier as readout for our superconductor-insulator-superconductor (SIS) receiver. This approach would allow the amplifier to operate at liquid helium bath temperatures, therefore a convenient integration is possible with SIS mixers. Higher gap temperature superconductor can be used as well in theory, but as mentioned earlier these type of high gap superconducting films will unavoidably have higher surface resistivity which could cause higher losses. Furthermore, fabrication of high-gap superconducting tunnel junction still remains a very challenging task [13], [14]. The use of Nb technology will also allow us to further integrate the TWPA with the SIS mixer on a single chip forming an integrated receiver in the future.

III. Simulation & Analysis

Based on the idea presented above, we have designed a periodically loaded JTWPA with a series of identical 1 μm^2 tunnel junctions embedded along a CPW line. The critical current of the junction is set to $I_C = 3.24\,\mu$A, with junction inductance $L_J = 101.48$ pH, and junction capacitance $C_J = 72.7$ fF. Each tunnel junction is shunted with a capacitor of 12 fF, increasing the impedance of the loading section from 50 Ω to about 93 Ω, creating a large and abrupt impedance perturbation. In each unit cell, the 50 Ω line is interrupted by two sets of 4-JJ arrays, and a third set of 2-JJ array loading sections. The distance between these sets of JJ arrays is approximately $0.15\lambda_c$ with the 4-JJ array occupying about $0.02\lambda_c$ and the 2-JJ array about $0.01\lambda_c$, resulting in a total of $\lambda_c/2$ long unit cell.

The combination of these design parameters produces deep stopbands in the S_{21} transmission profile and causes the dispersion near these gaps to diverge exponentially away from the otherwise linear relation, as shown in Fig 2(a). Here, we cascaded 70 unit cells together to form the JTWPA, which means a total of 700 junctions are required for each amplifier. The pump frequency is set to be near 8 GHz to obtain the extra dispersion required to compensate for the phase mismatch of all the waves travelling down the line. The large stopband near 24 GHz is created to suppress the third harmonic of the pump wave.

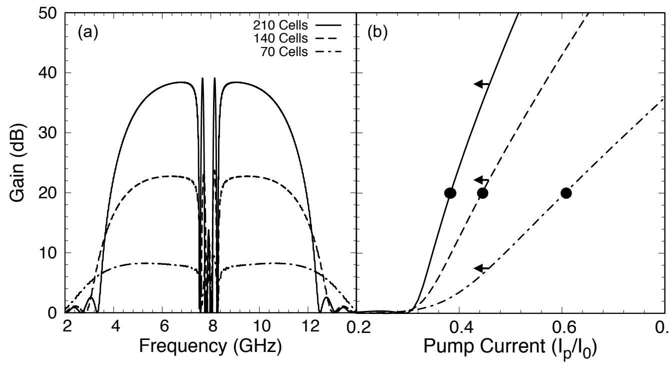

Fig. 3. (a) The gain-bandwidth product of periodically loaded JTWPAs with three different lengths: 70, 140 ,and 210 unit cells. The pump current is set to $I_p = 0.46 I_C$ for all curves simulated. As expected, with longer transmission length, the interaction time between the signal and the pump waves is longer, hence higher gain is achievable. (b) The parametric gain as the function of the pump current for the three different cases. The left-pointing arrows show the gain at 7 GHz, where $I_p = 0.46 I_C$. To achieve 20dB gain at 7 GHz, the 210-cell design requires only $I_p = 0.4 I_C$, whereas the 140-cell design requires $I_p = 0.48 I_C$, and the 70-cell case requires $I_p = 0.64 I_C$.

Fig. 2(b) shows the gain curve of our periodically loaded JTWPA. As can be observed, we managed to achieve more than 15 dB gain from 4–12 GHz, a 3:1 (100%) gain-bandwidth product, with the maximum gain approaching 25 dB. This high gain is made possible by setting the pump frequency near the first stopband, around 8 GHz, where the dispersion increases rapidly. By carefully choosing the position of the pump frequency (f_p=9.79 GHz), we can reduce the total phase mismatch to be close to zero ($\Delta_k \rightarrow 0$) across a large bandwidth, as shown in Fig. 2(c). The corresponding pump current required in this case is approximately $I_p = 0.65 I_C$. For comparison, we plot the gain-bandwidth product and the phase mismatch of the same JTWPA without the periodic loading scheme, i.e. uniformly spreading the same number of JJs along the 50 Ω line, while keeping the same pump wave settings. It is clear that without the periodic loading scheme, the mismatch Δ_k is too large for high gain at all frequencies, therefore not only the maximum gain is reduced from close to 25 dB to about 10 dB, but the relative bandwidth is reduced significantly as well.

