Rigorous Analysis and Design of Finline Tapers for High Performance Millimetre and Submillimetre Detectors

Chris North, Ghassan Yassin and Paul Grimes
Astrophysics, Oxford University, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH
Email: cen@astro.ox.ac.uk

Abstract—**Antipodal finline tapers have demonstrated excellent performance in conjuction with SIS mixers and have recently been used with TES detectors for the CMB polarisation instrument CLOVER. In this paper we present the computation of the finline parameters using the Transverse Resonance method and Spectral Domain Analysis and compare them with those obtained from Finite Element simulations. We also present a software package that can read an input file and then synthesise a minimum length taper for a requested return loss. The input file must contain the cutoff frequency as a function of the finline slot dimension which can be computed externally to the synthesis package.**

I. Introduction

Previous publications [1] have demonstrated the advantages of finline mixers at submillimetre wavelengths. An antipodal finline taper allows the mixer to be fed by a high-performance corrugated horn, easy fabrication of the mixer chip and can be fully integrated with other planar circuit technology.

Experimental investigation of the mixers at frequencies between 230-700GHz has shown that finline mixers have high bandwidth, high optical performance (low sidelobes and cross polarisation) and low noise temperature. The larger substrate allows the integration of important planar circuits for sideband separation and balanced mixers [2]. Their ease of fabrication and high performance makes them ideal for use in large-format arrays.

Figure 1 shows some schematics of finlines in waveguides, along with dimensions and axis conventions. The metallisation layer lies across the middle of a split-block waveguide in the form of two fins separated by 400nm of oxide and desposited on a 220μm thick substrate. Figure 2 shows a mask of a finline chip and the Electric fields at various points along the transition from waveguide to microstrip.

At THz frequencies, rigorous analysis of finline tapers is complicated by the large variation in lateral dimensions and relatively large metallisation thickness. Previous analysis methods have divided the taper into three sections. When the fins are not overlapping, the taper was approximated to a unilateral taper and analysed with Transverse Resonance (TR) or Spectral Domain Analysis (SDA). For the second section, where the fins are overlapping, SDA was used, though this ignores the metallisation thickness and the substrate carrying the fins, which is only a valid assumption if the overlap is larger than the fin separation. The third section is microstrip, which has been fully analysed using Conformal Mapping [3]. The taper itself can be synthesised using using the Optimum Taper Method (OTM), which tapers cutoff frequencies to give a required return loss for a minimum taper length.

In this paper we present and compare new procedures for analysing the the performance of finline tapers which can be applied along the whole taper. The Optimum Taper Method is still used, but the cutoff frequencies can be supplied

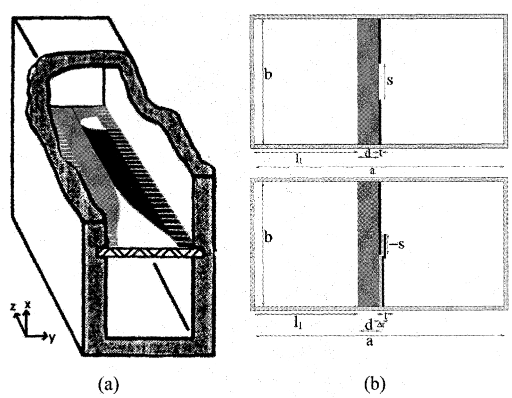


Fig. 1. (a) Schematic of a finline chip in a waveguide. (b) *Top*: Unilateral finline with dimensions; *Bottom*: Antipodal finline with dimensions.

