Effect of Lifetime Broadening of Superconducting Energy Gap on Quasiparticle Tunneling Current

Takashi Noguchi, Toyoaki Suzuki, Akira Endo, Masato Naruse, Yasunori Hibi,

Hiroshi Matsuo and Yutaro Sekimoto

Abstract—We numerically study the quasiparticle tunneling current of SIS junctions by taking into account the lifetime broadening of energy gap. It is demonstrated that calculated dc I-V curves agree well with those of the SIS junctions measured at temperatures below 4.2 K. It is predicted that the subgap current is strongly dependent on bias voltage at low temperatures unlike the prediction of the BCS tunneling theory. It is also interesting to note that the decrease of a subgap current always saturates at a certain temperature when the temperature goes down. These predictions on the subgap current at low temperatures are consistent with the behavior of the subgap current of SIS junctions experimentally obtained. It is indicated that it is essential to suppress the lifetime broadening of energy gap as low as possible to invent SIS junctions with an extremely low subgap current at low temperatures.

***Index Terms*— Leakage currents, Submillimeter wave detectors, Superconductor-insulator-superconductor devices, Superconducting device noise, Tunneling**

I. INTRODUCTION

WE have been developing Superconductor-Insulator Superconductor (SIS) junction detectors as single pixel elements for large direct detector arrays at submillimeter wavelengths [1]. Although up to date, SIS-junction detectors with $NEP \approx 10^{-19}\ \mathrm{W}/\sqrt{\mathrm{Hz}}$ have been demonstrated [2, 3], such detectors must have background-limited sensitivity in future space observatories and further effort is needed to achieve SIS-junction detectors with background-limited sensitivity, $NEP \approx 10^{-19}\ \mathrm{W}/\sqrt{\mathrm{Hz}}$. In order to achieve such high- sensitivity SIS detectors it is most essential to reduce electrical noise of SIS junctions.

The electrical noise of a SIS junction is proportional to square root of its subgap current I_{SG} as

$$NEP = \frac{h\nu}{\eta}\sqrt{\frac{I_{SG}}{e}} \quad \left(\frac{\mathrm{W}}{\sqrt{\mathrm{Hz}}}\right), \qquad (1)$$

where η is the quantum efficiency of the detector and I_{SG} is the quasiparticle current at an operating voltage V and is dependent on temperature as

$$I_{SG} \propto \exp\left(-\frac{\Delta}{k_B T}\right), \qquad (2)$$

where k_B and T are Boltzmann's constant and temperature, respectively. The large discrepancy between the sensitivity theoretically predicted and that experimentally obtained comes from the fact that the measured I_{SG} at low temperature is several orders in magnitude greater than that expected from (2). Such a discrepancy between measured and calculated I_{SG} at low temperatures has been generally reported in many papers on SIS junctions [2, 4, 5]. It has been widely recognized that extra current passing through defects or conductive paths in the tunneling barrier dominates over the intrinsic thermally-excited quasiparticle tunneling current at low temperatures. Thus a lot of efforts have been made to improve the quality of the tunneling barrier of SIS junctions to reduce the excess current. Nevertheless, such a SIS junction with a subgap current as low as that predicted by the BCS theory has never been invented. We have found evidences that the subgap leakage current at 4.2 K weakly depends on the quality of Nb films. To verify the hypothesis that the subgap leakage current is dependent on the quality of Nb films, we have been making theoretical calculations of the quasiparticle tunneling current in SIS junctions [6]. In this paper, we describe the procedure of the calculations of the quasiparticle tunneling current in a Nb/Al·AlO_x/Nb SIS junction with an ideal tunneling barrier, taking into account the imaginary part of the gap energy of Nb films as a parameter representing their quality. It will be shown that the calculated dc *I-V* characteristics for Nb/Al·AlO_x/Nb SIS junctions are consistent with those measured at 4.2 K. It will be also demonstrated that the calculated quasiparticle current at a subgap voltage in the SIS junction shows strong voltage dependence at low temperatures unlike the prediction of the BCS theory. Then we are investigating subgap tunneling current of Nb/Al·AlO_x/Nb SIS junctions experimentally and theoretically to find a way to lower it.