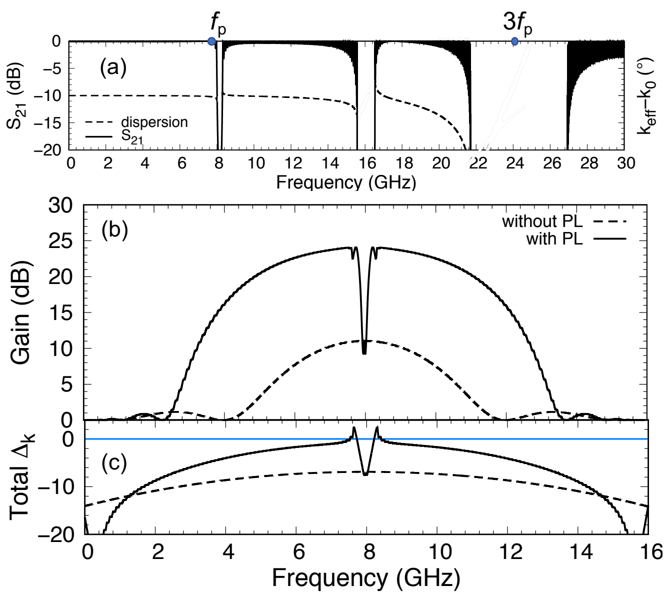

Fig. 2. (a) The dispersion relation of the amplifier shows divergencies where there is a stopband in the S_{21} profile. The ripple in the S_{21} curve is caused by the large mismatch between the loading section and the 50 Ω transmission line which is compounded due to the cascade of a large number of unit cells. (b) Gain profile simulated for the periodically loaded JTWPA comprising of 70 unit cells. It shows that at least 15 dB of gain is achievable across approximately 100% bandwidth. The pump frequency is set to 9.79 GHz, with a pump power of $I_p/I_C = 0.65$. The dashed curve shows the JTWPA without periodic loading scheme, but with the same number of JJs embedded uniformly along the CPW line. (c) The total phase mismatch between all waves i.e., the pump, signal, and idler, as the function of the frequency. Without the phase matching technique, the minimal phase difference is far from $\Delta_k = 0$, hence high exponential gain is not possible. By setting the $f_p = 9.79$ GHz, we obtain the additional dispersion correction required to achieve $\Delta_k \rightarrow 0$ across a broad bandwidth, hence achieving high gain over wide operational bandwidth.

As mentioned, high pump current is undesirable since it might incur high resistive loss in the form of I^2R, thus degrading the noise performance of the amplifier. For relatively thin Nb film at microwave frequency, the surface resistance should be minuscule at cryogenic temperature, compared to a high gap superconducting film of the same thickness. However, depending on the quality of the superconducting Nb film, the surface resistive loss could still be significant, especially in the case of a TWPA where the transmission length is long. This is particularly concerning for operation at liquid helium bath temperature, at only 40% of the critical temperature of Nb. A potential solution to eradicate this predicament is to reduce the pump power at the expense of a longer transmission length. This means the number of Josephson junctions needs to be increased as well. Fig. 3(a) shows the comparison of the achievable gain for three different periodically loaded JTWPAs with different lengths: the original design comprising of 70 unit cells, a 140-cell design, and a 210-cell design (700 JJs, 1400 JJs, and 2100 JJs, respectively). In this example, we fixed the pump power to be half of the value previously considered, i.e., $I_p/I_C = 0.46$, and adjusted the pump frequency for optimal gain. As shown in Fig. 3(a), with longer transmission length, we can achieve a higher gain, as expected. In the case of the 140-cell design, the gain-bandwidth product is almost the same as for the original design, but requiring only half the pump power. This is further illustrated in Fig. 3(b) where the left-pointing arrows show the peak gains of 8, 22, and 38 dB for the 70-cell, the 140-cell, and the 210-cell designs, respectively. Another way to depict the effect of longer transmission lengths is shown by the black dots in Fig. 3(b). To achieve 20 dB peak gain, a 210-cell JTWPA requires only about $I_p/I_C = 0.36$ while the 70-cell design requires close to $I_p/I_C = 0.6$, $2.78\times$ higher pump power. Nevertheless, it is worthwhile noting that the longer transmission length may complicate the fabrication of such a TWPA, as the number of tunnel junctions required becomes much higher which might affect the fabrication yield. However, with the mature Nb tunnel junction technology, we believe that TWPAs with such high number of tunnel junctions are realisable.

IV. Conclusion

We have presented a broadband Josephson junction travelling wave parametric amplifier with close to 15 dB-100% gain-bandwidth product. Phase matching is achieved by periodically embedding sets of JJs in the centre conductor of a linear 50 Ω coplanar waveguide. Because of the linearity of the main transmission medium, this amplifier has a great promise to display a ripple-free experimental gain curve. Although the design presented in this paper was focused on using Nb as the main superconducting material for both the transmission line and the tunnel junction, the simplicity of the design means that it can be adapted for other superconducting materials as well. For example, replacing niobium with aluminium would make the amplifier suitable for quantum bit (qubit) platforms that are being developed for quantum computation and information applications. These platforms generally use aluminium technology to construct the qubits and the corresponding superconducting circuitry, therefore on-chip integration of an aluminium TWPA is feasible without complicating the fabrication procedure.

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