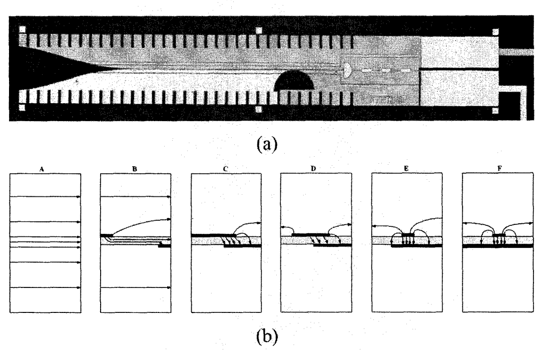


Fig. 2. (a) Mask of an SIS mixer chip utilising an antipodal taper at 230GHz. The metallisation layers are visible, as well as the semicirculare structure which converts overlapping fins into a microstrip. (b) The Electric field lines at various points along the transition: empty waveguide (A), unilateral finline (B), antipodal finline (C), the start (D) and end (E) segments of the semicircular transition to microstrip (F).

from finite-element simulation software or any other rigorous simulation package. We have developed a software package, in Fortran90, which reads an array of electrical parameters and outputs a taper profile in numeric and graphical formats. We descibe the current analysis methods in §II and the optimum taper method in §III. §IV outlines the proposed computational method, while §V discusses the results of the different methods.

II. Current Analysis Methods

Previous analyses [1] have used Transverse Resonance (TR) and Spectral Domain Analysis (SDA) to calculate cutoff frequencies and propagation constants. While TR can be accurate for thick metallisation, but doesn't take dispersion into account, SDA gives full-wave computation including dispersion, but assumes infinitely thin metallisation (see §II-B). Consequently, the analysis is least rigorous when the fins overlap slightly, since both metallisation thickness and dispersion are important. Moreover, there is always a difficulty in matching the solutions produced by two separate methods.

A. Transverse Resonance

Transverse Resonance calculates the cutoff wave number k_c by finding the first zero of the transcendental equation [4]:

$$-\cot(k_c l_1) - \cot[k_c(l_1 + d)] + \frac{b}{s}\tan(k_c t) + \frac{B}{Y} = 0 \quad (1)$$

where the dimensions b, d, t, l_1 are those defined in Fig. 1. The term B/Y is the normalised susceptance of the gap, calulated using the equivalent circuit of the finline from:

$$\frac{B}{Y} = \frac{b}{\pi} k_c [2P_1 + \epsilon(P_s + P_b)] \quad (2)$$

$$P_1 = \ln[\operatorname{cosec}(\frac{\pi}{2}\frac{s}{b})]$$

$$P_s = \frac{d}{s} \cdot \arctan(\frac{s}{d} + \ln\sqrt{1 + (d/s)^2})$$

$$P_b = \frac{d}{b} \cdot \arctan(\frac{b}{d} + \ln\sqrt{1 + (d/b)^2})$$

The propagation constant (β) can be related to the cutoff frequency by the following relations:

$$\beta = k_0\sqrt{\varepsilon_{eq}}\sqrt{1 - (f_c/f_0)^2} \quad (3)$$

where ε_{eq} is an equivalent dielectric constant given by:

$$\varepsilon_{eq} = (k_c/k_{c0})^2 \quad (4)$$

where k_{c0} is the cutoff for wave number $\varepsilon_r = 1$, so it satisfies (1) and (2) with that condition.

B. Spectral Domain Analysis

Spectral Domain Analysis (SDA) is based on finding a matrix equation which relates the Fourier transforms of the current and fields by the dyadic Green's function. The advantage of this method is that it converts the differential equations given by Maxwell's equations into a homogeneous set of algebraic equations. By setting the determinant of the coefficients to zero the propagation constant can be found, while finding the coefficients yields the characteristic impedance. The relationship between the Fourier transforms of the currents and fields may be written as:

$$\begin{pmatrix} \tilde{J}_y(\hat{k}_n) \\ \tilde{J}_z(\hat{k}_n) \end{pmatrix} = \begin{pmatrix} G_{yy}(\hat{k}_n,\beta) G_{yz}(\hat{k}_n,\beta) \\ G_{zy}(\hat{k}_n,\beta) G_{zz}(\hat{k}_n,\beta) \end{pmatrix} \begin{pmatrix} \tilde{E}_y(\hat{k}_n) \\ \tilde{E}_z(\hat{k}_n) \end{pmatrix} \quad (5)$$

where $\hat{k}_n = n\pi/b$ is the Fourier parameter of the y-coordinate.