II. CALCULATION OF QUASIPARTICLE CURRENT

A. Quasiparticle states inside the gap

The quasiparticle tunneling current I_T through an ideal tunneling barrier in a SIS junction is generally given by

$$I_T = A\int_{-\infty}^{\infty} N_r(E-eV)N_l(E)\{f_r(E-eV)-f_l(E)\}dE\,, \quad (3)$$

where A is a constant and $N_{r,l}(E)$ and $f_{r,l}(E)$ are densities of states and Fermi distribution function at energy E, respectively, and defined as

$$N_{r,l}(E) = \Re\left[\frac{E}{\sqrt{E^2-\Delta_{r,l}^2}}\right] \qquad (4)$$

and

$$f_{r,l}(E) = \left\{\exp\left(\frac{E}{k_B T}\right)+1\right\}^{-1}, \qquad (5)$$

where the subscripts r and l represent right and left superconductors and $\Delta_{r,l}$ are the gap energies for the superconductors on both sides [7]. Since the gap energy $\Delta_{r,l}$ are assumed to be real in the conventional calculations of tunneling current I_T, there is no quasiparticle state inside the energy gap and quasiparticle densities of states show an infinitely steep peak at the gap energy. Thus a very sharp step-like rise of current is predicted at the gap voltage on the I-V characteristics of a SIS junction. However, the current rise at the gap voltage is rounded in the actual SIS junctions. To explain the rounded I-V characteristics near the gap voltage, a complex superconducting energy gap, expressed as $\Delta_{r,l} = \Delta_1 - i\Delta_2$, has been introduced [8, 9], where Δ_1 and Δ_2 are real numbers. Calculated densities of states using (4) for superconducting Nb with (dotted and solid lines) and without (thin line) the imaginary part of gap energy Δ_2 are shown in Fig.1. It is clearly shown that the infinitely steep peak at the gap energy is suppressed by the existence of imaginary part of the gap energy Δ_2. It is also interesting to note that a little but finite numbers of quasiparticle states are caused at energy deep inside the gap if the gap energy has an imaginary part.

In Fig.1 calculated dc I-V curves using (1) and (2) with complex gap energies are plotted. It should be noted here that a finite amplitude of tunneling current through the quasiparticle states inside the energy gap is predicted at the subgap voltages. The amplitude of the quasiparticle tunneling current at a subgap voltage increases as the magnitude of the imaginary part of gap energy Δ_2 increases.

To determine the magnitude of the imaginary part of the gap energy Δ_2 of Nb films in actual Nb/Al·AlO_x/Nb junctions, curve fitting between calculated and measured I-V characteristics at 4.2 K is made. A small knee structure just above the gap voltage and a reduction of gap voltage are usually observed on the dc I-V curves of Nb/Al·AlO_x/Nb SIS junctions, which come from the superconducting proximity effect at the Nb/Al interface of the bottom electrode. The proximitized gap energy in the Al layer is obtained by solving McMillan's recursive equations [10], taking a complex number of energy gap for Nb into account, and is used to calculate the tunneling current. The energy gap of the top electrode is assumed to be free from the proximity effect and identical to that of the Nb film in the bottom electrode. The imaginary part of energy gap Δ_2 is determined so as to give a best fit to the measured I-V curve at 4.2 K.

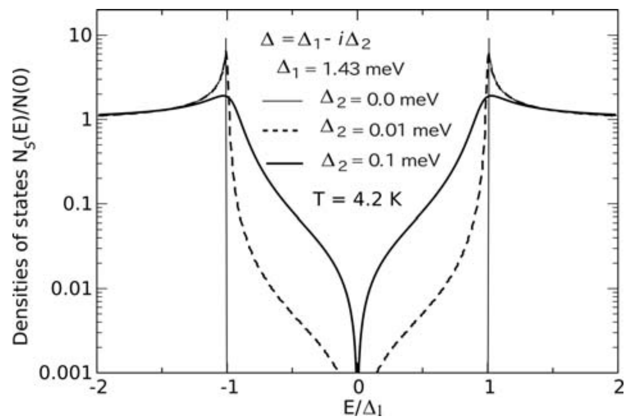

Fig. 1. Densities of states of Nb films having superconducting gap energy with (dotted and solid lines) and without (thin line) an imaginary part.

The measured dc I-V curve of a Nb/Al·AlO_x/Nb SIS junction at 4.2 K is plotted by open circles in linear (upper panel) and logarithmic (lower panel) scales in Fig.2. Solid lines in Fig.2 represent calculated quasiparticle tunneling current as a function of voltage. The measured dc *I-V* characteristics is quite consistent with calculated one by taking into account the imaginary part of gap energy Δ_2. It is also noted that amplitude of calculated quasiparticle tunneling current at a subgap voltage quantitatively agrees well with the measured one as shown in the lower panel of Fig. 2. This indicates that the measured subgap current is nearly identical to the theoretical lower limit of quasiparticle tunneling current in magnitude and indicates

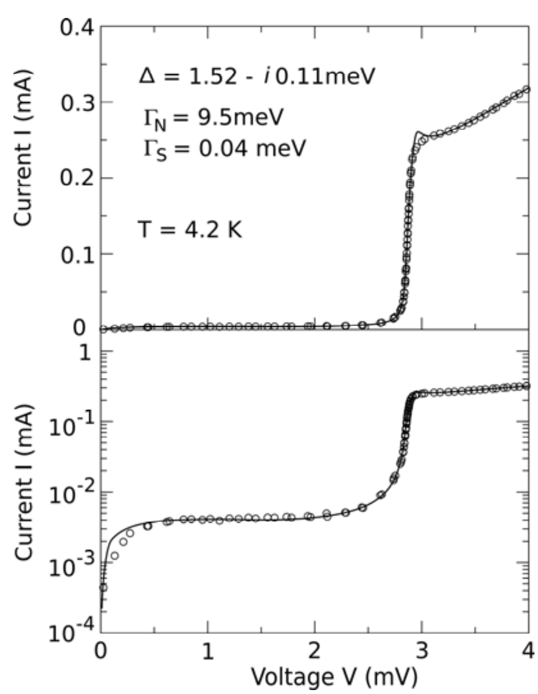

Fig. 2. Measured (open circles) and Calculated (solid lines) dc *I-V* curves of a Nb/Al·AlO_x/Nb SIS junction plotted in linear (upper panel) and logarithmic (lower panel) scale.

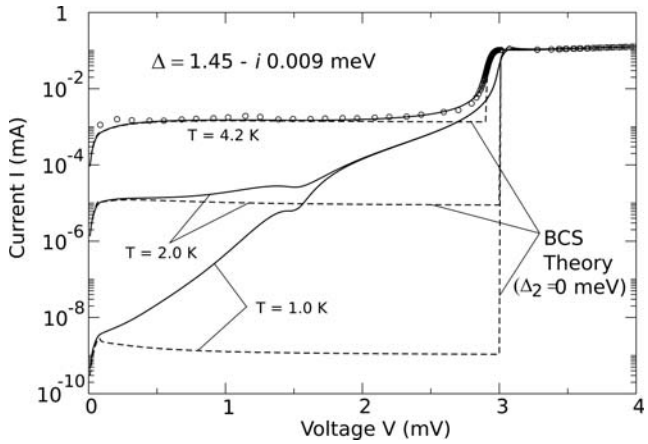

Fig. 3. Calculated dc I-V curves of a Nb/Al·AlO_x/Nb SIS junction at temperatures of 4.2, 2.0 and 1.0 K assuming that a Nb film has a complex superconducting gap energy. An imaginary part of the complex gap energy Δ_2 used in the calculation is determined from the fitting to the measured dc I-V curve at 4.2 K shown by open circles. dc I-V curves calculated by the conventional BCS theory at the respective temperatures are also shown by dotted lines.

that an almost ideal tunneling barrier is achieved in the present SIS junction.

B. Quasiparticle current at low temperatures

It has been shown by Dynes et al. that the imaginary part of the gap energy Δ_2 in strong-coupling superconductors is independent on temperature below $T_C/2$, where T_C is a superconducting transition temperature [8]. According to their result, the imaginary part of the gap energy Δ_2 of Nb can be assumed to be constant below 4.2 K and it would be possible to predict the quasiparticle tunneling current of a Nb/Al·AlO_x/Nb SIS junction at low temperatures using eq. (1) with the imaginary part of gap energy Δ_2 of Nb obtained from the measured I-V curve at 4.2 K, while temperature dependence of the real part Δ_1 is assumed to be given by the BCS gap equation [7].