The electric field is expanded in terms of basis functions such that

$$\tilde{E}_y(\hat{k}_n) = \sum_{i=1}^{M} a_i \tilde{\phi}_i(\hat{k}_n) \quad (6)$$

$$\tilde{E}_z(\hat{k}_n) = \sum_{j=1}^{N} b_j \tilde{\psi}_j(\hat{k}_n) \quad (7)$$

where $\tilde{\phi}_i(\hat{k}_n)$ and $\tilde{\psi}_j(\hat{k}_n)$ are Fourier transforms of the basis functions $\phi_i(y)$ and $\psi_j(z)$.

By substituting (6) and (7) into (5), and using Galerkin's method we obtain an $(M+N)\times(M+N)$ set of homogeneous linear equations with unknowns a_i and b_i. By solving the resulting equations we obtain the propagation constant β.

The choice of basis functions is important when using SDA, both in terms of the form and the number of terms M and N. In [5] rectangular and sinusoidal basis functions are discussed, while in [1] it is found that Legenre polynomials for $\phi(y)$ and sinusoidal functions for $\psi(z)$ give accurate results for both unilateral and antipodal finlines.

As mentioned previously, in our solution we did not take into account of the substrate, although it could easily be incorportated in the formulae.

III. The Optimum Taper Method

A. Parameters

The parameters required to perform the analysis are:

1) The frequency to be analysed (f_0); in this case 90GHz.
2) The required cutoff frequency at the start and end of the taper ($f_c(0)$ and $f_c(l)$); in this case around 53GHz and 23GHz respectively.
3) The return loss required (R_{max}); in this case -30dB

B. The Method

Following [4], the reflection coefficient of a taper of length l is given by:

$$R(\beta) = -\int_0^l \kappa^{-+}(z') \cdot \exp\left\{\int_0^{z'} -2\beta(f,z)dz\right\}dz' \quad (8)$$

If we only consider a given frequency f_0 (at which we want to analyse the taper) then by making the approximation that the exponent of (8) can be approximated to a product of a frequency-dependent (which is normailsed to 1 at $f = f_0$ and a z-dependent term, [4] show that (8) becomes

$$R(\eta) = \int_0^{2\theta} C \cdot K(\xi) e^{-i\eta\xi} d\xi \quad (9)$$

where ξ is a z-dependent phase-space variable given by [4]

$$\int_0^z 2\beta(f_0, z')dz' \simeq \xi(z) \quad (10)$$

and C is a normalising constant defined such that

$$\int_0^{2\theta} K(\xi)d\xi = 1 \quad (11)$$

The value of C is then given by:

$$C = \ln\left[\frac{f_c(0)}{f_c(l)} \cdot \left(\frac{1-(f_c(l)/f_0)^2}{1-(f_c(0)/f_0)^2}\right)^{\frac{1}{4}}\right] \quad (12)$$

where $f_c(z)$ is the z-dependent cutoff frequency.

The definition of θ is:

$$\theta = \operatorname{arccosh}(C/R_{max}) \quad (13)$$

where R_{max} is the maximum permissible return loss (one of the parameters mentioned above). The value of ξ is then in the range $0 \le \xi \le 2\theta$,with $\xi(0) = 0$ and $\xi(l) = 2\theta$.

C. Calculating the Cutoff Frequncies

The coupling distribution $K(\xi)$ is chosen to make sure the reflection coeffient is below R_{max} for all frequencies above f_0. Due to the normalisation of $K(\xi)$ given by (11) and (12), the integral

$$I(\xi) = \int_0^{\xi} K(\xi')d\xi' \quad (14)$$

has boundary conditions $I(0) = 0$ and $I(2\theta) = 1$. The cutoff frequency for a given value of ξ is given by:

$$f_c(\xi) = f_c(0) \cdot \left(F + \sqrt{F^2 + (1-2F)\exp(4CI(\xi))}\right) \quad (15)$$

where $F = \frac{1}{2}(f_c(0)/f_0)^2$.

IV. Proposed Method

A. Finite-Element Simulation

Finite-element simulation software such as Ansoft HFSS[1] can be used to accurately simulate sections of transmisison lines to calculate the electrical parameters. While the software returns the propagation constant acurately and straightforwardly, the cutoff frequencies must be obtained by scanning a range of frequencies around the expected value. In our calculation we made use of the fact that the cutoff frequency is usually accompanied by a sharp change in other parameters, such as the impedance or S-parameters.