In Fig.3 calculated dc I-V curves at temperatures of 4.2, 2.0 and 1.0 K are shown together with those calculated by the BCS theory without any imaginary part of gap energy. It is clear that the subgap current at a temperature below 2.0 K is strongly dependent on voltage unlike those predicted by the BCS theory. This is because there is tunneling current flowing through quasiparticle states inside the energy gap in addition to the thermally-excited quasipaticle current. At high temperatures (> 2 K) the subgap current of the SIS junction is dominated by thermally-excited quasiparticle tunneling current, while it is dominated by the tunneling current through quasiparticle states inside the energy gap at low temperatures. Because the thermally-excited quasiparticle tunneling current reduces exponentially as temperature goes down, while the tunneling current through the number of quasiparticle states inside the energy gap is approximately independent on temperature. It is another remarkable feature that the subgap current at a voltage

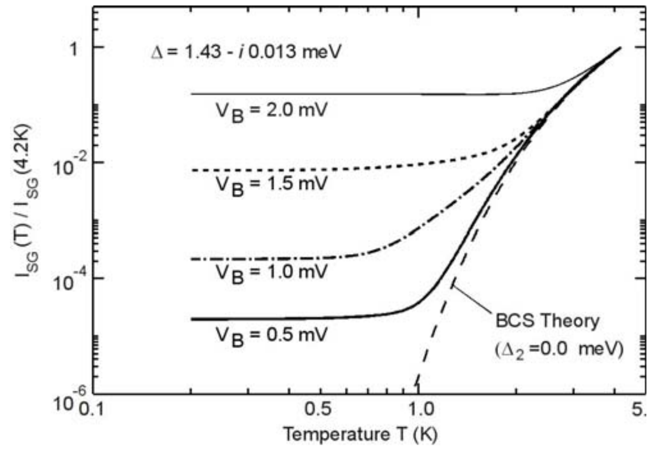

Fig. 4. Calculated subgap current of a Nb/Al·AlO_x/Nb SIS junction at bias voltages of 2.0, 1.5, 1.0 and 0.5 mV as function of temperature assuming that Nb film has complex superconducting gap energy. The dashed line represents a sugap current calculated by the conventional BCS theory as a function of temperature.

above $V = \Delta/e$ does not decrease as temperature goes down below ~2 K and seems to be saturated. Such a saturation of subgap current at low temperatures has been frequently found in SIS junctions [2, 4, 5]. In Fig.4 calculated subgap current at bias voltages of 0.5, 1.0, 1.5 and 2.0 mV are plotted as a function of temperature. It is clearly shown that any subgap current shows a saturation in magnitude at low temperatures, which is qualitatively consistent with the experimental results previously reported [2, 4]. It has been left unresolved why such a saturation of subgap current appears at low temperatures for a long time. Now it is clear that the saturation of subgap current occurs when the tunneling current through the quasiparticle states inside the energy gap dominates over the thermally-excited quasiparticle current at low temperatures.
The amplitude of the tunneling current through quasiparticle states inside the energy gap must be dependent on the magnitude of the imaginary part of the gap energy Δ_2, because

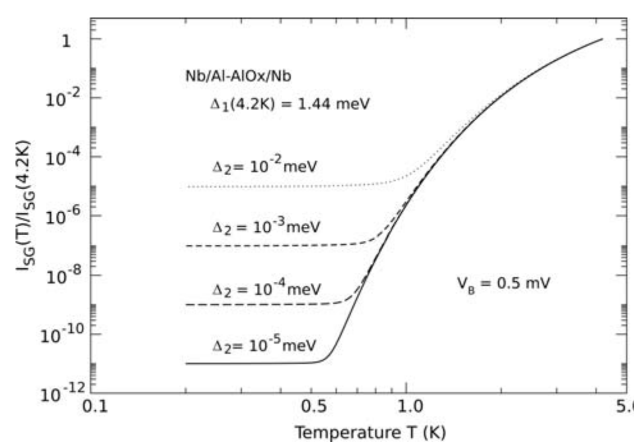

Fig. 5. Calculated subgap current of a Nb/Al·AlO_x/Nb SIS junction at a bias voltage of 0.5 mV as a function of temperature for different magnitudes of imaginary parts of gap energy Δ_2. Each subgap current is normalized to that at 4.2 K. The dashed line represents a sugap current normalized to that at 4.2 K predicted by the conventional BCS theory as a function of temperature.