[1] http://www.ansoft.com

B. Computational Procedure

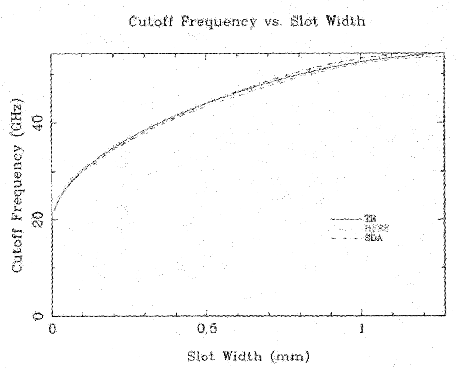
The computational procedure is as follows:

1) Select a return loss (R_{max}) and target frequency (f_0).
2) Determine initial and final cutoff frequencies ($f_c(0)$ and $f_c(l)$) from simulations.
3) Run simulations on a range of slot widths to obtain cutoff frequencies and propagation contants.
4) Input waveguide parameters:
   a) Waveguide width and height
   b) Substrate thickness and dielectric constant
   c) Metallisation thickness
5) Run the code, which does the following:
   a) Determine normalisation constant C from (12) and θ using (13).
   b) Choose a step size for ξ based on θ and the number of steps along the taper (e.g. 500)
   c) Set $z = \xi = 0$ and the inital slotwidth ($s(0)$).
   d) Step through ξ by one step and calculate the cutoff frequency ($f_c(\xi)$) using (15).
   e) Interpolate within the array of slot widths and cutoff frequencies from simulations to determine the new slot width ($s(\xi)$) and propagation contant ($\beta(\xi)$).
   f) Using the relation in (10), calulate the step size in z, from $\Delta z = \Delta\xi/2\beta(\xi)$.
   g) Repeat 5d) to 5f) until $\xi = 2\theta$.

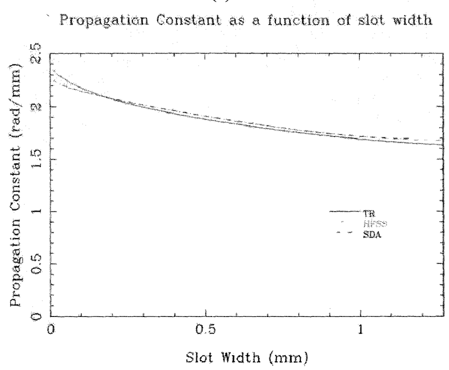
V. Results and Discussion

Figure 3 shows the results of the three different analysis methods combined with the OTM to synthesis the taper. The results are at 90GHz for a taper in a WR10 waveguide (a = 2.54mm, b = 1.27mm), with a final slot width of around 0.01mm and a maximum return loss of -30dB. The substrate was 220μm thick and had a dielectric constant of $\varepsilon_r = 2.2$. The taper produced is 3.8mm long, or 1.14λ (see Fig. 3(d)). While all the results are for unilateral finlines,the process simply generates a sequence of slot widths based on the cutoff frequencies returned by the OTM, and so would work for antipodal finlines. Finite-element simulations return accurate results for any slot width and will therefore be used as a benchmark. We are in the process of using HFSS to generate the parameter of the antipodal section.

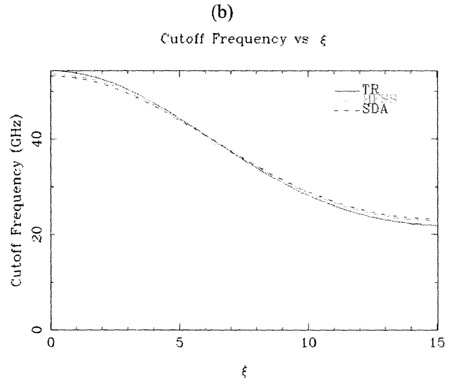
From Figure 3, it can be seen that the computations done using TR or SDA compare very well with the exact results computed using HFSS. This is because in the slotline geometry the effect of metallisation thickness and dispersion is not very large. It is interesting to notice that from 3(a) that there is a small deviation between HFSS and TR at large slot widths, which is to be expected since the TE approximation of a slotline fails when the slot is large. We also notice that there is a deviation between HFSS and SDA at large slot widths. This is, however, the result of the fact that the number of basis functions in (6) and (7) used in the computation was too small to give accurate results. In general, however, SDA and HFSS should agree very well at large slot widths.