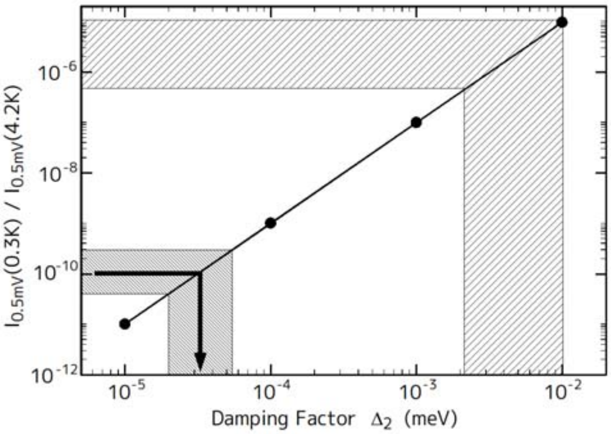

Fig. 6. Calculated subgap current of a Nb/Al·AlO_x/Nb SIS junction at T=0.3 K as a function of magnitudes of the imaginary part of gap energy Δ_2 at 4.2 K. Each subgap current is normalized to that at 4.2 K.

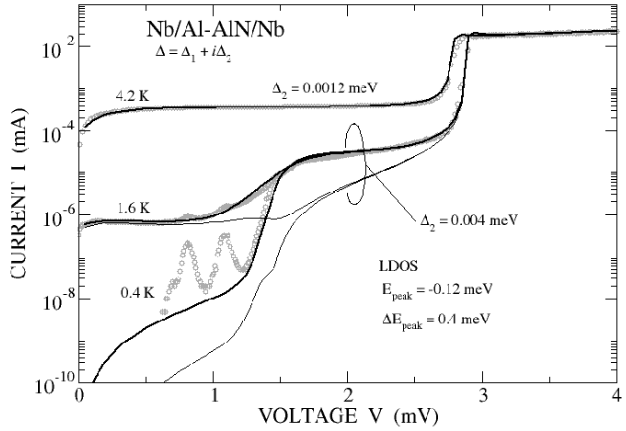

Fig. 7. Comparison between the calculated and measured I-V characteristics of a Nb/Al·AlN/Nb SIS junction at T=4.2, 1.6 and 0.4 K. Open circles represent measured data. Thin lines represent calculated ones using (3) by taking the imaginary part of the energy gap Δ_2. Thick solid lines represent calculated ones using (3) by taking the localized density of states (LDOS) in addition to the imaginary part of the energy gap Δ_2.

the number of quasiparticle states inside the energy gap strongly depends on the magnitude of Δ_2. In Fig.5 are plotted the calculated temperature dependence of the subgap current at a bias voltage V=0.5 mV for SIS junctions with different magnitudes of the imaginary part of the gap energy Δ_2 from 10^{-5} to 10^{-2} meV. Each subgap current is normalized to that at 4.2 K in order to clearly show the rate of change against the temperature. It is demonstrated that the saturation level of subgap current at 0.5 mV decreases as the magnitude of the imaginary part of the gap energy Δ_2 is reduced as expected.

Nb/Al·AlO$_x$/Nb .

In Fig. 6 calculated subgap current at 0.5 mV and 0.3 K is plotted as a function of the magnitude of Δ_2. The subgap current at 0.5 mV, which is sufficiently saturated, the upper right shaded region shows typical magnitude of Δ_2 of Nb-based SIS junctions reported up to date. The corresponding subgap current in this region is consistent with those ever achieved [2,5]. The lower left shaded region with thick arrow shows the our target value of the subgap current of SIS junctions for the background-limited sensitivity detectors. This indicates that is possible to reduce the saturated subgap current by 3 to 4 orders of magnitude, if the magnitude of Δ_2 can be achieved by approximately 2 orders of magnitude smaller than those of present Nb films.

III. COMPARISON TO EXPERIMENTAL DATA

We have made a high quality Nb/Al·AlN/Nb SIS junction with a quality factor at 4.2 K, where and are a subgap resistance at 2 mV and normal resistance of the SIS junction [11]. Measured dc I-V curves of the SIS junction at 4.2, 1.6 and 0.4 K are shown in Fig. 7 by open circles. Thin lines represent calculated ones using (3) by taking the imaginary part of the energy gap Δ_2 into account. The calculated dc I-V curve for 4.2 K shows a very good agreement with the measured one, whereas discrepancies between the calculated and measured ones at low temperatures are found. Especially, although there appears a remarkable structure just below 1.4 mV or a half gap voltage $V = \Delta_1 / e$ on the measured I-V curves at 1.6 and 0.4 K, no such a large structure is found on the calculated ones. It is also noted that the calculated subgap current below 1 mV at 0.4 K is an order of magnitude greater than measured one, whereas above 1.6 K the calculated subgap current below 1 mV is quite consistent with measured values. These discrepancies indicate that there are additional quasiparticle states inside the energy gap in addition to those induced by the imaginary part of the energy gap. In order to explain the measured dc I-V curves at low temperatures, we introduced a localized density of states (LDOS) inside the energy gap in addition to those induced by the imaginary part of the energy gap. The solid lines in Fig. 7 correspond to the calculated dc I-V curves including LDOS in addition to those induced by the imaginary part of the energy gap. The density of states used in the calculation is shown by the solid line in Fig. 8. The LDOS, shown by the broken line in Fig. 8, is assumed to have a Lorentzian-type energy dependence which is independent on temperature. It is found that calculated dc I-V curves agree well with measured ones.

Similar comparison between the calculated and measured dc I-V curves has been made for an Al/AlO$_x$/Al SIS junction reported by Teufel *et al* . [12]. In Fig. 9 calculated dc I-V curves with and without LDOS are shown by the solid and thin lines, respectively. The density of states used in the calculation is shown in Fig.10. It is noted here that there is a LDOS peak just below $E = 0$ or the Fermi level as in the case of Nb-based junction described above.

IV. DISCUSSION

We have shown that dc I-V characteristics of SIS junctions can be well described by the quasiparticle tunneling theory assuming that there are finite numbers of quasiparticle states inside the energy gap. We think that those quasiparticle states

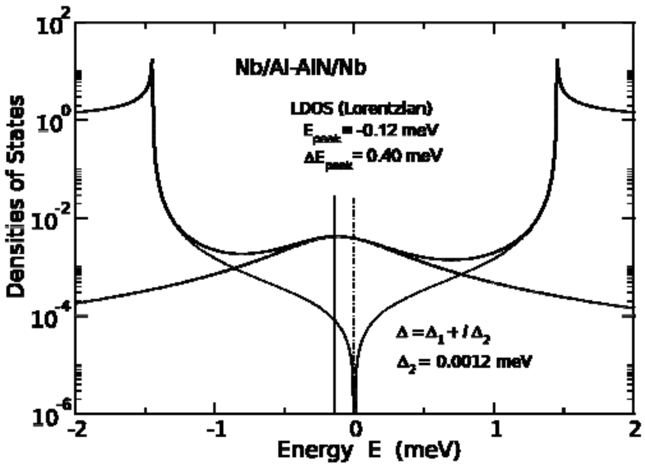

Fig. 8. Thick solid line represents the density of states which gives the best fit to the measured I-V curves of Nb/Al-AlN/Nb SIS junctions shown in Fig. 7. This density of states is decomposed into two parts; one is the quasiparticle density of states caused by the imaginary part of the energy gap (thin lines) and the other is a localized density of states (LDOS) inside the energy gap (broken line)..

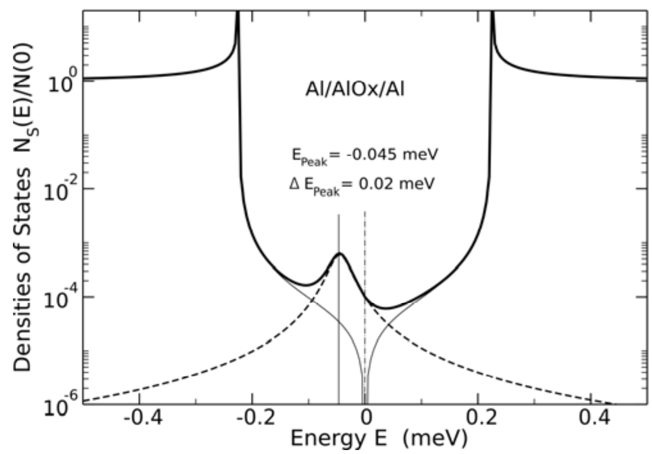

Fig. 10. Thick solid line represents the density of states which gives the best fit to the measured I-V curves of Al/AlOx/Al SIS junctions shown in Fig. 9. This density of states is decomposed into two parts; one is the quasiparticle density of states caused by the imaginary part of the energy gap (thin lines) and the other is a localized density of states (LDOS) inside the energy gap (broken line).