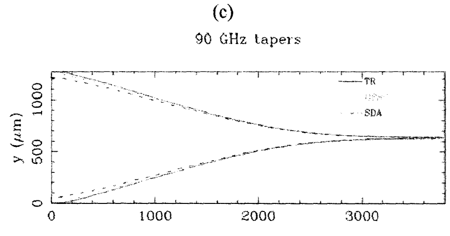
(a)



(b)



(c)



(d)

Fig. 3. Comparisons of the results of the three methods discussed in the text. Red solid: Transverse Resonance; Blue dot-dashed: Spectral Domain Analysis; Green dashed: HFSS Simulations. (a) Cutoff frequency vs. slot width, (b) Propagation constant vs.slot width, (c) Propagation constant vs cutoff frequency, (d) The taper produced by the three methods.

The results shown in Figure 3(c) show the cutoff frequency as a function of the dimensionless phase variable, ξ (see §II-A). This is sensitive to the initial and final cutoff frequencies ($f_c(z=0;\xi=0)$ and $f_c(z=l;\xi=2\theta)$ respectively), which accounts for the small deviations.

The tapers produced by the OTM are shown in Figure 3(d). The deviation of SDA from HFSS is discussed above. The TR taper also shows deviation at large slot widths, as discussed above, though it is within current fabrication tolerances of 5–10%.

VI. Conclusion

We have investigated the synthesis of finline tapers using TR and SDA and compared them with HFSS simulations. Our results show that the two methods can be used accurately even when small slot widths must be reached. It should be noted that the SDA results can be further improved by taking the metallisation into account using the Wheeler correction [6].

So far we have designed antipodal finline tapers using the SDA, therby neglecting metalisation thickness and the existance of the substrate carrying the fins. This approximation may not be accurate when the fins overlap is comparable to the thickness of the oxide that separate the fins. It can therefore be largly improved by including the substrate in the Dyadic Green function and the metalisation thickness using the Wheeler correction. Alternatively, accurate synthesis of the finline taper can be obtained by using the the taper synthesis code presented in this paper, in conjunction with an array containing the cutoff frequency as a function of the slot dimension.

References

[1] G. Yassin, S. Withington, M. Buffey, K. Jacobs, and S. Wulff, "A 350 ghz sis antipodal finline mixer," *IEEE Trans. Microwave Theory Tech.*, vol. 48, pp. 662–269, Apr. 2000.

[2] P. Grimes, G. Yassin, K. Jacobs, and S. Withington, "A 700 ghz single chip balanced sis mixer," in *16th International Symposium on Space Terahertz Technology*, Goeteborg, Sweden, May 2005, pp. 46–51.

[3] G. Yassin and S. Withington, "Electromagnetic models for superconducting millimetre-wave and sub-millimetre-wave microstrip transmission lines," *J. Phys. D: Appl. Phys.*, vol. 28, pp. 1983–1991, Apr. 1995.

[4] C. Schieblich, J. K. Piotrowski, and J. Hinken, "Synthesis of optimum finline tapers using dispersion formulas for arbitrary slot widths and locations," *IEEE Trans. Microwave Theory Tech.*, vol. 32, no. 12, pp. 1638–1644, Dec. 1984.

[5] L.-P. Schmidt and T. Itoh, "Spectral domain analysis of dominant and higher order modes in fin-lines," *IEEE Trans. Microwave Theory Tech.*, pp. 981–985, Sept. 1980.

[6] D. Mirchekar-Syahkal and J. B. Davies, "An accurate, unified solution to various finline structures of phase constant, characteristic impendance and attenuation," *IEEE Trans. Microwave Theory Tech.*, vol. 30, pp. 1854–1861, 1982.