are decomposed into two major components; one is a localized quasiparticle states inside the energy gap and the other is attributed to the broadening of the density of states caused by the imaginary part of the energy gap. It is theoretically predicted that such a localized quasiparticle state or LDOS can be formed by the magnetic impurity scattering in a superconductor [13]. Although origin of the LDOS is not fully understood at present, it is thought to be quite important to reduce the numbers of the LDOS in order to achieve extremely small subgap current at low temperatures.

The imaginary part of the gap energy was theoretically taken into account to explain Q-values of superconducting cavities [14], damping of the Riedel singularity [15, 16] and broadening of a current step at gap voltage in I-V characteristics of SIS junction [8]. However, little attention has been paid to the fact

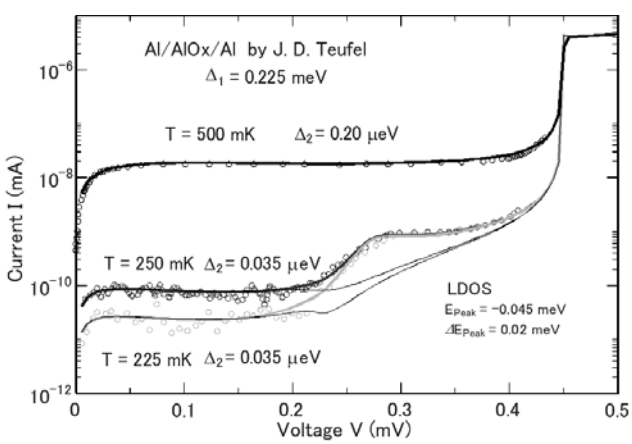

Fig. 9. Comparison between the calculated and measured I-V characteristics of a Al/AlOx/Al SIS junction at T=500, 250 and 225 mK. Open circles represent measured data. Broken lines represent calculated ones using (4) by taking the imaginary part of the energy gap Δ_2. Thick solid lines represent calculated ones using (4) by taking the localized density of states (LDOS) in addition to the imaginary part of the energy gap Δ_2.

that a small number of quasiparticle states are caused at energy deep inside the gap. Subgap current of SIS junctions at low temperatures are dominated by the tunneling current through those quasiparticle states, while a number of thermally-excited quasiparticles are extremely small at the temperature.

The imaginary part of the gap energy Δ_2 can be assumed to be inversely proportional to residual resistance ratio, *RRR*=*R*(300K)/*R*(10K), which represents quality of a superconducting film, where *R*(300K) and *R*(10K) are resistance of the superconducting film at 300 K and 10 K, respectively. The *RRR* of our Nb films is 4–5, whereas *RRR*>200 has been obtained in crystalline Nb films [17]. Thus, in principle, it is possible to reduce the magnitude of the imaginary part of energy gap of a Nb film approximately 2 orders of magnitude smaller than that of present Nb films. If such a high-quality Nb film would be available, extremely low subgap current of a SIS junction can be obtained at low temperature.

V. SUMMARY

We have demonstrated that the quasiparticle states inside the energy gap plays an important role in the behavior of subgap current of a SIS junction at low temperatures. It has been shown that the imaginary part of superconducting gap energy causes small but finite numbers of quasiparticle states even at an energy deep inside the gap and that localized quasiparticle states are induced inside the energy gap. Quasiparticle tunneling through those quasiparticle states occurs at subgap voltages and dominates over the thermally-excited quasiparticle tunneling at low temperatures. These results indicate that much effort must be made to improve the quality of electrode films of SIS junctions in addition to make a very good tunneling barrier, in order to invent a high-sensitivity SIS photon detector with an extremely low subgap current at low temperatures.

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